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Optimal close-to-home biases in asset allocation

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ABSTRACT

This article studies optimal portfolio decisions with (long-term) liabilities for small open economy based investors, including the optimality of currency hedging (Walker (2008a). Chile is the home country of the representative investor, but results are likely to hold more generally. The problem is set up as in Sharpe and Tint (1990) and Hoevenaars, Molenaar, Schotman and Steenkamp (2007). Hedging the liabilities and the consumption currency may imply optimal close-to-home biases, defined as overweighting asset classes which are highly correlated with local ones. The implementation challenges include: developing a methodology to estimate expected returns in local (real) currency; estimating the covariance matrix allowing for serial and crossed-serial correlations; and checking the results' robustness using a resampling method. The findings are: (i) portfolios always have optimal close-to-home biases, beyond the investment in local fixed income to hedge liabilities; (ii) currency hedging reduces investment in close-to-home asset classes, (iii) but has ambiguous effects on welfare — detected with the resampling method; (iv) currency hedged long-term US bonds are useful for hedging local interest rate risk; and (v) liabilities give access to high risk-return portfolios, not affecting otherwise the overall shape of the efficient regions. This article can be useful to investors based on small open economies, including pension funds, insurance companies, sovereign wealth funds and Central Banks.

1. Introduction

Institutional investors in emerging markets have been accumulating significant investable wealth, via pension funds, central bank reserves and especially sovereign wealth funds. According to *The Economist* (January 17, 2008), Sovereign Wealth Funds accumulate $2.9 trillion. Mandatory pension funds of countries (associated to FIAP) which have reformed their pension systems as Chile did in 1981 have accumulated about $400 billion only in mandatory savings as of June 2008 (FIAP, 2009). Thus, the issue of optimal asset allocation in this perspective has become increasingly important. Applied and even theoretical portfolio problems usually consider at least implicitly the perspective of developed market-based investors. For example, Campbell and Viceira (2001) develop a long-term asset allocation model assuming that interest rates follow a process as in Vasicek (1977), where future expected (local) short rates partly determine long-term interest rates. In contrast, in the case of small open economies, global markets should determine local interest rates (and therefore the present value of liabilities), so this portfolio problem may take a different form. In the case of small and open economies certain state variables (global and exogenous) should be more important, which implies that not just the implementation of asset allocation problem will be different. Still, setting up the problem properly and estimating the parameters are not trivial issues in this case.

Regarding the effect of liabilities, the benchmark in order to analyze asset allocation results is the Modigliani-Miller theorem as *a*. With non-binding short-sale constraints, leverage should not affect the investment opportunity set (or the efficient frontier) faced by an investor, because she can allocate the borrowed money to a dedicated portfolio, which (perfectly) hedges the investor's net worth from changes in the value of liabilities, and invest the rest as in the asset-only case.

For comparison purposes, the reference is a global (developed market-based) investor. Assuming that this investor finds long-term passive combinations of global equity and global fixed income to be (mean-variance) efficient, given her investment horizon (which should match the duration and currency of the bond portfolio), then an expected return–Beta relationship exists, which allows determining the required returns of all the asset classes considered in the asset allocation problem (local and global, assuming integrated markets). The global equity or market risk premium comes from the results of Dimson et al. (2006). A contribution of this paper is to develop a coherent methodology to obtain expected returns in local currency.

Our central hypothesis is that optimal portfolios have close-to-home biases, meaning that they will overweight the asset classes which are *highly* correlated with local ones. Local currency denominated fixed income has no close substitutes internationally, so almost by definition this particular home bias should increase with leverage

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in local currency. However, a bias toward local or close-to-home equity may exist, since they are better hedges against changes in the relevant state variables.

The article also studies currency hedging. Given the result in Walker (2008a), indicating that exchange rate movements in emerging markets act as a natural hedges for global investments (emerging market currencies tend to depreciate when global stock markets fall), the hypothesis is that currency hedging does not necessarily increase portfolio efficiency but may partly substitute the investment in closer-to-home asset classes.

The problem is set up in a way that is similar to Sharpe and Tint (1990) and Hoevenaars et al. (2007), considering different levels of long-term liabilities expressed in local (inflation-adjusted) currency. This study measures all returns in *local* (inflation-adjusted) currency, assumes a one-year investment horizon despite long-term liabilities, and considers the possibility that asset classes may be auto- and cross-correlated using a vector auto-regressive VAR(1) formulation to obtain the one-year covariance matrix, following Campbell and Viceira (2005a,b). Since autocorrelation is frequently significant in emerging markets (see Harvey (1995) or Bekaert and Harvey (1997)), this adjustment seems necessary.

A resampling methodology allows checking the results' robustness in the spirit of Michaud (1989, 1998), which is new in its application to VAR processes, as explained in Walker (2008b).

Finally, this article implements the above in the spirit of an annuity insurance company based on an emerging market (Chile), having long-term fixed rate liabilities expressed in inflation-adjusted local currency (see Walker (2009) for a description of this industry), and also presents the results for the no-liability case. Chile has an exceptionally long history of inflation-protected bonds, especially in the context of an emerging market, which makes this case study interesting. This information allows us to use a relatively long series of quarterly data for the period 1990:Q1 2008:Q1. The eligible asset classes considered are global and local bonds and stocks, in addition to emerging market equity. The implementation assumes that long-term liabilities are equivalent to shorting long-term local bonds.

This article finds the following. Leverage gives access to high risk-return portfolios otherwise unavailable, but does not affect the overall shape of the efficient regions. For all leverage levels close to home biases are indeed optimal. Beyond local fixed income, emerging market and local equity represent about half of the total investment in equity. These asset classes have hedging properties regarding currency and interest rate risk. Long-term US currency hedged bonds also have significant interest rate hedging properties.

Currency hedging reduces optimal investment in close to home asset classes but a significant bias remains, having ambiguous effects on expected utility due to larger sampling errors in the portfolio weights. The resampling methodology allows the detection of this effect. Without leverage, hedging increases expected utility for high risk tolerance, but reduces welfare in the other cases. With leverage, in general, currency hedging reduces expected utility.

In the rest of the paper, Section 2 presents the background and methodology; Section 3 presents the data and the inputs for the asset allocation problems; Section 4 presents the results; and Section 5 concludes the paper.

2. Background and methodology

2.1. The portfolio problem

For a given investment horizon (τ), the investor with long-term liabilities chooses a portfolio of financial instruments which maximizes the expected utility derived from her net worth:

$$U_t = E_t \frac{1}{1-\gamma}(W_{t+\tau})^{1-\gamma}. \tag{1}$$

$W_{t+\tau} = A_{t+\tau} - L_{t+\tau}$ represents the investor's net worth or equity, defined as the difference between the value of assets and liabilities in the investment horizon (τ), and γ is the relative risk aversion parameter. Here τ is four quarters. The problem is set up in a similar but not identical way to Hoevenaars et al. (2007). Their argument in the utility function is the funding ratio $A_{t+\tau}/L_{t+\tau}$. Defining preferences over the funding ratio ignores the present value of the future contributions that will be necessary in order to meet obligations. Defining expected utility in terms of the investor's final net worth seems intuitively sounder. With a fixed rate annuity insurance company in mind, for implementation purposes the asset–liability is held constant, e.g. the company issues new debt of similar characteristics (e.g., duration and convexity) to match exactly liability payments and asset growth. Similarly, the company reinvests all cash flows in the firm.

This study assumes log-normal returns with an opportunity set that may change when passing from a one-quarter to a one-year horizon, following the methodology of Campbell and Viceira (2005a). (Results in Table 2 discussed below generally do not allow us to reject the log-normality assumption in the data) Also, as Hoevenaars et al. (2007), portfolio proportions remain constant during the investment horizon.

Maximizing Eq. (1) is equivalent to maximizing (see for example Campbell and Viceira (2002), p. 34):

$$V_t = E_t\left(\sum_{k=1}^{\tau} r_{Wt+k}\right) + \frac{1-\gamma}{2} var_t\left(\sum_{k=1}^{\tau} r_{Wt+k}\right) \tag{2}$$

$\sum_{k=1}^{\tau} r_{Wt+k}$ corresponds to the cumulative log-return on the investor's net worth or equity. Appendix A presents the corresponding expressions for the cumulative return's expected value and variance. The equations use Campbell and Viceira's (2002) log-linear approximation. Although this problem may have an analytic solution (see Walker (2008b)), short-sale constraints imply that the researcher has to solve the problem numerically after substituting in Eq. (2) the expressions derived from Appendix A. Notice that, in contrast with Hoevenaars et al. (2007), who define utility as a function of the funding ratio, the solution obtained here will depend on leverage, as expected.

The Modigliani-Miller theorem is an interesting reference point for the optimal asset allocation problem for an asset–liability investor with positive net worth. A previously unnoticed – but perhaps evident – fact is that in perfect and complete markets, with no short-sale constraints, leverage should not affect the investment opportunity set (or the efficient frontier) faced by investors. The investor can buy a dedicated portfolio, which perfectly hedges the investor's net worth from changes in the value of liabilities with the borrowed money, and invest the rest as in the asset-only case. The article shows this result restating in this paper's context a similar result of Bazdarich (2006) (p. 64). Defining the surplus return as the return on assets minus the liability-to-assets ratio (L/A) times liability returns, the surplus optimal vector of portfolio proportions ($w_+(\mu_P)$) for a given level of expected returns (μ_P), is

$$w_+(\mu_P) = (1-L/A)w_M(\mu_P) + (L/A)w_L \tag{3}$$

where $w_M(\mu_P)$ is the vector of optimal proportions of the asset-only (Markowitz) case and w_L is the vector of weights of the portfolio assumed to perfectly replicate long-term liabilities. Using Eq. (3) to obtain the efficient frontier for a positive net worth investor ($W = A - L > 0$) yields exactly the same frontier as the asset-only case for the investor's net worth. This finding gives us a general benchmark to judge the asset allocation results.

2.2. Annualized covariance matrix

Significant autocorrelation and cross-serial correlation in returns exist, particularly in emerging markets (see for example Harvey (1995)

Table 1
Data and sources.

Series	Acronym	Description	Source
UF	UF	Inflation indexed unit of account, measured in CLP per UF	Central Bank of Chile
Variation in the USD/UF exchange rate	RUSD_UF	Log return of the exchange rate dollars per UF	GFD; __CLP_D
Chilean Treasury Bill returns	RCHL_TB	Chilean total short-term government bond returns measured in UF	GFD; Code: TRCHLBIM
Chilean Real Treasury Bond Index returns	RCHL_BND	Chilean total long-term government bond returns measured in UF. From 1986 to 1992, total return of holding a PDP10 or PRC10; 1993–2000 total return of holding a PRC20; 2001–2008, UF LVA long-term local government bond index	Estimated here and LVA Indices since 2001 Code: LVAXGU80
US Treasury Bills	RUS_02Y	Two-year US Treasury Bill returns measured in UF	GFD ; Code: TRUSG2M
US Treasury Bonds	R_US30Y	Thirty-year US Treasury Bond returns measured in UF	GFD; Code: TRUSG30M
GFD Global USD Total Return Government Bond Index	RGL_BND	Global bond index returns measured in UF	GFD; Code: _JPMGGUM
GFD Emerging Markets Return Index	REM_EQTY	Emerging market equity returns measured in UF	GFD; Code: TRGFDEM
GFD World Return Index	RW_EQTY	World equity returns measured in UF	GFD; Code: TRGFDEM

and Bekaert and Harvey (1997)). Thus, annualizing quarterly variances multiplying by four is probably wrong. Instead, following Campbell and Viceira (2005a,b), here excess quarterly returns follow a vector autoregressive VAR(1) process. This implementation considers four-quarter covariances, which will depend on the estimated VAR parameters as well as the covariance matrix of unexpected returns.

2.3. Expected returns

As explained, dollar returns in excess of a portfolio of global government bonds, jointly with the returns of the global bonds measured in US dollars, follow a VAR(1) process. The estimated VAR (1) parameters and the variances of unexpected returns allow estimating the unconditional or long-term covariance matrix. If y_t is the ($N\times 1$ column) vector consisting of the global bond returns and the other asset classes' excess returns, then

$$y_t = B_0 + B_1 y_{t-1} + u_t. \tag{4}$$

B_0 is an $N\times 1$ vector and B_1 an $N\times N$ matrix. The unconditional expected (excess) returns (μ_y) and covariance matrix of y (Σ_{yy}) come from the covariance of the error terms (Σ_{uu}) and from the coefficient matrices in Eq. (4) as:

$$\begin{aligned} \mu_y &= [I_N - B_1]^{-1} B_0 \\ vec(\Sigma_{yy}) &= [I_{N^2} - B_1 \otimes B_1]^{-1} vec(\Sigma_{uu}). \end{aligned} \tag{5}$$

The second and perhaps most important assumption used here, is that given their investment horizon, global investors consider a global government bond portfolio as riskless, and that the global equity portfolio is efficient in their perspective. A CAPM-like relationship exists between expected returns and Betas with respect to an efficient portfolio. (In this case, Betas are proportional to the covariances between returns in excess of global bonds and the corresponding excess returns of global equity). The authors estimated the Betas using the covariance matrix in Eq. (5). If the global equity premium over global bonds ($\mu_{GE}-\mu_{GB}$) is known, then for every asset class k the CAPM-like equation is:

$$\mu_k - \mu_{GB} = \beta_{kE}(\mu_{GE} - \mu_{GB}) \tag{6}$$

The question now is how to transform expected returns into local currency. For this purpose, knowing the expected return in local currency for any asset is enough because risk premia are unit-less. Assuming that the expected return for local long-term bonds in local currency is known (μ_L^*), by definition its risk premium with respect to global equity is $\beta_{LE}(\mu_{GE}-\mu_{GB})$. Thus, the premium of every asset class with respect to the local bond is:

$$\mu_k - \mu_L = (\beta_{kE} - \beta_{LE})(\mu_{GE} - \mu_{GB}). \tag{7}$$

Finally, Eq. (8) shows how to estimate the expected return of each asset class in local currency. Intuitively, the equation adds to the local reference interest rate the extra risk premium of the asset class.

$$\mu_k^* = \mu_L^* + (\beta_{kE} - \beta_{LE})(\mu_{GE} - \mu_{GB}). \tag{8}$$

Table 2
Descriptive Statistics. Sample 1990:Q1–2008:Q1.

	RUSD_UF	RUS_02Y	RUS_30Y	RGL_BND	RW_EQTY	REM_EQTY	RCHL_TB	RCHL_BND	RCHL_EQTY
A. Returns									
Mean	−0.01	0.00	0.01	0.01	0.01	0.02	0.01	0.02	0.03
Median	−0.01	0.01	0.02	0.01	0.01	0.02	0.01	0.02	0.02
Maximum	0.09	0.12	0.22	0.15	0.19	0.28	0.04	0.14	0.37
Minimum	−0.14	−0.11	−0.14	−0.11	−0.23	−0.26	−0.01	−0.12	−0.24
Std. Dev.	0.05	0.05	0.07	0.05	0.08	0.12	0.01	0.04	0.11
Skewness	−0.26	−0.15	0.09	0.07	−0.63	−0.20	−0.17	0.05	0.17
Kurtosis	2.97	2.86	2.73	3.06	4.03	2.86	2.81	4.06	3.01
Jarque-Bera	0.82	0.32	0.31	0.07	8.09	0.55	0.45	3.45	0.34
Probability	0.66	0.85	0.85	0.97	0.02	0.76	0.80	0.18	0.84
Sum	−0.91	0.09	0.71	0.48	0.40	1.16	0.77	1.51	2.15
Sum Sq. Dev.	0.15	0.17	0.38	0.20	0.48	1.07	0.01	0.14	0.95
Observations	73	73	73	73	73	73	72	73	73
B. Correlation matrix									
RUSD_UF	1	0.97	0.66	0.79	0.28	−0.10	0.25	0.15	−0.23
RUS_02Y		1	0.78	0.87	0.23	−0.17	0.25	0.18	−0.29
RUS_30V			1	0.83	0.16	−0.26	0.04	0.28	−0.37
RGL_BND				1	0.26	−0.22	0.11	0.23	−0.34
RW_EQTY					1	0.54	0.06	0.12	0.25
REM_EQTV						1	−0.15	0.18	0.61
RCHL_TB							1	−0.04	−0.14
RCHL_BND								1	0.09

Table 3
Global VAR(1) estimates of B_1. $y_t = B_0 + B_1\, y_{t-1} + u_t$. Sample 1990:Q2–2008:Q1.

Lagged:	RGLBOND	ERCHL_TB	ERUS_02Y	ERUS_30Y	ERW_EQTY	EREM_EQTY	ERCHL_BND	ERCHL_EQTY	Constant	R-squared
1) Global government bonds (RGLBOND)										
Coefficient Std. Error	0.05	0.17	0.05	−0.09	−0.10	−0.06	−0.02	0.04	0.02	0.20
	0.36	0.08	0.42	0.09	0.05	0.02	0.03	0.01	0.01	
2) Excess return of the Chilean T-Bill (ERCHL_TB)										
Coefficient Std. Error	0.07	−0.03	0.01	0.12	0.09	0.10	−0.07	−0.07	0.00	0.07
	0.66	0.17	0.78	0.17	0.10	0.06	0.11	0.05	0.01	
3) Excess return of the US 2-year T-Bill (ERUS_02Y)										
Coefficient Std. Error	0.44	−0.10	0.56	0.03	0.11				−0.01	0.14
	0.31	0.07	0.36	0.08	0.04				0.00	
4) Excess return of the US 30-year T-Bond (ERUS_30Y)										
Coefficient Std. Error	−0.06	−0.11	−0.17	−0.02	0.01				0.00	0.03
	0.55	0.12	0.64	0.13	0.07				0.01	
5) Excess return of global equity (ERW_EQTY)										
Coefficient Std. Error	−0.98	0.06	−1.16	0.40	−0.05				0.01	0.04
	1.07	0.24	1.25	0.26	0.13				0.02	
6) Excess return of emerging market equity (EREM_EQTY)										
Coefficient Std. Error	−1.42	−0.23	−2.55	0.46	−0.19	0.19			0.02	0.06
	1.84	0.41	2.16	0.45	0.25	0.12			0.03	
7) Excess return of Chilean equity (ERCHL_EQTY)										
Coefficient Std. Error	−0.21	−0.95	−0.22	−0.21	−0.20	0.15	0.69	0.02	0.01	0.12
	1.78	0.45	2.09	0.44	0.25	0.15	0.25	0.12	0.03	
8) Excess return of Chilean long-term real bonds (ERCHL_BND)										
Coefficient Std. Error	−0.29	−0.05	−0.78	0.05	−0.14	0.13	−0.04	−0.04	0.01	0.06
	0.80	0.21	0.94	0.20	0.12	0.08	0.14	0.07	0.01	

This paper takes the unconditional long-term expected returns as the starting point, which is equivalent to assuming that conditional and unconditional expected returns are the same, and does not deal with time-varying expected returns.

2.4. Resampled efficiency in VAR processes

As explained, in order to annualize the covariance matrix, this study assumes that returns of the local currency-denominated T-Bill and of the other asset classes' excess returns follow a VAR(1) process. This process should be consistent with the one estimated for global investors in Eq. (4), given that all regressions are linear. Letting x_t denote the vector of local T-Bill returns and excess returns, the process followed by the quarterly data is:

$$x_t = A_0 + A_1 x_{t-1} + \varepsilon_t. \tag{9}$$

The data allows estimating A_1 and the covariance matrix of the errors ($\Sigma_{\varepsilon\varepsilon}$). However, as explained, the adjusted A_0 reflects unconditional expected values (see Eq. (5)). These parameters allow estimating all the necessary inputs for the efficient frontiers, using different levels of leverage as in Campbell and Viceira (2005a) and Hoevenaars et al. (2007). However, this paper uses a numerical procedure in order to restrict portfolio weights to be positive.

Estimated optimal portfolio weights are subject to sampling error, as any other estimation. Jobson and Korkie (1981) notice that estimated optimal portfolio weights are highly sensitive to small changes in parameter values, which is why Michaud (1998) proposes a methodology named Resampled Efficiency which verifies the robustness of optimized portfolio weights. However, Michaud's solution does not consider the possibility of serial and crossed-serial correlation. On the other hand, Campbell and Viceira (2005a) assume a VAR process for returns but do not estimate standard errors for portfolio weights.

This paper considers that the estimates of A_0, A_1 and $\Sigma_{\varepsilon\varepsilon}$ are subject to sampling error by simulating multiple sample paths consistent with these estimates using Montecarlo. Each simulated path allows re-estimating the efficient frontier using the simulated means and variances. This methodology uses each vector of simulation-based portfolio weights jointly with the initial parameters to re-estimate means and standard deviations. A scatterplot illustrates these mean-standard deviation pairs jointly with the original efficient frontier. As in Michaud (1998), this procedure allows estimating efficient regions. Walker (2008b) explains this methodology in detail.

3. Data and inputs for asset allocation

This section presents the data and estimates the inputs for the asset allocation problems.

3.1. Data and sources

This work measures all returns in local inflation-adjusted currency (named the UF in Chile). Returns are logarithmic and excess returns are unit-less. The sample period is 1990:Q1 through 2008:Q1. Table 1 presents descriptive statistics for the series used in this study in.

3.2. The raw data

Table 2 presents descriptive statistics of the data used. Notice that, with respect to the Chilean inflation-adjusted unit (the UF), the dollar value depreciated at a rate of 1.24% per quarter (about 5% per year). The quarterly standard deviation is also quite large (9% annualized). These statistics illustrate that currency risk is significant. The most profitable asset class (ex post) is local equity, with an annualized real log-return of almost 36%. The following most profitable asset class (with an annualized real log return of about 24%) corresponds to local long-term bonds, illustrating another important risk (reinvestment risk) for the case of investors with liabilities in local (real) currency.

Table 4
Global Betas and expected returns.

	Estimated Betas	Risk premium (%)	Expected returns in local real currency (%)[a]
Chl T-Bill	0.28	1.12	3.21
US 2 Y T-Bill	0.06	0.24	2.34
US 30 Y T-Bond	−0.09	−0.35	1.75
Global Equity	1.00	4.00	6.10
Em. Mkt. Equity	1.27	5.07	7.17
Chl Equity	0.98	3.91	6.00
Chl long-term real bonds	0.25	0.98	3.08

[a] Small differences may exist due to rounding errors.

Table 5
VAR(1) estimates of A_1. $x_t = A_0 + A_1\, x_{t-1} + \varepsilon_t$. Sample 1990:Q2–2008:Q1.

Lagged:	RCHL_TB	ERUS_02Y	ERUS_30Y	ERW_EQTY	EREM_EQTY	ERCHL_EQTY	ERCHL_BND	CONSTANT	R-SQUARED
Return of the Chilean T-Bill (RCHL_TB)									
1) Coefficient Std. Error	0.55	−0.08	0.04	0.05	−0.01	0.01	−0.04	0.00	0.42
	0.10	0.03	0.02	0.02	0.01	0.01	0.02	0.00	
2) Excess return of the US 2-year T-Bill (ERUS_02Y)									
Coefficient Std. Error	0.32	0.08	−0.07	0.01	−0.05	−0.01	0.07	−0.01	0.03
	0.56	0.19	0.14	0.09	0.07	0.06	0.13	0.01	
3) Excess return of the US 30-year T-Bond (ERUS_30Y)									
Coefficient Std. Error	−0.36	0.11	−0.13	0.09	−0.20	0.01	−0.1.5	0.01	0.09
	0.84	0.29	0.20	0.14	0.11	0.09	0.20	0.01	
4) Excess return of global equity (ERW_EQTY)									
Coefficient Std. Error	−0.14	−0.23	0.06	−0.08	−0.17	0.09	0.08	0.00	0.08
	0.92	0.32	0.22	0.15	0.12	0.10	0.21	0.01	
5) Excess return of emerging market equity (EREM_EQTY)									
Coefficient Std. Error	−2.38	−0.27	0.20	−0.11	−0.08	0.11	0.34	0.03	0.10
	1.41	0.48	0.34	0.23	0.18	0.15	0.33	0.02	
6) Excess return of Chilean equity (ERCHL_EQTY)									
Coefficient Std. Error	−0.93	0.49	−0.26	−0.22	−0.08	0.20	0.89	0.02	0.18
	1.31	0.45	0.32	0.21	0.17	0.14	0.30	0.02	
7) Excess return of Chilean long-term real bonds (ERCHL_BND)									
Coefficient Std. Error	−0.48	0.00	−0.03	−0.12	0.00	−0.04	0.07	0.02	0.07
	0.56	0.19	0.13	0.09	0.07	0.06	0.13	0.01	

Finally, the orders of magnitude justify not using past returns as a guide for expected returns. For example, the high returns of local bonds are due to a significant and systematic drop in local interest rates. By definition, this trend in interest rates implies that long-term expected returns have fallen, at least for bonds.

3.3. Unconditional expected returns

As explained, estimating expected returns in local currency takes several steps. First, estimate the VAR represented in Eq. (**3**) from the perspective of a global investor. The VAR uses a combination of dollar returns of Global Bonds and excess returns with respect to these bonds for the other asset classes. Table 3 presents the corresponding results. Assuming that lagged equity returns of local and Emerging Markets have no correlation with excess returns in developed markets allows gaining degrees of freedom (a kind of pecking order).

Second, the results of Table 3 and the corresponding error covariance matrix allow estimation of the unconditional covariance matrix. Long-term Betas are covariances of excess returns with global equity excess returns, divided by the variance of global equity returns. The first column of Table 4 presents the estimated Betas.

Finally, consistent with the estimates of Dimson et al. (2006), this paper uses a 4% global equity premium with respect to long-term bonds. Assuming also that the long-term expected return on local real bonds equals its yield to maturity (3.08% at the moment these estimations were initiated), and using Eq. (**7**), gives the expected returns in real local currency. Table 4 also presents these results. Jensen's inequality requires subtracting 1/2 the historical variances from the expected returns presented in Table 4 in all estimations based on log-returns. Given that the estimations use the level of the local real interest rates that existed at one point in time, these expected returns are not exactly unconditional.

3.4. VAR process of the asset classes from a local perspective

Table 5 presents the results of estimating Eq. (8). In this case, the vector is composed of the local T-Bill's return measured in local real currency, and the rest to the asset class returns in excess of the local T-Bill. In this case all lagged variables can influence each other, because of the subtraction of the local T-Bill's return from all the other asset classes'. Table 5 shows that that in some cases lagged returns do seem to have predictive power. In addition to the covariance matrix of the errors, these VAR estimates allow us to estimate the annualized covariance matrix of the vector x_t. Table 6 presents these results, in the form of correlations and standard deviations.

The asset allocation problem now has all the necessary inputs.

4. Results

Each asset allocation problem assumes that the long-term local real bond perfectly represents long-term liabilities. Estimates for twenty points along each efficient frontier exist, starting with a risk aversion parameter (γ) of 0.1, ending with 1000. The cases consider three levels of leverage (L/W), 0, 4 and 8. The latter is close to the maximum allowed for annuity insurance companies. Each estimation considers frontiers with and without currency hedging. Here, currency hedging is equivalent to a short position in the US 2-year T-Bill and the corresponding positive allocation to local short-term T-Bills. As noticed in Walker (2008a), if local T-Bills have a positive risk premium with respect to US T-Bills, hedging increases expected returns. However,

Table 6
Annualized correlations and standard deviations for the asset allocation problem.

RCHI_TB	ERUS_02Y	ERUS_30Y	ERW_EQTY	EREM_EQTY	ERCHL_EQTY	ERCHI_BND	
Std. Dev. (%) correlations	3.20	9.35	13.96	13.01	24.49	26.25	9.64
RCHL_TB	1	0.02	−0.09	0.07	−0.44	−0.28	−0.46
ERUS_02Y		1	0.79	0.13	−0.23	−0.25	0.13
ERUS_30Y			1	0.08	−0.34	−0.37	0.16
ERW_EQTY				1	0.40	0.19	−0.03
EREM_EQTY					1	0.66	0.33
ERCH L_EQTY						1	0.33
ERCHL_BND							1

hedging may also increase portfolio volatility. In any case, allowing currency hedging removes a short-sale constraint, so the currency hedged frontier should not be interior to the unhedged one.

4.1. Efficient regions

As explained earlier, the purpose of using resampling is to estimate the effects of sampling error. For this purpose, after estimating each efficient frontier the resampling methodology used here simulates 1000 sample paths of 100 observations each consistent with the same parameters estimated originally (presented in Tables 4 and 6). For each simulated path, the authors used the VAR methodology described above to estimate the necessary parameters and thus a new frontier using the means and covariances of the corresponding simulation, as if they were the original ones. The procedure estimates 20 mean-standard deviation points for each particular simulation, by

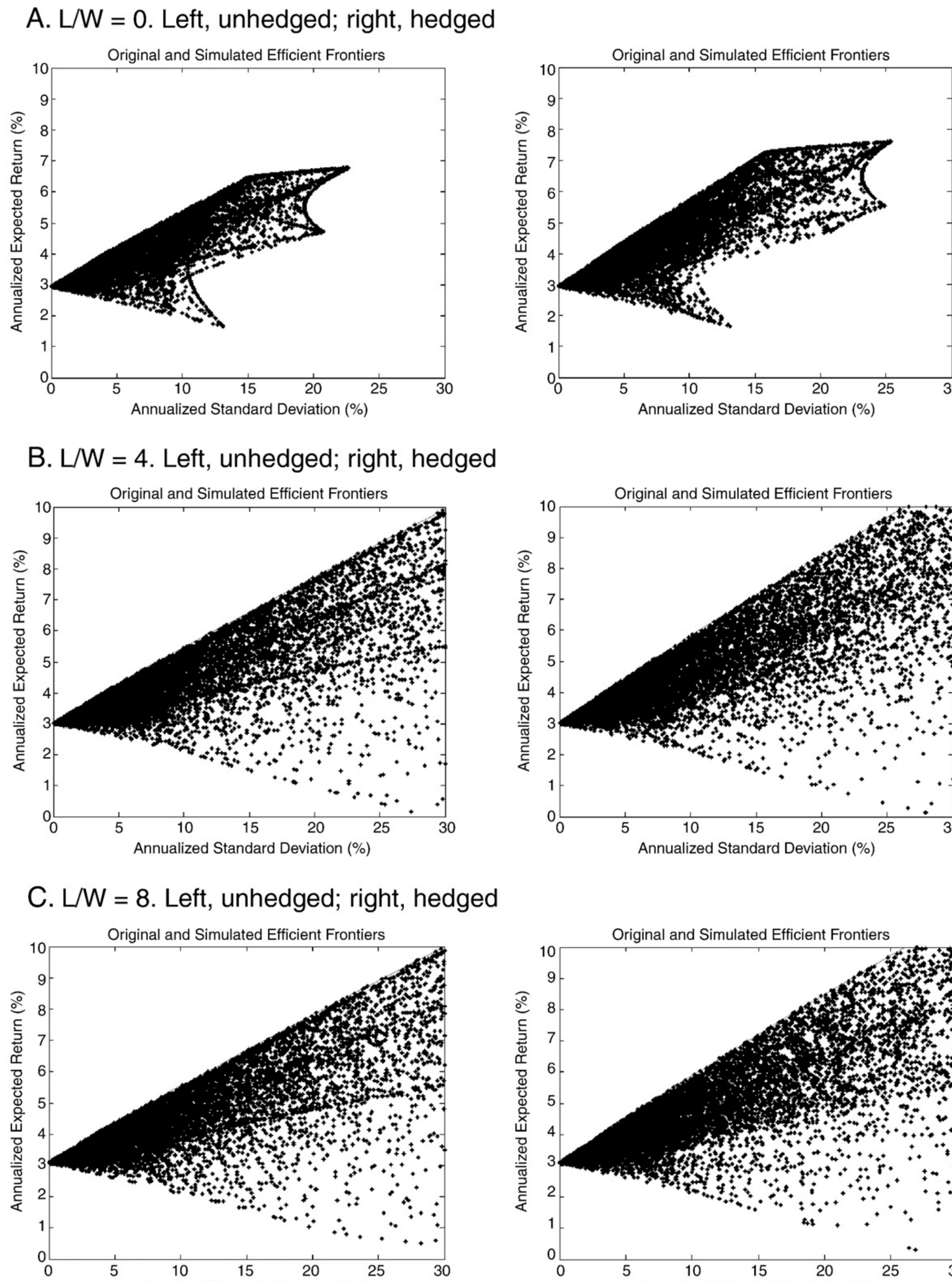

Fig. 1. Original and simulated efficient frontiers for net worth. For different levels of leverage, with and without currency hedging. (Real returns and standard deviations).

combining the corresponding vectors of portfolio weights with the original mean-variance parameters. Finally, the scatter plot that contains the original efficient frontier also includes these points. As in Michaud (1998), this procedure allows estimating efficient regions. However, here the regions consider the possibility of serial correlation and predictability, as explained in detail in Walker (2008b). The one-year risk-free real interest rate is 2.94%. This riskless rate is the linear projection on the zero-risk axis of the asset-only frontier, which is slightly below the short-term riskless rate in Table 4. This assumption does not significantly affect the results; without the one-year riskless asset the minimum variance portfolios have a small but positive variance. Fig. 1 illustrates the frontiers. Table 7 Panel A, presents the net worth's risk and expected return for the average optimal portfolio weights under different assumptions.

A first conclusion which is apparent, is that leverage clearly expands the investment opportunity set, allowing high risk-return portfolios otherwise unavailable due to short-sale constraints. Second, as expected, the risk-return frontiers look similar – they have similar slopes – independently of leverage, as expected in a Modigliani-Miller context. Third, contrary to one of this paper's hypotheses, currency hedging does seem to have an effect on frontier efficiency. Of course, this finding depends crucially on the assumption that hedging is equivalent to shorting the US T-Bill and investing the proceeds in the local T-Bill. Therefore, ignoring transaction costs, hedging increases expected returns. Fig. 1 illustrates that the high risk-return portfolios are the ones most affected by the possibility of currency hedging.

In order to assess the possible welfare gains associated with hedging, each simulation computes the utility value for each risk aversion level (as in Eq. (2)). Then, a simple statistical test checks for significant differences in the averages (which represent expected utility). Panel B in Table 7 summarizes these results. Results indicate (not surprisingly) that the higher the risk aversion, the lower the welfare improvements of currency hedging. In the case of zero leverage, only when risk aversion is below 2, hedging increases utility. With leverage, hedging tends to significantly reduce it. Since on average hedging increases expected returns, this effect must be due to higher sampling errors in the portfolio weights associated with hedging, which in itself involves levered positions.

Table 7
Net worth expected returns and risk and welfare gains of currency hedging (riskless return of 2.94%).

Gamma:	0.5	1	2	4	10	100	1000
A. Net worth annualized risk and return							
	Unhedged, L/W = 0						
Expected Return (%)	5.9	5.7	5.3	4.5	3.7	3.0	3.0
Return Standard Deviation (%)	13.6	12.6	10.3	6.9	3.0	0.4	0.0
	Hedged, L/W = 0						
Expected Return (%)	6.4	6.2	5.6	4.9	3.9	3.1	3.0
Return Standard Deviation (%)	14.2	12.9	10.5	7.3	3.4	0.4	0.0
	Unhedged, L/W = 4						
Expected Return (%)	12.6	8.7	6.0	4.6	3.7	3.0	3.0
Return Standard Deviation (%)	46.9	28.5	14.8	7.6	3.2	0.4	0.0
	Hedged, L/W = 4						
Expected Return (%)	14.6	10.0	6.8	5.1	3.9	3.1	3.0
Return Standard Deviation (%)	51.6	31.5	16.6	8.5	3.6	0.4	0.0
	Unhedged, L/W = 8						
Expected Return (%)	13.5	8.5	5.9	4.6	3.7	3.0	3.0
Return Standard Deviation (%)	56.5	29.3	14.6	7.4	3.1	0.4	0.0
	Hedged, L/W = 8						
Expected Return (%)	15.5	9.7	6.6	5.0	3.9	3.1	3.0
Return Standard Deviation (%)	59.2	31.3	15.6	8.0	3.4	0.4	0.0
B. Welfare gains	*T*-statistics						
(H, L/W = 0) minus (U, L/W = 0)	6.4	5.5	3.8	−0.8	−6.9	−4.9	−3.7
(H, L/W = 4) minus (U, L/W = 4)	0.8	−1.9	−2.7	−3.0	−4.0	−4.3	−2.0
(H, L/W = 8) minus (U, L/W = 8)	−2.4	−3.5	−3.1	−3.2	−4.4	−4.6	−4.8

4.2. Portfolio weights

Table 8 presents the simulation average portfolio weights and their standard errors in order to assess their statistical significance for different levels of risk aversion and leverage, assuming that that the first asset (Local 1 Yr T-Bill) is *e* is riskless in the investment horizon. In general, results are consistent with Eq. (3), that the levered portfolios are the unlevered ones plus the hedging component of long-term liabilities, but for the high risk portfolios the short-sale constraints are binding and this equality does not hold.

This paper argues that, in addition to the fixed income home biases due to liabilities, optimal close-to-home biases in portfolio weights exist. Expected returns take as reference an efficient portfolio of global equity and bonds. This assumption should imply zero net additional investment in other equity classes. On the contrary, results indeed tend to show that the expected biases are present.

4.1.1. Minimum variance portfolios

For all leverage levels, local bonds, either short or long-term, represent nearly 100% of the minimum variance portfolios as expected. Consistent with Eq. (3), the minimum variance investment in one-year local bills is the ratio of the net worth to total assets and the allocation to local bonds is its complement. The possibility of currency hedging only affects marginally the no-leverage case.

4.1.2. The unhedged portfolios

These portfolios have varying proportions invested in local fixed income, depending on the degree of risk aversion. Optimal home biases in these asset classes are evident. Excluding all local fixed income, the optimal composition of the rest of the portfolio is similar for risk aversion levels above 6 in the no-leverage case, and above 2 in the cases with leverage. This portfolio includes 20 to 25% in US T-Bills and T-Bonds, with a relatively higher proportion of US T-Bonds in the cases with leverage, 40 to 45% in global equity, and similar proportions in emerging market and local equity (about 20% each). Therefore, out of the total investment in equity, a clear close-to-home bias does exist.

4.1.3. Hedged versus unhedged portfolios

The effects of currency hedging are more important in the high risk portfolios. In general, hedging increases the investment in US T-Bonds, especially in the cases with leverage. Hedging increases investment in global equity, by reducing the investment in close-to-home equity. Still, even with currency hedging, close-to-home biases exist.

5. Conclusions

This article derives and applies a methodology for estimating optimal portfolio weights for positive net worth investors based on a small open economy with different levels of liabilities denominated in local currency. Regarding implementation, this work's contributions include a discussion of how to determine expected returns in local currency; how to consider possible serial- and cross-serial correlations in order to estimate the covariance matrix of the asset classes;

Table 8
Optimal portfolio weights. (Simulation averages and standard errors).

	A. Unhedged portfolio weights (%)							B. Hedged–unhedged portfolio weights (%)						
Gamma	0.5	1	2	4	10	100	1000	0.5	1	2	4	10	100	1000
A. Portfolio weights — L/W = 0														
Local 1YrTB	0.7	1.6	4.8	13.5	31.2	84.3	98.4	0.3	−0.1	−1.9	−5.7	−7.2	−2.5	−0.3
Std Err	*0.2*	*0.3*	*0.5*	*0.8*	*1.1*	*0.5*	*0.1*	*0.3*	*0.4*	*0.6*	*1.0*	*1.5*	*0.7*	*0.1*
Local STTB	2.2	3.7	8.8	20.6	37.1	12.1	1.2	66.0	63.2	57.0	46.7	27.0	4.6	0.5
Std Err	*0.4*	*0.5*	*0.7*	*0.9*	*1.1*	*0.5*	*0.0*	*1.7*	*1.7*	*1.8*	*1.8*	*1.7*	*0.7*	*0.1*
US Tbill	2.5	3.1	3.8	4.7	2.7	0.3	0.0	−65.3	−62.8	−57.8	−50.1	−29.9	−3.6	−0.4
Std Err	*0.5*	*0.5*	*0.5*	*0.4*	*0.2*	*0.0*	*0.0*	*1.7*	*1.7*	*1.7*	*1.6*	*1.1*	*0.1*	*0.0*
USTbonds	4.6	4.7	5.0	4.4	2.4	0.4	0.0	1.5	2.1	3.7	6.8	7.3	1.1	0.1
Std Err	*0.6*	*0.5*	*0.5*	*0.4*	*0.2*	*0.0*	*0.0*	*0.9*	*0.9*	*0.8*	*0.8*	*0.5*	*0.1*	*0.0*
World Eq	35.1	36.9	35.9	25.9	11.1	1.1	0.1	1.6	3.1	4.2	5.5	3.5	0.4	0.0
Std Err	*1.4*	*1.3*	*1.2*	*0.8*	*0.4*	*0.0*	*0.0*	*2.0*	*1.9*	*1.7*	*1.2*	*0.6*	*0.1*	*0.0*
Em Mkt Eq	33.3	28.0	18.7	10.3	4.3	0.5	0.0	−1.7	−2.3	−2.0	−1.3	−0.5	0.0	0.0
Std Err	*1.4*	*1.2*	*0.9*	*0.5*	*0.2*	*0.0*	*0.0*	*2.0*	*1.7*	*1.2*	*0.7*	*0.3*	*0.0*	*0.0*
Local Eq	17.0	16.2	14.1	9.5	4.2	0.4	0.0	−1.7	−2.7	−3.5	−2.8	−1.2	−0.1	0.0
Std Err	*1.1*	*1.0*	*0.8*	*0.5*	*0.2*	*0.0*	*0.0*	*1.5*	*1.4*	*1.1*	*0.7*	*0.3*	*0.0*	*0.0*
Local Bnds	4.5	5.9	8.8	11.1	6.9	0.9	0.1	−0.7	−0.3	0.4	1.0	0.9	0.1	0.0
Std Err	*0.6*	*0.6*	*0.7*	*0.7*	*0.4*	*0.0*	*0.0*	*0.8*	*0.9*	*1.0*	*1.0*	*0.6*	*0.1*	*0.0*
B. Portfolio weights — L/W = 4														
Local 1YrTB	1.7	2.5	2.8	3.6	6.5	17.1	19.7	−0.4	−0.9	−1.1	−1.0	−0.9	−0.4	0.0
Std Err	*0.3*	*0.3*	*0.3*	*0.2*	*0.2*	*0.1*	*0.0*	*0.1*	*0.1*	*0.1*	*0.0*	*0.1*	*0.0*	*0.0*
Local STTB	7.0	10.8	10.3	9.5	9.5	2.4	0.2	49.0	35.2	20.6	11.0	4.9	0.8	0.1
Std Err	*0.6*	*0.6*	*0.5*	*0.4*	*0.3*	*0.1*	*0.0*	*1.3*	*1.0*	*0.5*	*0.2*	*0.1*	*0.0*	*0.0*
USTb1II	3.3	3.2	2.1	1.3	0.6	0.1	0.0	−51.9	−40.1	−25.1	−14.4	−6.7	−0.8	−0.1
Std Err	*0.4*	*0.3*	*0.2*	*0.1*	*0.1*	*0.0*	*0.0*	*1.3*	*0.9*	*0.4*	*0.1*	*0.0*	*0.0*	*0.0*
USTbonds	5.8	5.0	3.0	1.7	0.8	0.1	0.0	3.3	5.2	4.8	3.3	1.9	0.3	0.0
Std Err	*0.5*	*0.4*	*0.2*	*0.1*	*0.1*	*0.0*	*0.0*	*0.3*	*0.3*	*0.1*	*0.0*	*0.0*	*0.0*	*0.0*
World Eq	30.2	18.7	10.0	5.4	2.2	0.2	0.0	4.5	3.3	2.2	1.5	0.7	0.1	0.0
Std Err	*1.1*	*0.7*	*0.3*	*0.2*	*0.1*	*0.0*	*0.0*	*1.2*	*0.5*	*0.1*	*0.0*	*0.0*	*0.0*	*0.0*
Em Mkt Eq	17.6	9.9	5.2	2.7	1.1	0.1	0.0	−1.1	−1.0	−0.6	−0.3	−0.1	0.0	0.0
Std Err	*0.8*	*0.4*	*0.2*	*0.1*	*0.0*	*0.0*	*0.0*	*0.7*	*0.2*	*0.1*	*0.0*	*0.0*	*0.0*	*0.0*
Local Eq	12.4	7.7	4.2	2.3	1.0	0.1	0.0	−2.9	−2.0	−1.1	−0.6	−0.2	0.0	0.0
Std Err	*0.7*	*0.4*	*0.2*	*0.1*	*0.1*	*0.0*	*0.0*	*0.5*	*0.2*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*
Local Bnds	22.1	42.2	62.4	73.6	78.3	79.9	80.0	−0.5	0.2	0.2	0.4	0.4	0.0	0.0
StdErr	*1.0*	*1.0*	*0.7*	*0.4*	*0.2*	*0.0*	*0.0*	*0.9*	*0.9*	*0.5*	*0.2*	*0.0*	*0.0*	*0.0*
C. Portfolio weights — L/W = 8														
Local 1YrTB	2.3	2.0	1.8	2.2	3.6	9.6	11.0	−1.0	−0.8	−0.6	−0.6	−0.5	−0.3	0.0
Std Err	*0.3*	*0.2*	*0.2*	*0.1*	*0.1*	*0.0*	*0.0*	*0.4*	*0.3*	*0.2*	*0.2*	*0.2*	*0.1*	*0.0*
Local STTB	9.2	7.5	5.6	5.1	5.3	1.3	0.1	33.5	20.0	10.7	5.8	2.6	0.5	0.0
Std Err	*0.6*	*0.5*	*0.3*	*0.2*	*0.2*	*0.0*	*0.0*	*1.4*	*1.0*	*0.6*	*0.4*	*0.2*	*0.1*	*0.0*
USTbill	3.7	2.2	1.4	0.9	0.4	0.0	0.0	−37.8	−23.5	−13.1	−7.6	−3.5	−0.4	0.0
StdErr	*0.4*	*0.2*	*0.1*	*0.1*	*0.0*	*0.0*	*0.0*	*1.4*	*0.9*	*0.5*	*0.3*	*0.1*	*0.0*	*0.0*
USTbonds	5.4	3.2	1.7	1.0	0.4	0.1	0.0	4.5	4.2	2.7	1.9	1.1	0.2	0.0
Std Err	*0.4*	*0.2*	*0.1*	*0.1*	*0.0*	*0.0*	*0.0*	*0.7*	*0.5*	*0.3*	*0.2*	*0.1*	*0.0*	*0.0*
World Eq	21.0	10.9	5.7	3.1	1.3	0.1	0.0	1.6	1.2	0.8	0.5	0.3	0.0	0.0
Std Err	*0.7*	*0.4*	*0.2*	*0.1*	*0.0*	*0.0*	*0.0*	*1.1*	*0.6*	*0.3*	*0.2*	*0.1*	*0.0*	*0.0*
Em Mkt Eq	9.9	5.2	2.7	1.4	0.6	0.1	0.0	−1.0	−0.5	−0.3	−0.1	0.0	0.0	0.0
Std Err	*0.5*	*0.2*	*0.1*	*0.1*	*0.0*	*0.0*	*0.0*	*0.7*	*0.3*	*0.2*	*0.1*	*0.0*	*0.0*	*0.0*
Local Eq	7.2	4.0	2.2	1.2	0.5	0.1	0.0	−1.9	−1.0	−0.6	−0.3	−0.1	0.0	0.0
Std Err	*0.4*	*0.2*	*0.1*	*0.1*	*0.0*	*0.0*	*0.0*	*0.6*	*0.3*	*0.2*	*0.1*	*0.0*	*0.0*	*0.0*
Local Bnds	41.3	65.0	78.9	85.3	87.9	88.8	88.9	2.1	0.4	0.4	0.3	0.3	0.0	0.0
StdErr	*1.0*	*0.7*	*0.4*	*0.2*	*0.1*	*0.0*	*0.0*	*1.4*	*1.0*	*0.6*	*0.3*	*0.1*	*0.0*	*0.0*

and the implementation of a methodology to estimate portfolio weight and expected utility sampling errors.

The methodology is applied to the case of a Chilean institutional investor, considering as eligible local and international asset classes: local and US short and long-term fixed, global, emerging market and local equity, plus the possibility of currency hedging. The paper uses a resampling methodology to estimate optimal portfolios for leverage levels of 0, 4 and 8, and different levels of risk aversion. The methodology is Campbell and Viceira's (2002) regarding the assumption that returns follow a VAR(1) process and that returns are log-normal. However, here the investment horizon is short (one year).

This article presents several interesting findings. First, long-term liabilities (equivalent to shorting long-term local bonds) give access to high risk-return portfolios otherwise unavailable. Second, for all leverage levels portfolio compositions exhibit optimal close to home biases. Beyond local fixed income, emerging market and local equity each represent about half the total investment in equity. These asset classes have hedging properties regarding currency and interest rate risk. Long-term US currency hedged bonds also have significant interest rate hedging properties.

Currency hedging is a substitute for optimal investment in close to home asset classes, but a significant bias remains. However, currency hedging has ambiguous effects on expected utility: without leverage, hedging increases expected utility for high risk tolerance investors but reduces welfare in the other cases. With leverage, in general, currency hedging reduces expected utility. This ambiguity is due to larger sampling errors in the portfolio weights with currency hedging. The resampling methodology implemented here allows detecting this effect.

The hedging properties of close-to-home asset classes (their correlations with local asset classes, particularly with local currency returns and local long-term interest rates) drive the findings presented here. Considering that regarding currency hedging and global portfolio investments Walker (2008a) finds similar results for Brazil, Chile, Colombia and Perú (due to the similar behavior of their local currencies) and that no a priori reasons exist to expect long-term local interest rates to behave differently, other emerging markets and/or small open economies should exhibit qualitatively similar results, but of course, this question is open.

The general lesson derived from this investigation is not to forget that risk, like beauty, is in the eye of the beholder, which sometimes has counterintuitive implications.

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Appendix A

Table A.1
Notation.

Symbol	Description	Symbol	Description
$W_t = A_t - L_t$	Net worth, equal to assets minus liabilities	Ω_{rx}	Column vector of covariances between one-period security excess returns and x
r_{Wt}	Logarithmic net worth rate or return	$\Omega_{rx}(\tau)$	Column vector of covariances between τ-period security excess returns and x
r_{ft}	One-period risk-free logarithmic rate	$\mu(\tau)$	Vector of cumulative expected returns of risky securities
r_t	Column vector of risky security logarithmic returns	$\mu_x(\tau)$	Cumulative expected return of x
σ^2	Column vector of one-period security variances	Σ	Matrix of one-period security return variances (measured in excess of the one-period riskless asset)
ι	Column vector of ones	$\Sigma(\tau)$	Matrix of τ-period security return variances (measured in excess of the one-period riskless asset)
$\alpha(\tau)$	Columns vector or risky security holdings for the investment horizon τ		

The geometric return on equity is given by

$$R_{Wt} = \left(1 + \frac{L}{W}\right)R_{At} - \frac{L}{W}R_{Lt}. \tag{A1}$$

Using the approximations

$$\begin{aligned} R_{Wt} &= r_{Wt} + \frac{1}{2}\sigma_W^2 \\ R_{Lt} &= r_{Lt} + \frac{1}{2}\sigma_L^2 \\ R_{At} &= r_{At} + \frac{1}{2}\sigma_A^2 \end{aligned} \tag{A2}$$

$$r_{At} = r_{ft} + \alpha(\tau)'\left(r_t + \frac{1}{2}\sigma^2 - r_{ft}\iota\right) - \frac{1}{2}\alpha(\tau)'\Sigma\alpha(\tau). \tag{A.3}$$

Following Campbell and Viceira all returns in excess of a one-period risk-free or benchmark rate (r_{ft}). This condition implies that:

$$\begin{aligned} r_{Wt} + \frac{1}{2}\sigma_W^2 = {} & r_{ft} - \frac{L}{W}\left(r_{Lt} - r_{ft} + \frac{1}{2}\sigma_L^2\right) \\ & + \left(1 + \frac{L}{W}\right)\alpha(\tau)'\left(r_t + \frac{1}{2}\sigma^2 - r_{ft}\iota\right). \end{aligned} \tag{A.4}$$

Assuming that a one-period conditionally risk-free rate exists, the variance becomes:

$$\sigma_W^2 = \left(1 + \frac{L}{W}\right)^2 \alpha(\tau)'\Sigma\alpha(\tau) + \left(\frac{L}{W}\right)^2\sigma_L^2 - 2\frac{L}{W}\left(1 + \frac{L}{W}\right)\alpha(\tau)' cov(r, r_L). \tag{A.5}$$

Thus,

$$\begin{aligned} r_{Wt} = {} & r_{ft} - \frac{L}{W}\left(r_{Lt} - r_{ft} + \frac{1}{2}\sigma_L^2\right) + \left(1 + \frac{L}{W}\right)\alpha(\tau)'\left(r_t + \frac{1}{2}\sigma^2 - r_{ft}\iota\right) \\ & - \frac{1}{2}\left[\left(1 + \frac{L}{W}\right)^2\alpha(\tau)'\Sigma\alpha(\tau) + \left(\frac{L}{W}\right)^2\sigma_L^2 - 2\frac{L}{W}\left(1 + \frac{L}{W}\right)\alpha(\tau)'\Omega_{rL}\right]. \end{aligned} \tag{A.6}$$

Considering that $W_{t+\tau} = W_t e^{\sum_{k=1}^{\tau} r_{Wt+k}}$, then

$$\begin{aligned} \mu_W(\tau) &= E_t\left(\sum_{k=1}^{\tau} r_{Wt+k}\right) \\ &= \mu_f(\tau) - \frac{L}{W}\left(\mu_L(\tau) + \frac{\tau}{2}\sigma_L^2 - \mu_f(\tau)\right) + \left(1 + \frac{L}{W}\right)\alpha(\tau)'\left(\mu(\tau) + \frac{1}{2}\tau\sigma^2 - \iota\mu_f(\tau)\right) \\ &\quad - \frac{\tau}{2}\left[\left(1 + \frac{L}{W}\right)^2\alpha(\tau)'\sum\alpha(\tau) + \left(\frac{L}{W}\right)^2\sigma_L^2 - 2\frac{L}{W}\left(1 + \frac{L}{W}\right)\alpha(\tau)'\Omega_{rL}\right] \end{aligned} \tag{A.7}$$

$$\begin{aligned} var(r_W(\tau)) = {} & \sigma_f^2(\tau) + \left(\frac{L}{W}\right)^2\sigma_L^2(\tau) - 2\frac{L}{W}\sigma_{fL}(\tau) \\ & + \left(1 + \frac{L}{W}\right)^2\alpha(\tau)'\Sigma(\tau)\alpha(\tau) \\ & + 2\left(1 + \frac{L}{W}\right)\alpha(\tau)'\left(\Omega_{rf}(\tau) - \frac{L}{W}\Omega_{rL}(\tau)\right) \end{aligned} \tag{A.8}$$

The problem is solved numerically by evaluating these expressions in the objective function (2). If a riskless rate exists in the investment horizon, then the elements of Eq. (A.8) with subscript f are zero.

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