Bargaining in Securities*

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Abstract

Many corporate negotiations involve contingent payments or *securities*, yet the bargaining literature overwhelmingly focuses on pure cash transactions. We characterize equilibria in a continuous-time model of bargaining in securities. A privately informed buyer and a seller negotiate the terms of a joint project. The buyer's private information affects both his standalone value and the net returns from the project. The seller makes offers in a one-dimensional family of securities (e.g., equity splits). We show how outcomes change as the underlying security becomes more sensitive to the buyer's information. We apply the framework to mergers and acquisitions under financial constraints, and to corporate restructuring.

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1 Introduction

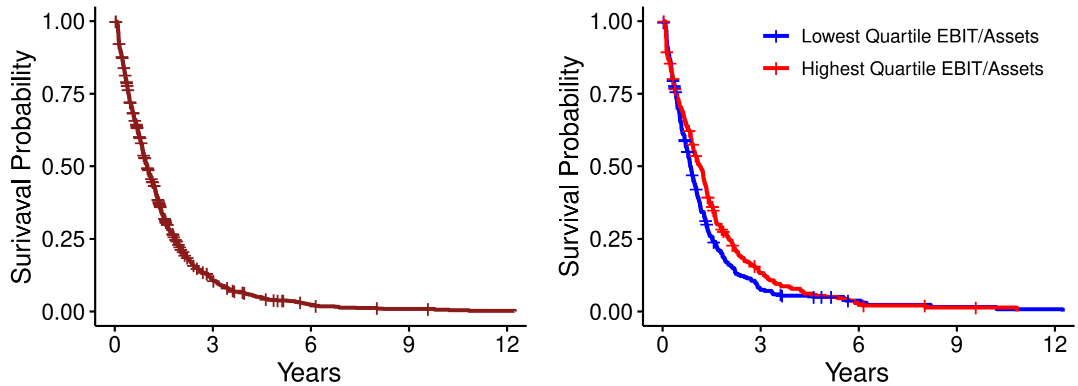
Many corporate negotiations involve payments other than cash. In merger and acquisitions (M&A), acquirers often pay the target using shares of their own companies (Malmendier et al., 2016). In Chapter 11 bankruptcy procedures, claim holders bargain not only over cash payments, but also over the terms of the restructuring plan such as the face value, maturity, and seniority of new debt (White, 1989). Similarly, when an individual land owner and a local oil and gas producer negotiate a lease agreement, they tend to specify an upfront cash payment and a pre-specified royalty over future revenues.[1] Likewise, procurement contracts, many of which are arrived at via negotiation with suppliers, specify some cost sharing rule.

In short, negotiating parties frequently make offers in contingent payments or *securities*, and yet, the bargaining literature overwhelmingly focuses on pure cash transactions. We therefore lack a theory that can explain the course of negotiations when, as is the case with contingent payments, the value of offers depends on the private information of parties.

For example, consider the data in Figure 1 on the length of Chapter 11 bankruptcy negotiations. In these negotiations, creditors and debtors negotiate over the assets and control rights of a troubled firm, but – by definition — the offers are not made in cash, which the debtor firm lacks, but in securities, such as the terms of new, renegotiated debt.

We highlight three notable features from Figure 1. First, long disagreements are very common in the data, with an average duration of 16 months, indicating substantial inefficiencies in these negotiations. A protracted bankruptcy negotiation imposes severe costs on a firm: it may miss valuable opportunities in the product market, suppliers and long-term customers may be spooked by the delayed reorganization and cancel important contracts, key personnel may leave, and so on. During Polaroid's Chapter 11 process, for example, management argued in favor of a quick resolution because "revenues [were] falling off" and "[the company] was like a melting ice cube" (LoPucki and Doherty, 2007, p. 54). Antill (2021) in fact finds that approximately 15% of Chapter 11 cases end in an inefficient liquidation. Second, there is substantial *variation* in the length of negotiations, with the 10th percentile-length negotiation lasting 57 days, and the 90th percentile one lasting 1,038 days. Dou et al. (2021) estimates a structural model of bankruptcy and finds that excessive delay is the primary source of inefficiency. Third, the length of negotiation is positively correlated with measures of firm viability and quality: firms that are more profitable going into bankruptcy, measured by their EBIT-to-assets ratio, took longer to reach a restructuring agreement. In other words, "better" firms seem to suffer higher delay costs, and failing to

[1] Government oil lease agreements are usually auctioned but individual lease agreements are commonly settled by negotiation.



(a) Survival function bankruptcy emergence for all firms

(b) Survival function bankruptcy emergence for low and high quality firms.

Figure 1: Survival curve for bankruptcy emergence of chapter 11 cases in the U.S. The survival curve is estimated using the Kaplan-Meier method, and adjusts for censoring arising due to dismissal of the case or transition to chapter 7 proceedings. In figure (b), firms are sorted by performance at the time of bankruptcy. The blue line are firms that were in the lowest quartile of ebitda/assets before bankruptcy, while the red line are firms that were in the highest such quartile. Source: UCLA LoPucki Bankruptcy Research Database.

agree appears to be a positive indicator of firm quality.

Why this correlation would arise in equilibrium is not immediate, since there are several paths through which an increase in the underlying value of the firm affects negotiation incentives. A debtor firm that is more valuable has a greater desire to restart regular operations, which makes it more willing to agree, but it will also have assets that are more valuable in liquidation, which makes the *creditor* less willing to agree. Moreover, since the value of any new risky debt covaries with the firm's value, a more valuable firm will find any debt terms more costly than a less valuable firm. Altogether, these negotiations entail significant efficiency costs, and there are important patterns to be explained in the size of those costs.

This paper presents a tractable continuous-time model of bargaining in securities that generates significant variation in delay and can rationalize some of the above patterns. We abstract from some institutional details in order to isolate the impact of security payments on the negotiation. In the model, a privately informed buyer and a seller negotiate over the terms of a joint project. The buyer has private information that affects both his assets in place (standalone value) and the net return of the project; the seller has a commonly known cost. The seller makes offers in a given one-dimensional family of securities (e.g., makes debt offers with different face values, or makes equity offers with different ownership shares) so that the value of an accepted offer depends on the buyer's private information. To capture the lack of commitment in a stark way, we assume that the seller can revise her offers infinitely frequently, and both players discount the future at the same exponential rate. We focus on a tractable class of Markovian "skimming" equilibria in which buyer types accept gradually in a given order.

We provide two sets of results: we characterize bargaining dynamics, and we show how outcomes depend on the security's sensitivity to the buyer's information. The first main finding is that the deals that fail to close will be overwhelmingly high value deals. The use of securities as means of payment can change the nature of selection: depending on the security, rejection of an offer (and therefore the length of the negotiation) may be a positive or a negative signal about the buyer's type and the value of a deal. Nevertheless, we show that, in equilibrium, the length of negotiation must be a positive signal of the buyer's type. Whenever there is delay, it is high types who take longer to agree, with very high types taking a possibly unbounded amount of time; hence, if we interpret the discount rate r as coming from an exogenous rate of negotiation breakdown, then the deals that fail to close before the negotiation breaks down come from the right tail of the value distribution. In particular, as in Figure 1(b), negotiation length must be *positively correlated* with measures of the value of a deal.

In general, the bargaining dynamics depend on the buyer's marginal rate of substitution between the time of trade and the offer amount.[2] If that rate is increasing in the type, then high types are (all else equal) more willing to suffer delay in order to fetch better offers; from the seller's perspective, selection is therefore "adverse." If on the contrary the rate of substitution is decreasing, selection is favorable. We show that, if selection is favorable, or if it is adverse but only "mildly" so, agreement is instantaneous and there is no inefficiency. On the other hand, if selection is *severely* adverse, there is delay of a very particular form, where types in the right tail of the value distribution may take an unbounded of time to agree. The equilibrium begins with a phase of gradual concessions, in which the seller's offer becomes more generous smoothly, and buyer types accept gradually in ascending order. Eventually, the negotiation reaches an impasse of random, possibly unbounded length, during which the seller intransigently refuses to improve his offers, even though they are continuously rejected. Finally, the impasse ends in a flash: the seller "submits," drops his ask discontinuously and trades immediately with all remaining types. What separates "mild" and "severe" adverse selection is whether or not, in a one-shot game, the seller would lose money at the offer that all types would be willing to accept; when the seller would lose money, adverse selection is severe, in the sense that the static market has a lemons problem.

Second, we show how the inefficiency of negotiation depends on primitive properties of

[2] Note that, unlike cash bargaining models, "offer" here is not synonymous with "payment." For example, when bargaining over equity splits, the "offer" is the share of the gross returns being proposed; the "payment" is the value of that share.

the security being negotiated. We focus on the *informational sensitivity* or "steepness" (DeMarzo et al., 2005) of the security: how closely the value of the payment covaries with the buyer's private information. Specifically, we look at security families that are ordered in their informational sensitivity according to a scalar parameter. For example, if there is a fixed royalty rate that must be paid to the seller, but the parties negotiate over an additional cash payment, then the higher the royalty rate, the more informationally sensitive the overall "cash + royalty" security family will be. That scalar parameter can capture, in M&A applications, the tightness of the buyer's financial constraints or the precision of the buyer's estimate of the synergies from a merger; in corporate restructuring applications, the parameter can capture the maturity of the debt being negotiated.[3]

More informationally sensitive securities tend to increase inefficiency (negotiation length) and harm the privately informed buyer. We measure bargaining inefficiency according to the (type-specific) *certainty-equivalent delay*: the deterministic delay that causes the same expected discounting cost as the random equilibrium delay. Making the security more informationally sensitive increases delay for low and high types, but the effect can sometimes be reversed for some intermediate types. The overall effect, however, tends to be to increase inefficiencies. Numerically we find that the delay ranking is often uniform, and even when it is not uniform, the distribution of negotiation lengths overwhelmingly implies longer delays from more sensitive securities.

Regarding utilities, we prove that low types (those below a threshold) are always harmed by bargaining in more informationally sensitive securities, even though their payments decrease. And we provide an easy-to-check sufficient condition under which using a more sensitive security harms all types.

We consider two applications of our theory to corporate negotiations. First, we apply our model to the case of M&A negotiations. We analyze the impact that financial constraints have on the outcome of these transactions. In the model, the parties negotiate in equity, but the buyer has pre-existing debt and maximizes the total value of debt and equity holders. The higher the leverage, the more financially constrained the buyer is; at the same time, the higher the leverage, *the more informationally sensitive levered equity will be as a security.* By our general results, tighter financial constraints tend to increase bargaining frictions and

[3]We note that the bulk of the results generalize to steepness comparisons that are not "parametrized." Equity offers, for instance, are more informationally sensitive than cash offers—the value of a cash offer to the seller does not depend on what the buyer knows about the project, but the value of an equity offer does—but there is no overarching (one-dimensional) parametrized security family that includes pure equity offers and pure cash offers as special cases. In the Online Appendix B, we show how the bulk of these results extend to steepness comparisons that are not parametrized; moreover, in the numerical simulations we have conducted, all the theoretical results for parametrized families seem to hold for non-parametric comparisons as well. See Remark 3.

negotiation length. If the discount rate reflects an underlying exogenous probability of deal failure, then our measure of bargaining frictions maps one-to-one to deal failure probabilities, so the model predicts that tightening financial constraints will raise deal failure probabilities. This prediction is consistent with empirical studies on means of payment in M&A, which have found that tighter financial constraints are associated with higher probabilities of deal failure and lower M&A activity by firms (Malmendier et al., 2016; Uysal, 2011).

Using a normal-linear model parametrization of the levered-equity model, we also show how the negotiation depends on the nature of the synergies or net returns generated by the project. In an M&A setting, merger synergies related to cost savings may be easier to estimate than merger synergies from market expansion or product market fit. A mean preserving-spread of future synergies raises the value of levered equity, but it also may dilute how tightly the buyer's signal covaries with the expected value of the project; signing the net effect requires an equilibrium analysis. We show that the net effect of raising the buyer's precision is equivalent to negotiating in a more informationally sensitive security. Hence, as the buyer's information becomes more precise, bargaining frictions tend to worsen. Our theoretical results imply that types below a threshold are always harmed by the increased precision, but in the numerical examples we have computed, the harm seems to be uniform across all types. In other words, we show that *more precise information can be value-destroying.*

Second, we apply our model to corporate restructuring negotiations. In this case, equity holders and creditors negotiate over the terms of the firm's new liabilities. In the model, the parties negotiate over the face value of the new debt, and delaying an agreement is costly because the firm might be forced into liquidation if the firm does not restructure its liabilities on time. First, we consider the impact that assets in place have on the negotiation. We show that firms that have more assets in place relative to their liabilities take longer to restructure those liabilities. This agrees with evidence by Teloni (2015), who finds that larger and more profitable firms (as measured by using EBIT-total assets ratio) take longer to reorganize. Second, we consider the impact that the maturity of the new debt has on the likelihood of a successful negotiation. It is common practice to increase the maturity of debt as part of a corporate restructuring, the idea being that a higher maturity spreads the firm's payments more thinly over time and gives the firm an opportunity to improve performance before its debt obligations come due. However, our analysis shows that increasing the maturity of debt also increases bargaining frictions, and reduces the probability of reaching an agreement. Thus, while increasing maturity can be optimal ex-post, it can also increases the probability that a restructuring fails and the firm is inefficiently liquidated.

The remainder of this section briefly outlines related work. Section 2 then presents the model setup and our continuous-time equilibrium notion, and Section 3 presents our equilibrium construction and our results on equilibrium uniqueness for general securities. Section 4 presents our comparative statics results on informational sensitivity. Section 5 applies the theoretical results to equity bargaining under financial constraints and to corporate restructuring. Section 6 concludes. All omitted proofs are in the appendix.

Related Literature:

We contribute to a nascent bargaining literature that considers bargaining over objects richer than cash. Strulovici (2017) considers a two-type Coasean bargaining model where parties negotiate over the terms of contracts, including, for instance, the quantity or quality of goods traded. Hanazono and Watanabe (2018) study the splitting of a stochastic pie in a setting where both parties have noisy signals about the size of the pie. de Clippel et al. (2019) consider a two-stage Nash demand model where the utility possibility sets depend on privately known types, and they provide a non-cooperative foundation to Myerson's axiomatic solution for these problems.

We also contribute to work in corporate finance on the interaction between asymmetric information and contingent payments.[4] The literature has studied security payments in financing decisions (Myers and Majluf, 1984; DeMarzo and Duffie, 1999), bidding wars for mergers and acquisitions (Fishman, 1989; Hansen, 1987; Rhodes-Kropf and Viswanathan, 2004), and auctions (DeMarzo et al., 2005; Axelson, 2007; Che and Kim, 2010), but overwhelmingly these models are static/one-shot interactions, or they feature full commitment. There are two competing forces highlighted by those papers. On the one hand, as first noted by Myers and Majluf (1984), informationally sensitive securities (steep securities) are costly to use due to adverse selection. On the other, as noted by DeMarzo et al. (2005) work on security auctions, steeper securities create a tighter link between the informed party's payment and his type and can therefore help surplus extraction. We use DeMarzo et al. (2005)'s definition of informational sensitivity ("steepness"). We also adopt Che and Kim (2010)'s extension of DeMarzo et al. (2005) that allows bidders' private information to affect their standalone value; part of our contribution is showing how dynamic considerations interact with, and may overwhelm, these linkage-principle forces at work in the security auction literature. Section 6 elaborates on this connection.

The use of contingent payments makes the seller and buyer values interdependent. Since Vincent (1989), it has been recognized that interdependence in values can lead to bargaining

[4]Outside of the corporate finance literature, Lam (2020) studies the impact of steepness in a directed search environment with owners with heterogeneous assets and workers of privately known productivity. Lam (2020) characterizes the inefficiencies that arise as the market moves (exogenously) from cash transfers to output share (equity) payments; when asset owners are free to choose among securities, competition drives them to offer only cash payments.

inefficiencies and delay (see also Deneckere and Liang (2006)). Relative to those papers, we provide (i) an approach for solving bargaining models in non-cash offers, and (ii) a tractable model that allows for rich comparative statics with respect to the source of interdependence in values. (i) and (ii) together make it possible to analyze many real-world issues in corporate negotiations that are beyond the scope of previous bargaining models. Finally, our paper belongs to a recent literature that adds tractability and richness to discrete-time models of bargaining with asymmetric information (e.g., Fudenberg et al., 1985; Gul et al., 1986; Fuchs and Skrzypacz, 2010) by reformulating them directly in continuous time. The original contributions by Ortner (2017) and Daley and Green (2020) used models with discrete types and driving Brownian process (changing costs in the former, news about the informed party's type in the latter). These formulations were adapted to continuous-type Coasean bargaining (without driving Brownian processes) in Chaves (2019). While some of the dynamics in Daley and Green (2020) resemble ours (smooth trade, followed by an atom of trade), their model does not generate an impasse phase. The forces leading to gradual trade are also different.

2 Setup

Players, Protocol, Payoffs: A buyer (he) and a seller (she) negotiate over the terms of a joint project, the rights to which initially rest with the seller, but which only the buyer can undertake. Both players discount the future at rate $r > 0$. The buyer has a privately observed type $\theta \sim U[0, 1]$ that affects both his disagreement payoff and his payoff from agreeing. Before agreement is reached, the seller enjoys a flow payoff of $rc, c > 0$ and the buyer enjoys a flow of $rA(\theta)$. A is mnemonic for "assets in place," which are the source of the buyer's disagreement flow payoffs. The project, once undertaken, generates random cash flows with present value $\tilde{V}$ that is affiliated with θ. Hence, once the buyer owns the project, he enjoys a present value $V(\theta) := \mathbb{E}[\tilde{V}|\theta]$ in expectation, gross of payments.

As in other models of Coasean bargaining, the seller makes offers of payment terms to the buyer, who at each point in time chooses whether to accept or reject. Unlike those models, the seller makes offers in a one-dimensional family of contingent payments, which we call the *security family* (sometimes shortened to "the security"). Concretely, when the buyer accepts an offer $\alpha \in [\underline{\alpha}, \bar{\alpha}] \subset \mathbb{R}$, he commits to a contingent payment $S(\alpha, \tilde{V})$ as a function of the project's value. Conditional on his type, he then expects to pay $\bar{S}(\alpha, \theta) := \mathbb{E}[S(\alpha, \tilde{V})|\theta]$. The function S is fixed throughout the negotiation; different offers by the seller therefore correspond to different indices in $[\underline{\alpha}, \bar{\alpha}]$.

Altogether, if the buyer with type θ accepts an offer α at time t, the seller's expected payoffs are

$$(1 - e^{-rt})c + e^{-rt}\bar{S}(\alpha, \theta), \tag{1}$$

while the buyer's are

$$(1 - e^{-rt})A(\theta) + e^{-rt}\left(V(\theta) - \bar{S}(\alpha, \theta)\right) \tag{2}$$

Let $R(\theta) := V(\theta) - A(\theta)$ denote buyer θ's net return on the project. We impose the following restrictions:

Assumption 1.

1. $R(\theta) \geq c \, \forall \theta$, strictly so for $\theta > 0$.
2. $\bar{S}(\bar{\alpha}, \theta) \geq R(\theta) \geq c > \bar{S}(\underline{\alpha}, \theta) \, \forall \theta$.
3. $\bar{S}$ is strictly increasing in α for all θ, and in θ for all $\alpha > \underline{\alpha}$.
4. $V(\theta)$ and $A(\theta)$ are C^1 and $\bar{S}$ is C^2.

Condition 1 says that there are gains from trade with every type of buyer. Below we distinguish between the *gap* ($R(0) > c$) and *no gap* ($R(0) = c$) cases (Fudenberg et al., 1985; Gul et al., 1986). Condition 2 is a non-degeneracy assumption. It ensures that the expected payment is sufficiently variable as a function of the offer: in a one-shot game, there exist offers so unfavorable that any player would definitely reject, and offers so favorable that any player would definitely accept. The assumption would be trivially satisfied if the parties were bargaining in an unrestricted amount of cash. Condition 3 is DeMarzo et al. (2005)'s notion of ordered securities with minor modifications. The first part ($\bar{S}_\alpha > 0$) is just an ordering assumption on offers, such that higher offers correspond to strictly higher expected payments; the second ($\bar{S}_\theta(\alpha, \theta) > 0, \alpha > \underline{\alpha}$) says that higher types are strictly good news for the seller.[5]

An important object for the analysis is $\alpha^f(\theta)$, the solution to

$$V(\theta) - \bar{S}(\alpha^f(\theta), \theta) = A(\theta). \tag{3}$$

This is the highest take-it-or-leave-it offer that type θ would consider accepting. As a mnemonic, the superscript on α^f stands for "final."

<u>Adverse vs Favorable Selection</u>: Next, we describe the parameters that govern the sorting of types in the model.

Definition 1 (Selection)**.** Say "selection is adverse under S" if

[5] Conditions 3-4 are satisfied, for example, if $S(\alpha, \tilde{V})$ is weakly increasing in both arguments, $\tilde{V} - S(\alpha, \tilde{V})$ is weakly increasing in $\tilde{V}$, and $\tilde{V}|\theta$ admits a conditional density $g_V(v|\theta)$ that satisfies strict MLRP and is twice-differentiable in both arguments, with $vg_V(v|\theta)$, $\left|\frac{\partial}{\partial\theta}g_V(v|\theta)\right|$, and $\left|\frac{\partial^2}{\partial\theta^2}g_V(v|\theta)\right|$ integrable on $v > 0$. This is Lemma 1 in DeMarzo et al. (2005). Under those assumptions, V is also strictly increasing and smooth.

$$\iota^S(\theta,\alpha) := -\frac{R(\theta)-\bar{S}(\alpha,\theta)}{\bar{S}_\alpha(\alpha,\theta)} \tag{4}$$

is strictly increasing in θ for every α. The selection is *severely* adverse if, moreover,

$$\mathbb{E}[\bar{S}(\alpha^f(1),\theta)] < c, \tag{5}$$

in which case we can define the "critical type" k^{SL} as

$$k^{SL} = \inf\{k \le 1 : \mathbb{E}\left[\bar{S}(\alpha^f(1),\theta)|\theta \in [k,1]\right] \ge c\}.$$

If instead ι^S is decreasing in θ for every α, selection is *favorable*. The game satisfies the "skimming property" if selection is either adverse or favorable.

The quantity $\iota^S(\theta,\alpha)$ is the marginal rate of substitution between delay (t) and offer (α) in the buyer's utility in (2). When ι^S is increasing in θ, the indifference curves of low types in (α,t) space cross the indifference curves of high types from below, and high types are more willing to trade off additional delays in order to get better offers. Selection is therefore "adverse" in that the types least valuable to the seller are more likely to accept (vice-versa for "favorable" selection when $\theta \mapsto \iota^S$ is decreasing).[6] Condition (5) for severe adverse selection says that, at the take-it-or-leave-it offer that all types would accept, the seller cannot break even in a one shot game. Having all types accept is, from the seller's perspective, the best possible combination of types that she could hope for under adverse selection, so selection is "severely adverse" when (5) holds.

Equilibrium Notion:

For environments that satisfy the skimming property, the seller's posterior is always a truncation of the prior, and the truncation point forms a natural state for the game. When selection is adverse, the state k is a *left* truncation: if the state at time t is $K_t = k$, then, given the history of offers and rejections, the seller believes $\theta > k$. When selection is favorable, k is a *right* truncation, i.e., the seller believes $\theta < k$. We focus on a tractable class of equilibria that are Markovian in the truncation (henceforth, "the cutoff"). To simplify the exposition, we describe the equilibrium notion for the adverse selection case, later explaining the minor changes needed when selection is favorable.

We first give a brief verbal description of the equilibrium notion. Following recent for-

[6] With cash bargaining, a standard argument (Fudenberg et al., 1985) establishes a "skimming" property: high types dislike delay more than low types but like cash equally well; therefore if a type θ is indifferent between accepting and rejecting an offer p after history H_t, all types $\theta' > \theta$ strictly prefer to accept p at H_t. This argument breaks down when bargaining in non-cash securities because the buyer's true type affects his expected payment, so "skimming" must be rederived from scratch (Lemma 4 in the Online Appendix). When bargaining in equity, for example, high buyer types dislike delay more, but they also dislike giving up their equity more; whether high or low θ's are more willing to reject offers depends on the primitives, as first noted by Che and Kim (2010). See Example 1.

mulations of Coasean bargaining in continuous time (see Ortner (2017), Daley and Green (2020), and, most relevant for the current setup, Chaves (2019)), the buyer's chooses a (type-specific) reservation offer strategy $\alpha(\cdot)$. On path, the seller chooses how fast to screen through buyer types, knowing that to screen through types $\theta < k$, she must offer $\alpha(k)$. Hence, the seller chooses paths of belief cutoffs $t \mapsto K_t$, which result in paths of offers $t \mapsto \alpha(K_t)$. We also give the seller an option to "give up on screening": she can make a pooling offer $\alpha(1)$ that would be accepted by all remaining types, thereby ending the game. Finally (unlike Ortner (2017), Daley and Green (2020), and Chaves (2019)), the buyer's strategy $\alpha(\cdot)$ must sometimes be discontinuous in equilibrium. We therefore augment the seller's strategy space *off-path*: after the rejection of an off-path offer $\alpha' \notin \alpha([0,1])$, we give the seller the ability to randomize over offers in a way that depends on α'. In the lingo of the discrete time literature, our equilibrium will be "weak Markov" (see Fudenberg et al. (1985) and Gul et al. (1986) for the origins of this "Weak Markov" approach).

The technical detals are as follows. First, the set of all measurable non-decreasing paths is an unmanageably large strategy space for the seller. Using the approach in Chaves (2019), we impose some restrictions on seller strategies that make the analysis tractable while still allowing a rich set of dynamics.

Definition 2 (Seller Strategy Space)**.**

1. A *plan* of on-path offers by the seller consists of a non-decreasing cutoff path $t \mapsto K_t$ and a stopping time T at which to make a pooling offer $\alpha(1)$. We denote an entire cutoff path $(K_t)_{t\geq 0}$ by K. K is *admissible* if it has no singular-continuous parts. We allow for mixed strategies in the stopping time T, which are represented by a CDF $F = (F_t)_{t\geq 0}$. Thus a plan for the seller is given by a pair (K, F), and we denote by $\mathcal{A}_k^U$ the set of admissible plans (K, F) satisfying $K_{0^-} = k$, i.e., with initial value k, and generic element K^k. The stopping time T is *Markov* if its hazard measure $dF_t/(1-F_{t-})$ is a function of K_{t-}.
2. Time intervals $[\underline{t}, \bar{t})$ where K and F are absolutely continuous are *smooth trade regions*. For such regions, $\dot{K}_t$ is the (a.e.) *trading speed* and γ_t denotes the (a.e.) hazard rate $\frac{dF_t}{dt}(1-F_t)^{-1}$. A special case of a smooth trade region is a *quiet period*, i.e., an interval $[\underline{t}, \bar{t})$ with $\dot{K}_t = \Delta K_{t^-} = 0$.
3. An on-path plan is supplemented by an off-path plan. For any off-equilibrium offer $\alpha' \notin \alpha([0,1])$ made at time t, we let $\sigma_t(\alpha') \in \Delta([0,1])$ be the randomized offer that "immediately" follows the rejection of α'.

In the third item, we "stop the clock" after an off-equilibrium offer is made, and we allow the seller to respond to the rejection of an off-path offer before restarting the clock.[7]

Definition 3 (Buyer and Seller Problems)**.** At state k, a buyer type θ takes $\alpha(\cdot)$ and K as given, and solves

$$\sup_{\tau\in\mathcal{T}} \mathbb{E}\left[(1-e^{-r(\tau\wedge T)})A(\theta)+e^{-r(\tau\wedge T)}\left(V(\theta)-\bar{S}(\alpha(K_{\tau\wedge T}),\theta)\right)\right] \tag{6}$$

where by definition $K_T = 1$, and $\mathcal{T}$ is the set of stopping times adapted to the filtration generated by T. Meanwhile, the seller takes $\alpha(\cdot)$ as given. Given any path Q and realization of the stopping time T, her payoff is

$$\begin{aligned}\Pi(Q,T) := &\int_0^T e^{-rt}\mathbb{E}\left[\bar{S}\left(\alpha(Q_t),\theta\right)\Big|\theta\in[Q_{t-},Q_t]\right]dQ_t\\ &+e^{-rT}\mathbb{E}\left[\bar{S}\left(\alpha^f(1),\theta\right)\Big|\theta\in[Q_{T-},1]\right]+\left(1-(1-Q_T)e^{-rT}-\int_0^T e^{-rt}dQ_t\right)c,\end{aligned}$$

and, at each k, her strategy (Q,F) solves

$$\sup_{(Q,F)\in\mathcal{A}_k^U}\int_0^\infty \Pi(Q,T)dF(T). \tag{7}$$

We can now fully define a weak Markov equilibrium.

Definition 4 (Equilibrium)**.** A weak *Markov Equilibrium* of an adverse selection game consists of a tuple

$$(\{K^k\}_{k\in[0,1]},F,\alpha(\cdot),\sigma(\cdot|\cdot,\cdot))$$

together with a value $J(\cdot)$ for the the seller and a value $B(\cdot,\cdot)$ for the buyer such that

1. For all $\theta\in[0,1]$, $k\in[0,1]$, accepting at $\tau^*=\inf\{t:\alpha(K_t^k)\le\alpha(\theta)\}$ solves the buyer's problem (6) and delivers value $B(\theta,k)$.
2. $\alpha(1)=\alpha^f(1)$, where α^f is defined in (3).
3. For all $k\in[0,1]$ and T in the support of F, K^k is an admissible path and T is a Markov stopping time that together solve (7) and deliver value $J(k)$.
4. For any point of discontinuity of $\alpha(\cdot)$, k', and any off-equilibrium offer $\alpha'\in(\alpha(k'+),\alpha(k'-))$, $\sigma(\cdot|k',\alpha')$ maximizes[8]

[7] The discrete-time "Weak Markov" equilibria in Fudenberg et al. (1985) and Gul et al. (1986) sustain Markovian behavior on path by prescribing randomization immediately following the rejection of an off-path offer. In continuous time, there is no "next" period immediately after a seller deviation, which is why a different formalization, somewhat outside the continuous time formulation, is needed. See Smith and Stacchetti (2002) and Fanning (2016), who use the technique of "stopping the clock" to allow for multiple sequential moves in a continuous time bargaining games.

An alternative approach is to follow the formalization in Fudenberg and Tirole (1985) and to consider "intervals of consecutive atoms."

[8] Here, $\alpha^{-1}(\cdot)$ represents the generalized inverse defined as $\alpha^{-1}(y)\equiv\sup\{x>0:\alpha(x)\ge y\}$.

$$\int_0^1 \left\{(\alpha^{-1}(\tilde{\alpha}) - k')^+ \mathbb{E}\left[\bar{S}\left(\tilde{\alpha}, \theta\right) \middle| \theta \in [k', \alpha^{-1}(\tilde{\alpha}) \wedge k']\right] \right.$$
$$\left. + (1 - \alpha^{-1}(\tilde{\alpha})) J\left(\alpha^{-1}(\tilde{\alpha})\right)\right\} d\sigma(\tilde{\alpha}|k', \alpha')$$

5. For any point of discontinuity of $\alpha(\cdot)$, k', and any off-equilibrium offer $\alpha' \in (\alpha(k'+), \alpha(k'-))$, $\sigma(\cdot|k', \alpha')$ satisfies

$$V(k') - \bar{S}(\alpha', k') \leq B(k', k') \int_0^{\alpha(k')} d\sigma(\tilde{\alpha}|k', \alpha')$$
$$+ \int_{\alpha(k')}^1 \left(V(k') - \bar{S}(\tilde{\alpha}, k')\right) d\sigma(\tilde{\alpha}|k', \alpha')$$

Condition 2 is a natural refinement inspired by the corresponding discrete time game.[9] In a stationary equilibrium of the discrete time game, for any positive period length, the seller would never offer more than $\alpha^f(1)$ when her beliefs are concentrated at $\theta = 1$. (And $\alpha(1)$ can never be above $\alpha^f(1)$, since $\theta = 1$ would strictly prefer to reject). Conditions 4 and 5 say that, when the seller makes an off-path offer "by mistake", the buyer still accepts according the reservation offer curve $\alpha(\cdot)$, and after making the mistake, the seller randomizes in way that justifies the buyer's choice of accepting according to $\alpha(\cdot)$ (Fudenberg et al., 1985; Gul et al., 1986).

Finally, to streamline the derivation of necessary conditions, we restrict our search for equilibria to an amenable subclass, along the lines in Chaves (2019):

Definition 5 (Regularity)**.** A weak Markov Equilibrium is *regular* if

1. J is continuous.
2. In any interval (k', k'') for which smooth trade is prescribed at all $k \in (k', k'')$, J is C^1 and α is continuous.
3. $\dot{K}_t$ is continuous in the interior of smooth trade regions.
4. Jump discontinuities in cutoff paths are *isolated*.

Below, we refer to regular weak Markov Equilibria as simply "equilibria."

Remark 1 (Modifications for Favorable Selection)**.** When selection is favorable, equilibria are defined almost identically, with the following changes:

1. Admissible paths $t \mapsto K_t^k$ are non-*increasing* and satisfy $K_0^1 = 0$. The admissible set at state k is denoted $\mathcal{A}_k^D$.
2. Condition 2 in Definition 4 becomes $\alpha(0) = \alpha^f(0)$.

[9]See the discussions in Ortner (2017) and Daley and Green (2020), who impose conditions similar to our Condition 1; Ortner (2017) shows that, absent this kind of refinement, continuous time equilibria can violate this natural discrete-time property.

3. The seller's objective for given (Q, T) is now written

$$\Pi(Q,T) = \int_0^T e^{-rt}\mathbb{E}\left[\bar{S}\left(\alpha(Q_t),\theta\right)\middle|\theta \in [Q_t, Q_{t-}]\right] d(1-Q_t)$$
$$+ e^{-rT}\mathbb{E}\left[\bar{S}\left(\alpha^f(0),\theta\right)\middle|\theta \in [0, Q_{T-}]\right]$$
$$+ \left(1 - Q_T e^{-rT} - \int_0^T e^{-rt} d(1-Q_t)\right) c, \quad (8)$$

3 Dynamics for General Securities

For our class of equilibria, we can fully characterize equilibrium dynamics. Here we provide an informal derivation when selection is severely adverse, relegating the full proof of necessary conditions and equilibrium verification to the appendix.

We construct an equilibrium where the game starts with smooth trade. By the usual Coasean logic, whenever the seller is trading smoothly, her payoff is pinned down at c: otherwise, she would have strict incentives to speed up trade. To wit, the HJB equation in the smooth trading region is given by

$$rJ(k) = \sup_{\dot{k}\geq 0}\left\{\left(\bar{S}(\alpha(k),k) - J(k)\right)\frac{\dot{k}}{1-k} + J'(k)\dot{k} + rc.\right\} \quad (9)$$

The choice variable $\dot{k}$ enters the HJB in an affine way. Hence, if trade is happening at a positive speed ($\dot{k} > 0$ is optimal), the coefficients on $\dot{k}$ must cancel. It follows that $J(k) = c$, and $\bar{S}(\alpha(k), k) = c$, i.e., the seller exactly breaks even, conditional on trading with type k.

The seller is therefore indifferent among speeds of trade in a smooth trading region, and the speed of trade at state k is pinned down instead by the marginal buyer $\theta = k$ and his incentives to delay. Ignoring second-order effects, if the buyer $\theta = K_t$ waits an additional dt units of time before accepting, he suffers discounting costs

$$rdt\left(V(K_t) - \bar{S}(\alpha(K_t), K_t)\right).$$

While waiting he receives flow utility $rdtA(K_t)$, and the price he faces improves by

$$\alpha'(K_t)\bar{S}_\alpha(\alpha(K_t), K_t)\dot{K}_t dt.$$

Setting marginal costs equal to marginal benefits, the speed $\dot{K}_t$ must satisfy

$$\dot{K}_t = r\frac{R(K_t) - c}{-\alpha'(K_t)\bar{S}_\alpha(\alpha(K_t), K_t)}.$$

The numerator represents the gains from trade with type K_t, while the denominator represents the (absolute value of) the slope of expected payments with respect to the state.

Trade is therefore faster (i.e., types are skimmed more quickly) when the gains of trade are larger, and it is slower when the equilibrium expected payment changes more quickly with respect to the Markov state. Intuitively, when expected payments are more sensitive to the state, the buyer has a stronger incentive to "move the state along" by rejecting offers and misrepresenting his type. Incentive compatibility then requires that trade is slower.

We obtain a useful re-interpretation of the speed of trade by leveraging the seller's break-even condition $S(\alpha(K_t), K_t) = c$, which holds when trade is smooth. Totally differentiating on both sides with respect to K_t,

$$0 = \alpha'(K_t)\bar{S}_\alpha(\alpha(K_t), K_t) + \bar{S}_\theta(\alpha(K_t), K_t),$$

Plugging into our expression above, we have that, starting at a state k with smooth trade, K_t evolves according to the ODE

$$\dot{K}_t = r\frac{R(K_t) - c}{\bar{S}_\theta(\alpha(K_t), K_t)}, K_0 = k. \tag{10}$$

Hence—foreshadowing our steepness results—the speed of trade depends on the sensitivity of expected payments to the true type.

A natural guess is that, using (10), one can construct an equilibrium in smooth trade. However, smooth trading cannot persist indefinitely. If the seller were to continue screening more and more types, eventually the state would reach *and cross* k^{SL}. At that point, trading instantly with all remaining types at an offer they would all accept would become strictly more profitable than trading smoothly with the marginal type: for $k > k^{SL}$, $\mathbb{E}[\bar{S}(\alpha^f(1), \theta)|\theta \in [k, 1]] > c$.

Our construction therefore specifies smooth trade at $k < k^{SL}$, with each type trading separately at different offers, and an atom of trade at $k > k^{SL}$, with all remaining types $[k, 1]$ trading simultaneously at the (strictly lower) pooling offer $\alpha^f(1)$.[10] In order to make the types just below k^{SL} willing to trade at the higher offer rather than wait for the offer to drop discontinously, the seller imposes some additional delay. In equilibrium, she delays just long enough to make $\theta = k^{SL}$ indifferent between accepting $\alpha(k^{SL})$ "now" and rejecting it in hopes of receiving the pooling offer $\alpha^f(1)$ "later." The expected discount until $\alpha^f(1)$ is offered, denoted by δ^{SL}, must then solve

$$\underbrace{V\left(k^{SL}\right) - \bar{S}\left(\alpha(k^{SL}), k^{SL}\right)}_{\text{payoff from accepting } \alpha(k^{SL})} = \underbrace{(1 - \delta^{SL})A\left(k^{SL}\right) + \delta^{SL}\left(V\left(k^{SL}\right) - \bar{S}\left(\alpha^f(1), k^{SL}\right)\right)}_{\text{payoff from waiting for } \alpha^f(1)}, \tag{11}$$

[10]On the one hand, since the seller trades smoothly for $k < k^{SL}$, $\alpha(k^{SL}-)$ must satisfy $\bar{S}(\alpha(k^{SL}-), k^{SL}) = c$. On the other, given that all types in $(k^{SL}, 1]$ trade at the final offer $\alpha^f(1)$, $\mathbb{E}[\bar{S}(\alpha(k^{SL}+), \theta)|\theta \in (k^{SL}, 1]] = c$. It follows that $\alpha(k^{SL}+) < \alpha(k^{SL})$.

where $\bar{S}(\alpha(k^{SL}), k^{SL}) = c.$ [11] The seller can implement this delay in a Markov way by withholding the final offer until the first tick of a Poisson clock with a rate λ given by $\lambda/(r+\lambda) = \delta^{SL}$.[12]

Figure 2 illustrates typical realized paths of outcomes for this equilibrium.The cutoff rises gradually from 0 until it reaches k^{SL}, with the seller gradually dropping her offers from $\alpha(0)$ to $\alpha(k^{SL})$. When the state arrives at k^{SL}, the game reaches an impasse, with the cutoff frozen at k^{SL} for a random amount of time $T - \tau(k^{SL})$. During the impasse, the seller "stubbornly" refuses to move her offer from $\alpha(k^{SL})$, until finally, at a random time, she concedes, dropping her offer to $\alpha^f(1)$. At that point, all remaining types $\theta \in (k^{SL}, 1]$ accept suddenly, and the cutoff jumps to $k = 1$.

We have outlined the construction of one equilibrium, but in fact, this is the *only* equilibrium outcome. We prove the following characterization:

Theorem 1. *There exists a (regular weak Markov) equilibrium. If selection is severely adverse, there is a unique on-path equilibrium triple* $\left(\{K^k\}_{k\in[0,1]}, F, \alpha(\cdot)\right)$*:*

- *The buyer's acceptance strategy* $\alpha(\cdot)$ *is given by* $\bar{S}(\alpha(k), k) = c$ *for* $k \leq k^{SL}$*, and* $\alpha(k) = \alpha^f(1)$ *for* $k > k^{SL}$*.*
- *There is smooth trade for* $k \in [0, k^{SL})$*, with the cutoff path* K_t *given by* (10)*.*
- *At state* k^{SL}*, there is a breakdown of trade of random length. The seller keeps the cutoff constant, but at a Poisson rate* $\lambda = r\delta^{SL}/(1-\delta^{SL})$*, with* δ^{SL} *given by* (11)*, makes the pooling offer* $\alpha^f(1)$*.*
- *For* $k > k^{SL}$*, the seller immediately offers* $\alpha^f(1)$*.*

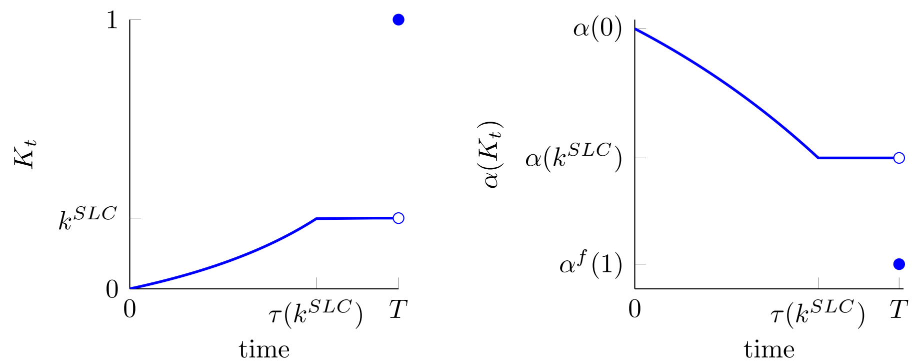
If selection is favorable, or if it is adverse but not severely so, all equilibria have instant trade at an offer of $\alpha^f(0)$ *and* $\alpha^f(1)$*, respectively.*

Remark 2. Notice that, if $R(0) = c$ (there is no gap at the "bottom"), the equilibrium outcome has a *complete breakdown of trade.* If $R(0) = c$, the unique solution to (10) with $K_0 = 0$ is $K_t = 0$, so the on-path, states above $k = 0$ are never reached.

Example 1 (Bargaining in Equity)**.** One has $\iota^S(\theta, \alpha) = -\left(R(\theta)V(\theta)^{-1} - \alpha\right)$). If R/V is increasing, selection is favorable, and there is instant trade at an offer of $\alpha^f(0) = \frac{R(0)}{V(0)}$. If R/V is decreasing, selection is adverse; when $\frac{\mathbb{E}[V(\theta)]}{c} \geq \frac{V(1)}{R(1)}$, there is instant trade at an offer of $\alpha^f(1) = \frac{R(1)}{V(1)}$. If R/V is decreasing but $\frac{\mathbb{E}[V(\theta)]}{c} < \frac{V(1)}{R(1)}$, there is gradual trade, with offers

[11] Such a $\delta^{SL} \in (0,1)$ always exists: since there are strict gains from trade, we have $\bar{S}(\alpha^f(1), k^{SL}) < c = \bar{S}(\alpha(k^{SL}), k^{SL}) < R(k^{SL}) = V(k^{SL}) - A(k^{SL})$.

[12] For these equilibrium dynamics, one can then use standard mechanism design arguments to show that it is globally incentive-compatible for buyers to accept from lowest to highest according to $\alpha(\cdot)$, and a verification approach shows that these screening dynamics are also optimal for the seller, given $\alpha(\cdot)$.



(a) Equilibrium path of cutoff K_t (b) Equilibrium path of offers $\alpha(K_t)$

Figure 2: Illustration of a realized equilibrium path in an environment with severe adverse selection environment. $\tau(\theta)$ denotes the realized time at which state θ is reached on-path.

$\alpha(\theta) = \frac{c}{V(\theta}$, followed by an impasse. Intuitively, if R/V is decreasing, then, as the buyer's type grows, his disagreement motive (i.e., the assets in place) grows proportionally faster than his agreement motive (i.e., the net surplus), and he becomes more likely to reject. The importance of the direction of increase of R/V was highlighted in Che and Kim (2010).

Next, we discuss some features of the equilibrium. Having seen the form of equilibrium, we can better understand the benchmark case of cash bargaining, and why cash cannot lead to delay. Gradual trade (more generally delay) requires simultaneously

- offers getting gradually more generous, so as to give the marginal buyer an reason to wait, and
- the seller's payoffs being pinned down to her outside option, offer by offer.

Heuristically, if types are trading gradually, the following relationship must hold across time:

$$\bar{S}(\overbrace{\alpha(K_t)}^{\text{offer decreasing over time}}, \underbrace{K_t}_{\text{type increasing over time}}) = c$$

The offers $\alpha(K_t)$ are decreasing over time, which cause $\bar{S}$ to decrease through its first argument. For the left hand side to be constant in time, $\bar{S}$ must therefore increase through its second argument to compensate. From this fact stem two conclusions. One, $\bar{S}$ must be strictly increasing in type, i.e., it cannot be cash, which is constant in type, and two, the marginal type must be increasing over time, i.e., selection must be adverse.

The impossibility of delay under favorable selection can also be seen a different way. When selection is favorable, the seller strictly prefers making a pooling offer and ending the game to getting her outside option: for any $k > 0$,

$$\mathbb{E}[\bar{S}(\alpha^f(0),\theta)|\theta \in [0,k]] > \bar{S}(\alpha^f(0),0) = R(0) \geq c,$$

where the strict inequality uses Assumption 1. This rules out the possibility of any smooth trade or quiet periods, since the seller's payoff in such cases would be exactly c. (Formal details in the appendix).

Next, since the equilibrium time at which any θ trades could be random, it is useful to have different statistical summaries of how much delay takes place on average. We use two different measures throughout. On the one hand, for our theoretical results, we define a type's *certainty-equivalent delay*, $\tau^{CE}(\theta)$. Let $\tau(\theta)$ denote the (possibly random) time at which time θ trades; then $\tau^{CE}(\theta)$ is the scalar that solves

$$\mathbb{E}[e^{-r\tau(\theta)}] = e^{-r\tau^{CE}(\theta)}.$$

That is, $\tau^{CE}(\theta)$ is the (deterministic) delay that imposes the same expected discounting costs on θ as the equilibrium time of trade; in that sense, it provides a welfare-relevant measure of bargaining frictions. Using Theorem 1, we obtain explicit expressions for τ^{CE} that come in handy below:

$$\tau^{CE}(\theta;L) = \begin{cases} \int_0^{\theta} \frac{\bar{S}_\theta(\alpha(s),s)}{r(R(s)-c)}ds, & \theta \leq k^{SL}, \\ \int_0^{k^{SL}} \frac{\bar{S}_\theta(\alpha(s),s)}{r(R(s)-c)}ds - \frac{\log \delta^{SL}}{r}, & \theta > k^{SL} \end{cases} \tag{12}$$

where δ^{SL} is given by (11). Alternatively, rather than measure delay from the buyer's perspective, we can summarize equilibrium delay from an outside observer's perspective through the *survival function*, i.e., the probability that the negotiation has not ended by time t. In the model, that probability is given by

$$\Pr(\tau \geq t) = \begin{cases} 1 - K_t & \text{if } t \leq \tau(k^{SL}) \\ (1-k^{SL})e^{-\lambda(t-\tau(k^{SL}))} & \text{if } t > \tau(k^{SL}) \end{cases} \tag{13}$$

where $\lambda = r\delta^{SL}/(1-\delta^{SL})$. In Section 5 we use the surival function to connect delay in the model to the length of negotiation in the data.

4 Means of Payment and Bargaining Dynamics

Our purpose is to understand how changes in the shape of the security affect the inefficiency and dynamics of the negotiation. Let *supp* $\tilde{V}$ denote the support of $\tilde{V}$.

Definition 6. Say a function $S : [\underline{\alpha}, \bar{\alpha}] \times \textit{supp}\, \tilde{V} \times [\underline{\gamma}, \bar{\gamma}]$ satisfies $S \in \mathcal{OAS}$ (short for "is a security class with ordered adverse selection") if

1. For every $\gamma \in [\underline{\gamma}, \bar{\gamma}]$, $S(\cdot, \cdot; \gamma)$ is a security that satisfies Assumption 1 and induces

severe adverse selection.

2. For any α_1, α_2, and $\gamma_1, \gamma_2 \in [\underline{\gamma}, \bar{\gamma}]$ with $\gamma_1 > \gamma_2$,

$$\bar{S}(\alpha_1, \theta; \gamma_1) = \bar{S}(\alpha_2, \theta; \gamma_2) \Rightarrow \bar{S}^1_\theta(\alpha_1, \theta; \gamma_1) > \bar{S}^2_\theta(\alpha_1, \theta; \gamma_2) \tag{14}$$

Point 1 ensures that, as the informational sensitivity parameter γ increases, the resulting security family continues to cause delay (a minimal condition for studying comparative statics on delay). Point 2 says that, as the one-dimensional parameter γ increases, $S(\cdot, \cdot, ; \gamma)$ becomes "steeper"—more responsive to the true type—in the sense defined by DeMarzo et al. (2005). Hence, in the main text, and for all our key applications, we study *parametric* increases in information sensitivity.[13]

In Section 5, we show how the sensitivity parameter γ and the class $\mathcal{OAS}$ can capture changes in various real world fundamentals, including (i) a buyer's pre-existing leverage in an M&A transaction, (ii) the precision of the buyer's information about synergies in such a transaction, and (iii) the maturity of the debt in a restructuring setting. Other tractable examples include[14]

Example 2 ($\mathcal{OAS}$ Examples)**.**

- Equity plus a fixed cash component $L - \gamma \in [0, c)$: $S(\alpha, \tilde{V}; \gamma) = \left(\bar{L} - \gamma\right) + \alpha\tilde{V}$.
- Cash plus royalty rate $\gamma \in [0, 1]$: $S(\alpha, \tilde{V}; \gamma) = \alpha + \gamma\tilde{V}$.

For both of these settings, if there is severe adverse selection for some γ', (i) there is severe adverse selection for every $\gamma'' > \gamma'$, and (ii), as γ rises, the security $S(\cdot, \cdot; \gamma)$ becomes more informationally sensitive in the sense of (14). In the first example, if c is sufficiently high and R/V is decreasing, there exists $\underline{\gamma} < L$ for which there is severe adverse selection. In the second, if c is sufficiently high and A is strictly increasing, there exists $\underline{\gamma} < 1$ for which there is severe adverse selection. Hence, starting from such a $\underline{\gamma}$ and taking $\bar{\gamma} = L$ in the first case and $\bar{\gamma} = 1$ in the second, $S \in \mathcal{OAS}$.

Proposition 1 (Inefficiency and Delay)**.** *Fix a security family $S \in \mathcal{OAS}$. Let $\tau^{CE}(\theta; \gamma)$ denote the* certainty-equivalent *delay for θ and $k^{SL}(\gamma)$ denote the critical type when the sensitivity parameter is γ. Then $\tau^{CE}(\theta; \gamma)$ is strictly increasing in γ for "low" and "high" types: for an increase from γ to $\gamma + \Delta$,*

- *$k^{SL}(\gamma + \Delta) < k^{SL}(\gamma)$, and*
- *all types $\theta \notin (k^{SL}(\gamma + \Delta), k^{SL}(\gamma)) \cup \{0\}$ suffer strictly greater delay.*

[13] In Online Appendix, we describe how many of our results extend to "non-parametric" comparisons of informational sensitivity (such as between equity and debt).

[14] It follows from Lemma 5 in DeMarzo et al. (2005) that higher γ's correspond to steeper securities in these examples.

The full proof requires some technical lemmas and is postponed to the appendix. The fact that $k^{SL}(\gamma) > k^{SL}(\gamma+\Delta)$, and that types below $k^{SL}(\gamma+\Delta)$ suffer greater delay, follow quickly from $S \in \mathcal{OAS}$. To avoid clutter, let $\bar{S}^1(\alpha,\theta) = \bar{S}(\alpha,\theta;\gamma+\Delta)$ and $\bar{S}^2(\alpha,\theta) = \bar{S}(\alpha,\theta;\gamma)$, with $\alpha^{f,i}(1), i = 1,2$ denoting the respective pooling offers under the two securities. By definition, the pooling offers $\alpha^{f,1}(1), \alpha^{f,2}(1)$ satisfy $\bar{S}^1(\alpha^{f,1}(1),1) = \bar{S}^2(\alpha^{f,2}(1),1) = R(1)$, which, by the greater steepness of S^1, implies that

$$\bar{S}^1(\alpha^{f,1}(1),\theta) < \bar{S}^2(\alpha^{f,2}(1),\theta) \text{ for all } \theta < 1. \tag{15}$$

Therefore, for all $k < 1$,

$$\overbrace{\frac{1}{1-k}\int_k^1 \bar{S}^1(\alpha^{f,1}(1),\theta)d\theta}^{\mathbb{E}[\bar{S}^1(\alpha^{f,1}(1),\theta)|\theta\in[k,1]]} < \overbrace{\frac{1}{1-k}\int_k^1 \bar{S}^2(\alpha^{f,2}(1),\theta)d\theta}^{\mathbb{E}[\bar{S}^2(\alpha^{f,2}(1),\theta)|\theta\in[k,1]]}, \tag{16}$$

and $k^{SL}(\gamma) < k^{SL}(\gamma+\Delta)$ follows.

Likewise, consider the gradual concessions phase. The buyer's incentive to reject an offer is that, by rejecting, he can affect the seller's beliefs about his type and can obtain a better price in the future. However, because of the Coasean force the seller's expected payment is constant in the state. Letting α^1 and α^2 denote the reservation offer curves for the bargaining games with securities S^1 and S^2, we have $\bar{S}^i(\alpha^i(k),k) = c$. *The change in price from a change in the seller's beliefs therefore exactly offsets the change in price from a change in the buyer's type*: for $i = 1,2$ and $\theta \le k^{SL}(S^2)$,

$$\bar{S}^i(\alpha(k),k) = c \Rightarrow \underbrace{-\overbrace{(\alpha^i)'(k)\bar{S}^i_\alpha(\alpha(k),k)}^{<0}}_{\text{Price improvement from changing seller's belief}} = \underbrace{\frac{\partial}{\partial\theta}\bar{S}^i(\alpha(k),\theta)\Big|_{\theta=k}}_{\text{sensitivity to }\theta\text{ at }\theta=k}$$

Hence, the more sensitive price is to the buyer's private information, the greater the price improvement that he expects from rejecting an offer, the greater his incentives to reject, and the slower trade must be. (Formally,since $\bar{S}^i(\alpha^i(k),k) = c$ implies $\bar{S}^1_\theta(\alpha^1(k),k) > \bar{S}^2_\theta(\alpha^2(k),k)$, the denominator in the speed of trade (10) is larger state-by-state for the more sensitive security.)

Propositon 1 shows that more informationally sensitive securities often destroy surplus. Our next result shows that this happens at the buyer's expense:

Proposition 2 (Buyer's payoffs)**.** *Fix a security family $S \in \mathcal{OAS}$. Increasing γ*

1. *Strictly hurts low types: an increase from γ to $\gamma+\Delta$ makes types*

$$\theta \in (0, \min\{k^*, k^{SL}(\gamma+\Delta)\}]$$

stictly worse off, where k^* solves $\bar{S}(\alpha^{f,\gamma}(1),k^*;\gamma) = c$;

2. *Strictly hurts all types* $\theta \in (0,1)$ *if either of these two conditions hold:*

   (a) $\varphi(\theta,\gamma) := R(\theta) - \bar{S}(\alpha^f(1;\gamma),\theta;\gamma)$ *is log-supermodular in* $(\theta,-\gamma)$*; or*

   (b) $R(0) > c$ *and* $R(1) = c$.

Said differently, types who pay less under the less sensitive security—those beneath k^*–always prefer to bargain with it, no matter whether they suffer higher or lower delay under that security. Easy-to-check sufficient conditions ensure that *all* types prefer bargaining in the flatter security, even when equilibrium requires them to pay more or suffer more equilibrium delay.[15]

The first sufficient condition does not follows directly from $S \in \mathcal{OAS}$, but it can be readily verified for particular families of securities. The second sufficient condition says that there is a gap at the bottom ($\theta = 0$) *but not at top* ($\theta = 1$). To see why it suffices, notice that when $c = R(1)$, $\bar{S}(\alpha^f(1,\gamma),1) = c$, so it must be that $\mathbb{E}[\bar{S}(\alpha^f(1,\gamma),\theta,\gamma)|\theta \in [k,1]] < c$ for all $k < 1$, and $k^{SL} = 1$, regardless of γ. The seller would never want to make a pooling offer, and all the non-monotonicities that the pooling brings about vanish. Hence, all types trade smoothly regardless of γ; since they pay c in either case, but suffer more delay when γ is higher, they are worse off with a more informationally sensitive security.

The condition $R(0) > R(1) = c$ requires the net return $R(\cdot)$ to be at least partly decreasing, which is somewhat uncommon in the literature. However, in many applications, there is nothing pathological about a strictly decreasing net return.[16] In the case of M&A, $R(\theta)$ measures the synergies from the merger, which can be higher or lower for high types. Suppose, for instance, that the buyer is acquiring a seller in order to gain access to the seller's proprietary technology, and θ measures how close the buyer is to the technological frontier. A higher θ would raise the expected value of assets in place $A(\theta)$ and may even raise the total value $V(\theta)$, but the marginal value of the seller's technology $R(\theta) = V(\theta) - A(\theta)$ can be lower the closer the buyer is to the technological frontier.

Remark 3. Many, but not all, of our theoretical results extend to arbitrary non-parametric comparisons between two severe-adverse-selection securities S^1, S^2 that satisfy a single-crossing "steepness"/informational sensitivity ranking of the form in (14). The ranking on delay always holds in the regions with smooth trading, the more sensitive security separates more types (has a higher critical type), and types below the critical type of the more

[15] Note that, under the conditions of Proposition 2, there are no types for whom the steeper security leads to strictly more certainty-equivalent delay *and* strictly higher payments. In that sense, there is a "delay-payment trade-off."

[16] We are grateful to Brett Green for suggesting this possibility.

sensitive security are made worse off by the higher sensitivity. Online Appendix B gives the precise results.

In our numerical examples, more sensitive securities decreases the buyer's payoffs uniformly even in circumstances where the sufficient conditions of Proposition 2 do not hold, and even when the ranking on delay is not uniform. That is, in our simulations, intermediate types prefer a less sensitive security even when this imposes greater negotiation delays on them.

5 Applications

5.1 Mergers and Acquisitions with Financial Constrains

Here we use our general results to study mergers and acquisitions under financial constraints. We consider a seller and a financially constrained acquirer who negotiate over the equity split in the merged entity. The acquirer carries some pre-existing debt that the merged company will have to assume. The offers therefore consist of *levered equity*:

$$S_{lev}(\alpha, \tilde{V}; d) = \alpha(\tilde{V} - d)^{+}, \alpha \in [0, 1], d \geq 0.$$

To match the utility specification in (2), we assume that the acquirer maximizes *total firm value* and not just the value of equity holders. This would be the case if there are covenants that require approval from debt holders. We write S^{d}_{lev} as a shorthand for $S_{lev}(\cdot, \cdot; d)$.[17]

We first study how the negotiation changes as the buyer/acquirer's financial constraints become tighter. Then, we show how the negotiation changes as the buyer's signal about future synergies becomes more precise.

For some of our results, we use the following convenient parametrization:

Example 3 (Normal-Linear Primitives)**.** The stand alone value is $A(\theta) = \chi\theta$, and the synergy value, conditional on θ, is distributed $\tilde{R}|\theta \sim \mathcal{N}(c + \Delta + \beta\theta, \eta^{-2})$, where $\Delta, \chi, \beta > 0$.

[17] Identical results hold in an alternative model for financial constraints, in which the acquirer has a limited amount of cash, fixed at the outset of the negotiation, that is added to the equity payment to the target (details available upon request). The impact of tighter financial constraints is the same as in the levered-equity model.

Insofar as cash is costly for firms—and costlier for the financially constrained ones— the amount of cash added to the equity payment parametrizes the acquirer's liquidity constraints. (For example, the acquirer might not have enough cash on hand to complete the transaction, or external financing might be prohibitively expensive). Even if the company has sufficient cash, the opportunity cost of depleting its cash reserves may outweigh the efficiency benefits from negotiating in a less informationally sensitive security. Indeed, there is empirical evidence that financial constraints limit the use of cash. For example, Alshwer et al. (2011) finds that financially constrained acquirers rely more on stock as a method of payment than financially unconstrained ones. Other empirical studies have found that, even when acquirers have enough cash to complete a transaction, they tend to use stock as a means of payment if they are financially constrained.

η is therefore the *precision* of the buyer's signal about synergies.

Effects of financial constraints on deal failure and M&A activity: Increasing the buyer's leverage (tightening its financial constraints) makes the underlying equity more informationally sensitive according to Definition 6.

Lemma 1. *Assume S_{lev} induces severe adverse selection for some debt level $\underline{d}$. Then $S_{lev} \in \mathcal{OAS}$ from Definition 6 with $\gamma = d$, $[\underline{\gamma}, \bar{\gamma}) = [\underline{d}, \infty)$.*

Lemma 1 and Proposition 1 imply that tighter financial constraints tend to worsen bargaining frictions and increase negotiation lengths. While the effect of informational sensitivity on bargaining frictions in Proposition 1 can be ambiguous for some intermediate types, but that ambiguity is greatly reduced when it comes to observable implications. This is illustrated in Figure 4, where we show the effect on delay from increasing leverage from $d = 5$ to $d = 10$. In the first panel, the higher leverage increases certainty-equivalent delay for types outside the shaded region, but decreases that delay—significantly—for types in the shaded region). In the second panel, however, one sees that for $d = 10$ the survival curve mostly lies above the one for the $d = 5$. The survival curves can cross in the tail of distribution. However, the tail has low probability mass and the difference between them is small. Therefore, in a sample of negotiation durations, we would still see negotiation length increase with leverage.

With a slight reinterpretation of the discounting cost r, our results also generate predictions for deal failures and M&A activity. If negotiations break down at a Poisson rate r, then $e^{-r\tau^{CE}(\theta)}$ is the probability of a negotiation failure for an acquirer of type θ. Our results then imply that a marginal tightening of financial constraints increases the probability of deal failure for all types outside a small intermediate region. This is broadly consistent with empirical studies that have looked at financial constraints and M&A activity, e.g. Malmendier et al. (2016) and Uysal (2011). The former study shows that successful acquisitions have a larger cash component, while the latter shows that over-levered firms (firms that are more levered than predicted by other covariates) are less likely to make an acquisition in the observation period.

The impact of rising uncertainty about synergies: We now show how bargaining outcomes change as the acquirer's signal about potential synergies becomes more precise. For example, it is typically easier for a merged firm to achieve cost reductions, say by economizing on fixed costs, than it is for the firm to raise revenue by expanding into new markets (Berk and DeMarzo, 2013). An acquirer who hopes to achieve cost efficiencies through a merger will therefore tend to have a more precise estimate of the potential synergies than one who hopes to exploit a particular kind of product market fit.

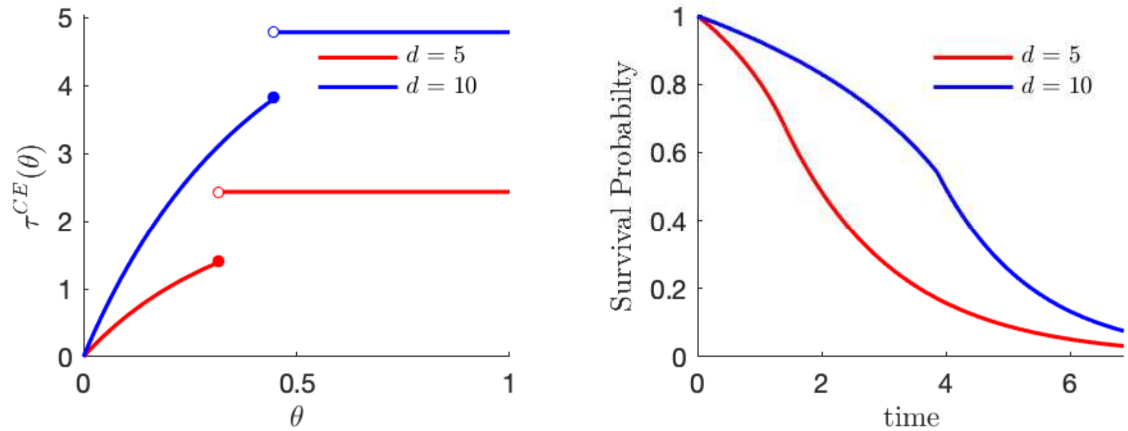


Figure 3: Impact of leverage on probability of acquisition. Parameters are $c = 5, \Delta = 1/2, \chi = 10, \beta = 1, \eta = 1/7, r = 1$.

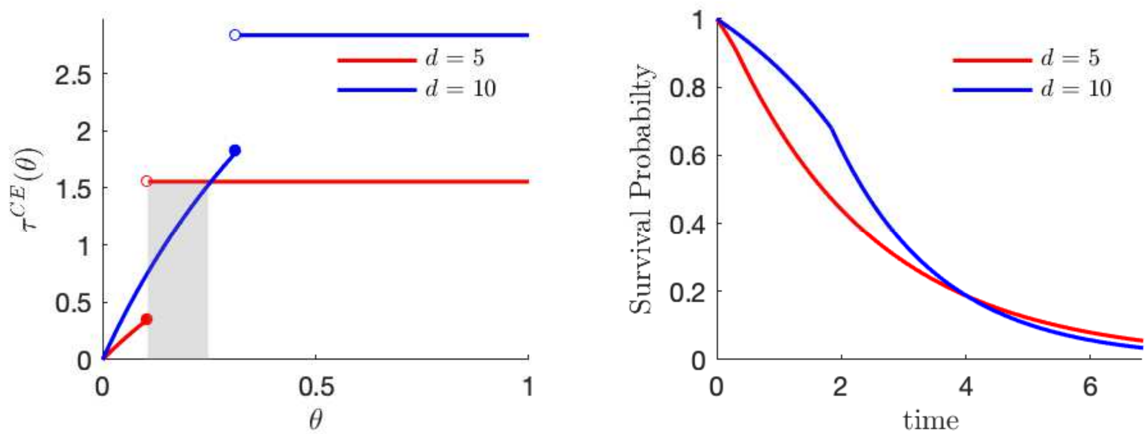


Figure 4: Impact of leverage on negotiation length. Parameters are $c = 5, \Delta = 1/2, \chi = 5, \beta = 1, \eta = 1/7, r = 1$.

Rising precision about synergies affects the negotiation through two different channels. On the one hand, for any offer α, a higher precision lowers the value of the levered equity: $\max\{\tilde{V} - d, 0\}$ is convex in $\tilde{V}$, and lowering η causes a mean-preserving spread of $\tilde{V}|\theta$. On the other, an initial intuition would suggest that a higher precision η will raise the signal-to-noise ratio for the buyer's signal θ, thereby making the levered equity more sensitive to θ. *A priori* it is not clear how what the full repercussions of these two effects may be. We show that, even though the slope of levered equity with respect to the buyer's type may go up or down in *absolute* terms, the net effect of increased precision is indeed a "heightened sensitivity": for a given debt level, raising the precision of the buyer's signal is equivalent to bargaining in a more informationally sensitive security:

Proposition 3. *Consider the levered equity model, with normal-linear primitives. Let*

$\bar{S}^d_{lev}(\alpha, \theta; \eta) := \alpha\mathbb{E}[(\tilde{V} - d)^+|\theta]$ *denote the expected payment as a function of offer, type, and precision.*

1. *For any $\eta_1 > \eta_2$, and α_1, α_2,*
$$\bar{S}^d_{lev}(\alpha_1, \theta; \eta_1) = \bar{S}^d_{lev}(\alpha_2, \theta; \eta_2) \Rightarrow \frac{\partial}{\partial\theta}\bar{S}^d_{lev}(\alpha_1, \theta; \eta_1) > \frac{\partial}{\partial\theta}\bar{S}^d_{lev}(\alpha_2, \theta; \eta_2) \tag{17}$$
2. *If there is non-trivial delay for some η, there is non-trivial delay for every $\eta' > \eta$.*
3. *Let $h(\cdot)$ be the hazard rate of the standard normal distribution. A sufficient condition for selection to be adverse for all d is*
$$\frac{\beta}{\beta + \chi} < \frac{\eta(c + \Delta)}{\eta(c + \Delta) + \eta(\beta + \chi) + h(-\eta(c + \Delta))}.$$

Points 1 and 2 imply that all the results from Section 4 apply unchanged: bargaining frictions rise for types outside an intermediate region and types below a threshold are harmed by the increase in precision.

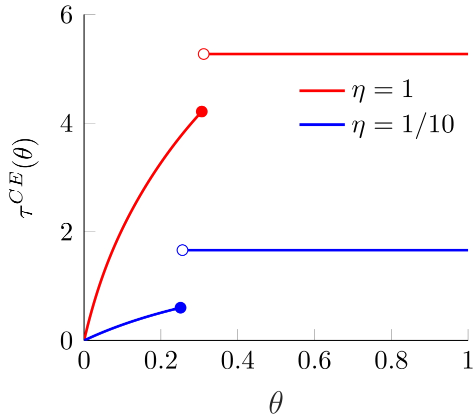
Figure 5 shows the changes in certainty-equivalent delay, expected payments $\bar{S}_{lev}(\alpha(\theta; d), v; d)$, and indirect buyer utilities as η increases from $\eta = 1/10$ to $\eta = 1$. The other parameters are held fixed at $c = 5, \Delta = 1/2, \beta = 1, \chi = 10, r = 1, d = 4$. With a higher precision about synergies, bargaining frictions—measured by τ^{CE}—rise for all types, and all types are made worse off.

Put differently, increasing the precision of the buyer's signal *destroys value*. The seller's equilibrium payoff is unaffected (since the game starts with smooth trade, the seller gets c regardless of the precision level), so this destruction is all at the buyer's expense. The buyer, in particular, would not want to invest in a technology that improves its prediction about synergies. If instead we interpret different η's as modeling different possible mergers—as in the cost reduction vs market expansion example—then our results imply that, all else equal, the buyer prefers to pursue mergers with more uncertain synergies because of their lower bargaining costs.

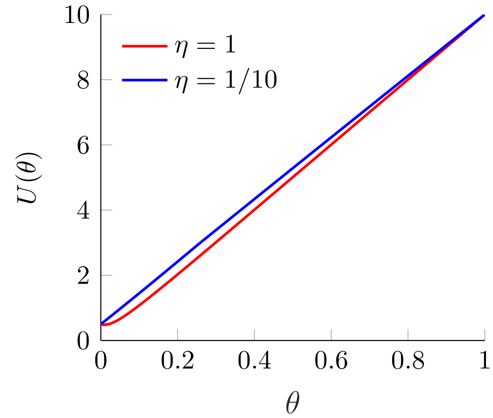
5.2 Corporate Restructuring

With minor modifications, our model applies to corporate restructuring negotiations of firms in financial distress. Even though there is significant evidence of inefficiencies in corporate restructuring negotiations, the literature by and large uses bargaining models that only generate efficient negotiations.[18] In contrast, with our model one can study the effect of

[18]Most models consider either Nash Bargaining or alternating offer with symmetric information (Bebchuk and Chang, 1992; Bernardo et al., 2016; Antill and Grenadier, 2019). Some models allow for inefficient liquidation/bankruptcy filing choices, but the negotiation conditional on those choices has no inefficient delay. As shown by Dou et al. (2021), most of the value destruction in restructuring comes from inefficient



(a) Certainty-Equivalent Delay



(b) Expected Payoff

Figure 5: Discounting costs and buyer payoffs as a function of signal precision. Parameters are $c = 5, \Delta = 1/2, \chi = 10, \beta = 1, r = 1, d = 4$.

different factors on the efficiency of negotiation. We consider the impact of the debtor's financial structure going into a restructuring (its assets in place) and of the maturity of the debt being negotiated.

We briefly elaborate on the institutional setting, and we describe how the model can be adapted to study restructuring. The identity of the negotiation parties depends on the circumstances. In some circumstances the negotiation mostly involves equity holders and debt holders, while in others, equity holders are wiped out and the negotiation is between junior and senior creditors. These negotiations can occur in a formal bankruptcy procedure (a Chapter 11 process), or outside of one (an out-of-court restructuring). The main advantage of an out-of-court restructuring is to avoid the costs (direct and indirect) of going through a formal bankruptcy procedure, which include the possibility of liquidation. There are multiple restructuring options depending on the severity of financial distress. The first option, when the firm is not severely distressed, is for equity holders to infuse new capital or repurchase debt, in exchange for some reduction in the value of current debt. The second option, when firms are severely distressed, is to exchange the current debt for fresh new debt and equity. The new debt usually has a lower face value and longer maturity (Altman et al., 2019).

As a particular example, consider a negotiation between equity and debt holders, and let the seller represent debt holders, and the buyer represent the current equity holders. Equity holders have current debt of face value f_0, and the negotiation is over the face value α of new debt that will be issued with a fixed maturity m. The type θ represents the equity holders' private information that determines both the value after restructuring and the expected recovery that can be obtained from a failed restructuring; this failure can

delay.

represent liquidation in an in-of-court negotiation, say from having to convert from Chapter 11 to Chapter 7 in an in-court negotiation, or it can represent to having to file for bankruptcy in an out-of-court negotiation. As a shorthand, we henceforth refer to a failed restructuring as "liquidation." The discount rate r can capture the probability that negotiations break down, leading to a failed restructuring, or it can capture in reduced form the deterioration in the value of the firm due to ongoing financial distress.

Specifically, the value from a failure to restructure is $\tilde{L} = L(\theta)e^{-\frac{\nu^2}{2}+\nu\hat{Z}}$ where $\hat{Z} \sim \mathcal{N}(0,1)$, and $L(\theta) = \mathbb{E}[\tilde{L}|\theta]$ is therefore the expected liquidation value conditional on θ. Thus, if the restructuring fails, creditors expect to receive $D^O(L(\theta), f_0) := \mathbb{E}[\min(\tilde{L}, f_0)|\theta]$, while equity holders expect to receive $L(\theta) - D^O(L(\theta), f_0)$, where

$$D^O(L(\theta), f_0) = L(\theta)\left(1 - \Phi\left[d_0(\theta, f_0)\right]\right) + f_0\Phi\left[d_0(\theta, f_0) - \nu\right],$$

$\Phi[\cdot]$ is the CDF of the standard normal and

$$d_0(\theta, f_0) := \frac{\ln\left(\frac{L(\theta)}{f_0}\right) + \frac{1}{2}\nu^2}{\nu}.$$

If equity holders agree to restructure the debt, the value of assets after the restructuring follows a geometric Brownian motion[19]

$$dV_t = \sigma V_t dB_t,\ V_0 = V(\theta)$$

where B_t is a Brownian motion independent of θ and $\hat{Z}$; at the time of debt maturity, equity holders default if the value of assets is below the face value α of the new debt. Hence, the value of the new debt conditional on θ is

$$D^N(V(\theta), \alpha) = \mathbb{E}[\min(V_m, \alpha)|\theta] = V(\theta)\left(1 - \Phi\left[d(\theta, \alpha)\right]\right) + \alpha\Phi\left[d(\theta, \alpha) - \sigma\sqrt{m}\right],$$

where

$$d(\theta, \alpha) = \frac{\ln\left(\frac{V(\theta)}{\alpha}\right) + \frac{1}{2}\sigma^2 m}{\sigma\sqrt{m}},$$

and the value of equity after the restructuring is $V(\theta) - D^N(V(\theta), \alpha)$. The expected payoff of debt holders from agreeing on a face value α at time t is therefore

$$(1 - e^{-rt})D^O(L(\theta), f_0) + e^{-rt}D^N(V(\theta), \alpha),$$

while the expected payoff of equity holders is

$$(1 - e^{-rt})\left(L(\theta) - D^O(L(\theta), f_0)\right) + e^{-rt}\left(V(\theta) - D^N(V(\theta), \alpha)\right).$$

[19]This specification for the value of debt and equity follows the valuation model of corporate risky debt with log-normal cash flows developed by Merton (1974).

We can map this setting to our general model. The security is debt $S(\alpha, \tilde{V}) = \min\{\alpha, \tilde{V}\}$, and the negotiation is over the face value α, in which case $\bar{S}(\alpha, \theta) = D^N(V(\theta), \alpha)$. The return $R(\theta) = V(\theta) - L(\theta) + D^O(L(\theta), f_0)$ then captures the equity holder's benefits from restructuring relative to liquidation. The outside options $A(\theta) = L(\theta) - D^O(L(\theta), f_0)$ and $c(\theta) = D^O(L(\theta), f_0)$ capture the liquidation value that the parties expect to obtain if the negotiation fails, which depend on their pre-existent claims to the assets.

Notice that, unlike our benchmark model, the seller's cost c is now a function of the equity holder's type θ. Our results can easily accommodate this situation with minor changes.[20] The critical type k^{SL} at which a pooling offer breaks even is now

$$k^{SL} = \inf\left\{k \leq 1 : \mathbb{E}\left[\bar{S}(\alpha^f(1), \theta) - c(\theta) | \theta \in [k, 1]\right] \geq 0\right\}.$$

Using the above expressions for $\bar{S}$ and c, k^{SL} therefore solves

$$\int_{k^{SL}}^{1} \left(D^N(V(k), \alpha^f(1)) - D^O(L(k), f_0)\right) dk = 0,$$

where the pooling offer satisfies

$$V(1) - D^N(V(1), \alpha^f(1)) = L(1) - D^O(L(1), f_0).$$

In the smooth trading region, $\theta \in [0, k^{SL}]$, the offer equilibrium offer $\alpha(\theta)$ is determined by the seller's break even condition $\bar{S}(\alpha(k), k) - c(k) = 0$, which amounts to

$$D^N(V(\theta), \alpha(\theta)) = D^O(L(\theta), f_0).$$

By now familiar arguments, delay for $\theta \in [0, k^{SL}]$ is given by

$$\tau(\theta) = \int_0^\theta \frac{V'(s) D_v^N(V(s), \alpha(s))}{r(V(s) - L(s))} ds = \int_0^\theta \frac{V'(s)\left(1 - \Phi\left[d(s, \alpha(s))\right]\right)}{r(V(s) - L(s))} ds, \tag{18}$$

and the duration of the final quiet period is given by

$$\delta^{SL} = 1 - \frac{D^N(V(k^{SL}), \alpha(k^{SL})) - D^N(V(k^{SL}), \alpha^f(1))}{V\left(k^{SL}\right) - L(k^{SL}) + D^O(L(k^{SL}), f_0) - D^N(V(k^{SL}), \alpha^f(1))}.$$

Finally, since expected payoffs are homogenous of degree one in $(V(\theta), L(\theta), f_0, \alpha(\theta))$, we normalize $f_0 = 1$ and re-interpret $V(\theta)$, $L(\theta)$, and $\alpha(\theta)$ as the continuation value of the firm, the liquidation value, and the restructured debt, relative to the company's initial liabilities of the company.

When is bargaining inefficient? Comparing sources of private information: Before looking at comparative statics in the terms of the debt contract and the firm's pre-existing financial structure, we begin by pinpointing the kinds of private information that

[20] An additional requirement is that $\bar{S}_\theta(\alpha(k), k) > c'(k)$, so that $\alpha(k)$ is decreasing. If $\alpha(k)$ is non-monotone, a Markovian equilibrium can be constructed using the techniques in Chaves (2019).

cause inefficient delay. In the model, private information affects both the expected value of restructuring, $V(\theta)$, and the expected recovery/liquidation value if negotiations fail, $L(\theta)$. These two sources of private information impact the equilibrium in different ways.

Proposition 4. *Consider the debt restructuring model, assuming that $L(\theta)$ and $V(\theta)$ are both non-decreasing, with at least one of them strictly increasing.*

- *If $L(\cdot)$ is constant, so that equity holders only have private information about the value of restructuring, then selection is favorable. There is no delay and debt is immediately restructured at a face value $\alpha^f(0)$*
- *If $V(\cdot)$ is constant, so that equity holders only have private information about the value of liquidation, then selection is adverse, but not severely so. There is no delay and debt is immediately restructured at $\alpha^f(1)$.*

The proof, which follows directly from plugging into ι^S, is omitted.

The negotiation is inefficient *only* if equity holders have private information about *both* the value of restructuring *and* the value of liquidation: private information about liquidation value makes high-type equity holders willing to delay negotiations, while private information about the value of restructuring makes a pooling offer that attracts all types of equity holder too costly for the debt holders.

The impact of assets in place:

Next, we study how negotiation outcomes are affected by changes to the debtor's assets in place at the time the restructuring starts. In the model, assets in place (net of liquidation costs) are measured through the liquidation value function $L(\cdot)$. Since $L(\cdot)$ is a high-dimensional parameter, we "increase" this parameter in two different ways: a uniform shift upwards, and a clockwise rotation around the function's endpoint $(1, L(1))$. Both shifts increase the liquidation value for all types, but the latter captures the possibility that raising the assets in place increases the liquidation value more strongly for low types than for high types.[21]

Figure 7 plots the impact of these changes on the negotiation survival function (calculated from (13)). In Panel (a), we raise the liquidation value $L(\theta)$ uniformly for all types, while in

[21] By similar arguments to the ones above, increasing $L(\cdot)$ will raise equilibrium delay for types below k^{SL}. The integrand in (18), $\frac{V'(\theta)\left(1-\Phi\left[d(\theta,\alpha(\theta))\right]\right)}{r(V(\theta)-L(\theta))}$, is increasing in $L(\theta)$ because

$$\frac{\partial}{\partial L(\theta)}\Phi\left[d(\theta,\alpha(\theta))\right] = \phi\left[d(\theta,\alpha(\theta))\right] d_\alpha(\theta,\alpha(\theta)) \frac{\partial \alpha(\theta)}{\partial L(\theta)} < 0.$$

Likewise, the size of the separating region increases with pointwise increases in $L(\cdot)$. Suppose that $L(\theta)$ is increasing in some parameter l. Differentiating implicitly the equation for k^{SL} we get

Panel (b), we *rotate* the liquidation value curve $L(\theta)$ clockwise around the point $(1, L(1))$. Both changes in the liquidation value $L(\theta)$ increase the length of negotiations, but the effect in Panel (b) is much more pronounced.

We can compare the model-predicted effects of increasing the liquidation value to its emprical counterpart of increasing the assets in place. Figure 6 shows the empirical survival function of a sample of Chapter 11 negotiations for large public companies in the U.S. We see that firms in the highest quartile of assets-to-debt ratios take significantly longer to emerge from bankruptcy than firms in the lowest quartile. The empirical survival function is uniformly higher for the first-quartile firms; within the high assets-in-place group, the median firm takes 391 days to emerge from bankruptcy, while the median firm in the low assets-in-place group takes 296 days. Directionally, either of the comparative statics with respect to liquidation value in Figure 7 is consistent with the empirically observed assets-in-place effect. The size of the empirical effect on negotiation length, however, is more in line with the comparison in Panel (b). Hence, we interpret the data as confirming that assets in place affect the liquidation value more strongly for low types, i.e., higher assets in place correspond to a clockwise rotation of $L(\theta)$.

The impact of debt maturity: Finally, we note that our model uncovers an unintended consequence of raising the maturity of debt. Diamond and He (2014) show that debt with longer maturity is more informationally sensitive.[22] Proposition 1 implies that *increasing the maturity increases bargaining frictions.* Figure 8 illustrates how increasing the maturity of debt increases delay (that is, it reduces the probability of reaching and agreement).

Raising the maturity of debt is typically seen as a way to spread the firm's payments more thinly over time, giving the firm a greater opportunity to improve performance and meet its debt obligations. However, our analysis shows that increasing the maturity has an indirect cost: it increases the frictions in the negotiation, reduces the probability of reaching an agreement, and therefore increases the probability of a failed restructuring including possibly liquidating the firm inefficiently.

$$\frac{\partial k^{SL}}{\partial l} = \frac{\int_{k^{SL}}^{1} \left(D_{\alpha}^{N}(V(k), \alpha^f(1)) \frac{\partial \alpha^f(1)}{\partial l} - D_L^O(L(k), f_0) \frac{\partial L(k)}{\partial l} \right) dk}{D^N(V(k^{SL}), \alpha^f(1)) - D^O(L(k^{SL}), f_0)}$$

The numerator is clearly negative. The denominator is negative as well, since $D^O(L(k^{SL}), f_0) = D^N(V(k^{SL}), \alpha(k^{SL}))$ and $\alpha(k^{SL}) > \alpha^f(1)$. Hence, $\frac{\partial k^{SL}}{\partial l} > 0$. The effect for types outside the separating region, and therefore the overall effect on the survival functions, is ambiguous.

[22] Diamond and He (2014) do not state their results directly in terms of informational sensitivity/steepness. However, the fact that debt with longer maturity is more sensitive follows from their Proposition 1, which states the following: If we let $D(V; F, m)$ be the value of debt with face value F and maturity m when the current value of assets is V, then, for any $m_2 > m_1$, $D_V(V; F_1, m_1) < D_V(V; F_2, m_2)$ whenever $D(V; F_1, m_1) = D(V; F_2, m_2)$. This corresponds exactly to the definition in (14).

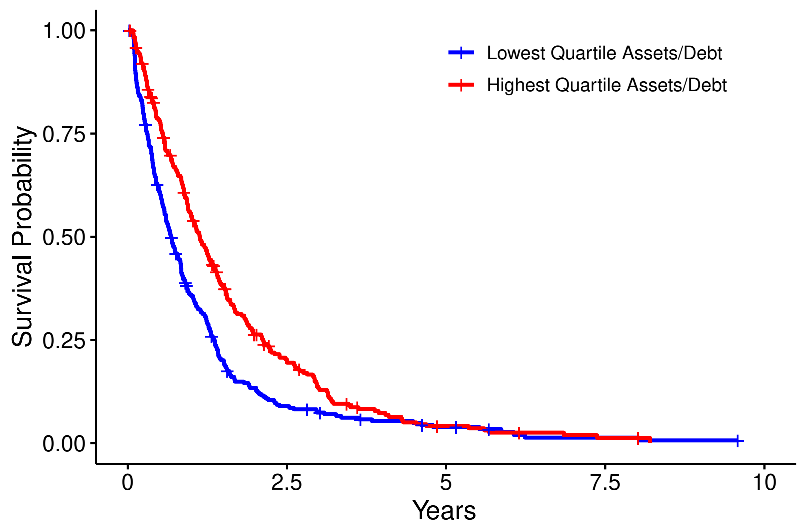
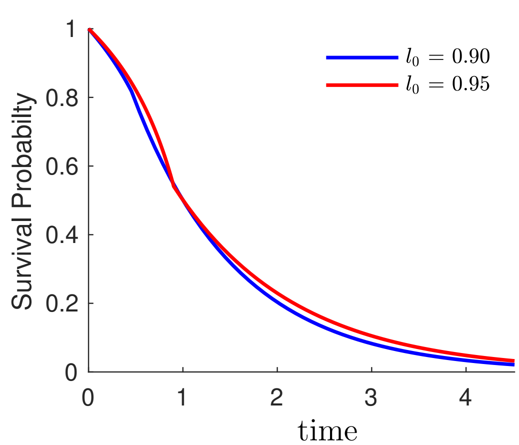
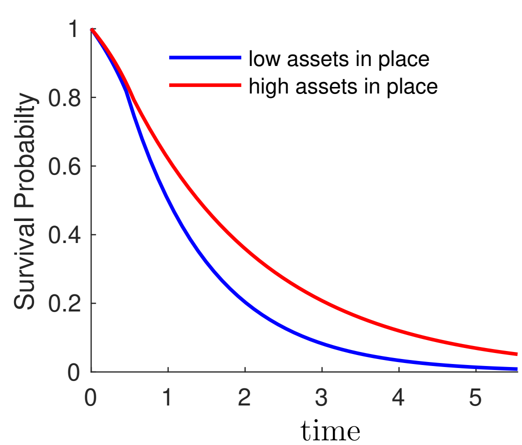


Figure 6: Effect of assets on survival probability for emergence from chapter 11 cases in the U.S. The survival curve is estimated using the Kaplan-Meier method, and adjusts for censoring arising due to dismissal of the case or transition to chapter 7 proceedings. Firms are sorted by asset/liabilities. Source: UCLA LoPucki Bankruptcy Research Database.

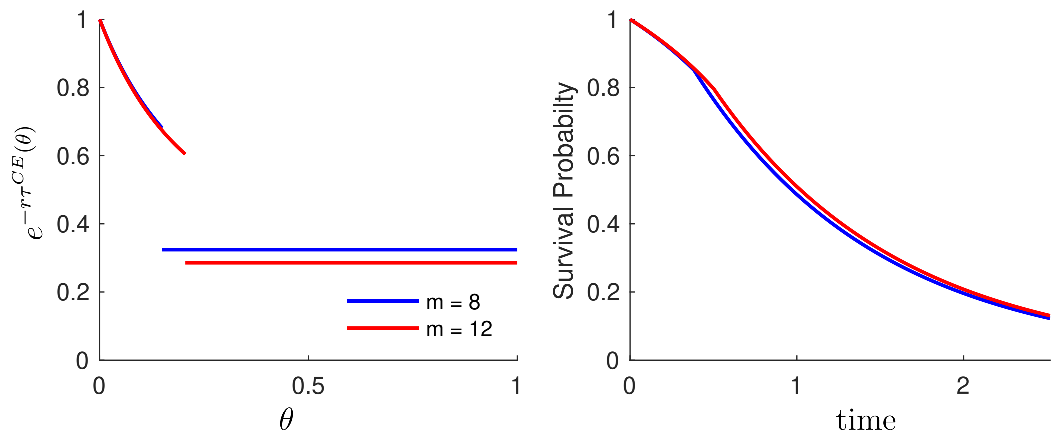


(a) Uniform increment in $L(\theta)$. Liquidation value $L(\theta) = l_0 + \theta$, $V(\theta) = 1 + 1.05 \times \theta$



(b) Increment in $L(0)$ keeping $L(1)$ constant. Liquidation value $L(\theta) = 0.9 + \Delta + (1 - \Delta)\theta$. The case with low assets in place corresponds to $\Delta = 0$ while the case with high assets in place corresponds to $\Delta = 0.05$

Figure 7: Impact of assets in place on survival probability. Parameters: $\sigma = 1$, $\nu = 0.2$, $m = 10$, $r = 1$, $f_0 = 1$, $V(\theta) = 1 + 1.05 \times \theta$.



(a) Probability of agreement (b) Survival Probability

Figure 8: Bargaining equilibrium and maturity. Parameters: $\sigma = 1$, $\nu = 0.2$, $f_0 = 1$, $r = 1$, $L(\theta) = 0.9 + \theta$, $V(\theta) = 1 + 1.05 \times \theta$.

6 Final Remarks

In this paper, we have studied the impact of securities in negotiations. While most of the bargaining literature focuses on pure cash transactions, many real-world negotiations are over the terms a contingent contract, that is, over the terms of a security. We explored two applications where such securities play a fundamental role: M&A and corporate restructuring. We have shown that, depending on the primitives, a security may generate adverse selection screening (high types like delay more) or favorable selection (low types like delay more). Delay only occurs when selection is not only adverse but *severely* so. In such a case, more informationally sensitive securities increase the amount of delay.

We have taken the security used as given, focusing on the bargaining dynamics while abstracting from the security design. Our results imply that using more informationally sensitive securities raises inefficient delay in negotiations. It is then natural to ask why parties would specify contingent payments using those securities and not others, and we return to that question now. First, there are often first-order institutional aspects that favor the use of an informationally sensitive security. For example, in the case of mergers and acquisitions, there are sizeable tax benefits to using equity rather than cash as a means of payment (Brown and Ryngaert, 1991). Similarly, even though cash transactions would be efficient, firms' financial/liquidity constraints can make it infeasible to negotiate over cash. The case of corporate restructuring is a clear example of this. A firm in a Chapter 11 bankruptcy proceeding—by definition—does not have enough cash to settle with creditors, and significant cash injections from equity holders (current or new) are unlikely due to debt overhang. Thus, restructuring necessarily involves a renegotiation of the terms of the firm's existent liabilities.

Second, the buyer and seller disagree often disagree over the use of inefficient securities, and (some types of) the buyer may prefer a more informationally sensitive security to a less informationally sensitive one. For instance, were the parties to choose a security that leads to favorable selection, there would be instant trade at an offer of $\alpha^f(0)$, and the lowest types would receive payoffs close to $\approx V(0) - \bar{S}(\alpha^f(0), 0) = V(0) - R(0)$. Meanwhile, if the parties were to choose a security with severe adverse selection, there would be delay, but the lowest types would still trade almost instantly, with an expected payment of $\approx c$ and expected payoffs around $V(0) - c > V(0) - R(0)$. The lowest types of buyer would therefore prefer a security that leads to inefficient negotiation.

Third, as we have shown, even securities that are very informationally insensitive, such as debt, can cause inefficient delay.

Fourth, in many circumstances, informationally sensitive securities might be necessary for incentives. For example, if the seller of the firm is needed to run it (as in the case of an entrepreneur selling a startup), then an informationally sensitive security might be needed to align the seller's ex-post incentives. If the seller posses some private information, then an informationally sensitive security can help signal that information to the market (Hansen, 1987).

Since the security in our model is fixed at the outset, the negotiation is only over the specific terms of that security. For example, in the case of a debt contract, the negotiation concerns the face value of debt but not other term of the contract such as the maturity. A more complex model would allow the buyer to offer different securities in each round. The buyer could initially offer to pay using equity, and if this offer is rejected by the seller, the counter-offer might entail the use of debt. What would the security family chosen by the buyer be, and whether the choice of family would change over time, are open questions for future work.

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Appendix: Proofs for Section 4

Throughout this section, S^1 and S^2 refer to two order securities in $\mathcal{D}_{A,\tilde{V},c}$, with S^1 steeper. An outline for this section is as follows. The ranking of critical types $k^{SL}(S^i)$ was proved in the text (Proposition 1.1). With that in hand, we prove Proposition 5, which ranks equilibrium expected payments by type. Using an envelope-like representation of equilibrium payoffs (Lemma 2), we prove Proposition 2, our ranking on equilibrium utilities by type. Finally, with the ranking on equilibrium utilities, we return to the issue of certainty-equivalent delay and prove Proposition 1.2, which gives ranks delay for types outside an intermediate region. Auxiliary lemmas are proved in Online Appendix C.

Proposition 5 (Expected payments are "single-crossing")**.** *Take S^1 and S^2 in $\mathcal{D}_{A,\tilde{V},c}$, with S^1 steeper. Let $\pi_i(\theta) := \bar{S}^i(\alpha^i(\theta),\theta)$, θ's equilibrium expected payment under S^i.*

1. *Under S^1, high types pay strictly less, low types pay more: there exists a unique $k^{cross} \in (k^{SL}(S^2),1)$ such that*

$$\begin{aligned} \pi_1(\theta) &= \pi_2(\theta), \quad \theta \in [0,k^{SL}(S^2)]. \\ \pi_1(\theta) &> \pi_2(\theta), \quad \theta \in (k^{SL}(S^2),k^{cross}). \\ \pi_1(\theta) &< \pi_2(\theta), \quad \theta \in (k^{cross},1). \end{aligned} \tag{19}$$

2. *Let k^* solve $\bar{S}^2(\alpha^{f,2}(1),k^*) = c$. Then $k^{cross} = \min\{k^*, k^{SL}(S^1)\}$.*

Since the expected payment of the highest type is the same under both securities ($\bar{S}^i(\alpha^{f,i}(1),1) = R(1)$), and high enough types accept $\alpha^{f,i}(1)$ in either case, they must pay less under the steeper security. At the same time, since flatter security has a larger pooling region, there are types at the bottom of the interval that would be separated under a steep security (and pay c), but get cross-subsidized by very high types when they face the flat security. They must therefore pay less strictly than c under the flat security. Finally, types at the bottom of the distribution are separated and pay c in either case.

Proof of Proposition 5. The first line of (19) follows from smooth trading: $\pi_i(\theta) = \bar{S}^i(\alpha^i(\theta),\theta) = c$ on $[0,k^{SL}(S^2))$. To show the latter two lines, for $\theta \in (k^{SL}(S^1),1)$, the equilibrium payment under S^i satisfies $\pi_i(\theta) = \bar{S}^i(\alpha^{f,i}(1),\theta)$; the inequality in (15) then becomes

$$\pi_1(\theta) < \pi_2(\theta), \theta \in \left(k^{SL}(S^1),1\right).$$

Meanwhile, by definition $k^{SL}(S^2)$ must satisfy

$$\mathbb{E}[\bar{S}^2(\alpha^{f,2}(1),\theta)|\theta \in (k^{SL}(S^2),1]] = c,$$

so there exists a $k^* \in (k^{SL}(S^2),1)$ such that

$$\bar{S}^2(\alpha^{f,2}(1),\theta)\begin{cases}< c, \theta \in (k^{SL}(S^2), k^*), \\ = c, \theta = k^* \\ > c, \theta \in (k^*, 1]\end{cases} \tag{20}$$

Since $\pi_1(\theta) = c$ for $\theta \le k^{SL}(S^1)$, if $k^* < k^{SL}(S^1)$, then (15) and (20) yield $k^{cross} = k^*$. On the other hand, if $k^* \ge k^{SL}(S^1)$, (15) and (20) yield $k^{cross} = k^{SL}(S^1)$. □

We use the following convenient representation of equilibrium payoffs:

Lemma 2. *Let $U_i(\theta)$ be the equilibrium indirect utility under a security S^i with adverse selection, and let α^i be the associated equilibrium offer. Define*

$$\nu^i(\theta,\alpha) := \frac{R'(\theta) - \bar{S}^i_\theta(\alpha,\theta)}{R(\theta) - \bar{S}^i(\alpha,\theta)}$$

Then for $\theta < 1$,

$$U_i(\theta) = A(\theta) + (R(0) - c)\exp\left\{\int_0^\theta \nu^i(y,\alpha^i(y))dy\right\}. \tag{21}$$

With (28), we can prove Proposition 2. We split the proof in two parts.

Proof of Proposition 2.1. Let $\tilde{\alpha}^i(\theta)$ denote the locus $\bar{S}^i(\tilde{\alpha}^i(\theta),\theta) = c$, and $\alpha^i(\theta)$ denote the equilibrium offer accepted by type θ under security i. Using Propositions 1 and 5, we have

$$\begin{aligned}&\alpha^1(\theta) = \tilde{\alpha}^1(\theta), \theta \in [0, k^{cross}] \\ &\alpha^2(\theta)\begin{cases} = \tilde{\alpha}^2(\theta), \theta \in [0, k^{SL}(S^2)] \\ < \tilde{\alpha}^2(\theta), \theta \in (k^{SL}(S^2), k^{cross})\end{cases}\end{aligned} \tag{22}$$

In addition, let $U_i(\theta)$ denote θ's equilibrium expected utility when bargaining with security S^i.

Since S^1 is steeper,

$$\nu^1(\theta,\tilde{\alpha}^1(\theta)) = \frac{R'(\theta) - \bar{S}^1_\theta(\tilde{\alpha}^1(\theta),\theta)}{R(\theta) - \bar{S}^1(\tilde{\alpha}^1(\theta),\theta)} < \frac{R'(\theta) - \bar{S}^2_\theta(\tilde{\alpha}^2(\theta),\theta)}{R(\theta) - \bar{S}^2(\tilde{\alpha}^2(\theta),\theta)} = \nu^2(\theta,\tilde{\alpha}^2(\theta)).$$

Integrating with respect to θ for $\theta < k^{cross}$, and using (22), one obtains

$$\begin{aligned}U_1(\theta) = A(\theta) + (R(0) - c)e^{\int_0^v \nu^1(y,\alpha^1(y))dy} &= A(\theta) + (R(0) - c)e^{\int_0^v \nu^1(y,\tilde{\alpha}^1(y))dy} \\ &< A(\theta) + (R(0) - c)e^{\int_0^v \nu^2(y,\tilde{\alpha}^2(y))dy} \\ &< A(\theta) + (R(0) - c)e^{\int_0^v \nu^2(y,\alpha^2(y))dy} = U_2(\theta),\end{aligned}$$

where the second inequality uses

$$\frac{\partial}{\partial\alpha}\nu^2(\theta,\alpha) \propto \frac{\partial}{\partial\alpha}\left(R'(\theta) - \bar{S}^2_\theta(\alpha,\theta)\right)\left[R(\theta) - \bar{S}^2(\alpha,\theta)\right] + \bar{S}^2_\alpha(\alpha,\theta)\left[R'(\theta) - \bar{S}^2_\theta(\alpha,\theta)\right] \propto -\frac{\partial}{\partial\theta}\iota^{S^2}(\theta,\alpha) < 0 \quad (23)$$

□

To prove the second part of Proposition 2, we need a quick technical lemma:

Lemma 3. *Let $S(\cdot,\cdot,\gamma)$ be a security belonging to a parametrized steepness class. Let $k^{SL}(\gamma)$ be the critical type under security $S(\cdot,\cdot;\gamma)$, and let $k^*(\gamma)$ solve $\bar{S}(\alpha^f(1;\gamma), k^*(\gamma);\gamma) = c$, where $\alpha^f(1;\gamma)$ is final offer under $S(\cdot,\cdot;\gamma)$. For $\hat{\gamma} > \gamma$ sufficiently close to γ, $k^*(\gamma) > k^{SL}(\hat{\gamma})$.*

Proof of Proposition 2.2. By Lemma 3, $k^*(\gamma) > k^{SL}(\hat{\gamma})$ for all $\hat{\gamma} > \gamma$ sufficiently close to γ, $k^*(\gamma) > k^{SL}(\hat{\gamma})$. By Proposition 2.1, buyer types in $[0, k^*(\gamma)]$ must then prefer $S(\cdot,\cdot;\gamma)$ to $S(\cdot,\cdot;\hat{\gamma})$, strictly so for $\theta > 0$.

Using the payoff representation in Lemma 2, we can write the indirect utility for type $\theta \in (k^*(\gamma), 1)$ under security $S(\cdot,\cdot;\gamma^\dagger), \gamma^\dagger \in \{\gamma,\hat{\gamma}\}$ as

$$U(\theta;\gamma^\dagger) = A(\theta) + U(k^*(\gamma);\gamma^\dagger)\exp\left\{\int_{k^*(\gamma)}^{v} \nu(y,\alpha^f(1;\gamma^\dagger);\gamma^\dagger)dy\right\}. \quad (24)$$

It follows that from the assumption that $\nu(\theta,\alpha^f(1;\gamma);\gamma)$ is decreasing in γ, and from $U(k^*(\gamma);\gamma) > U(k^*(\gamma);\hat{\gamma})$, that

$$U(k^*(\gamma);\gamma)\exp\left\{\int_{k^*(\gamma)}^{v} \nu(y,\alpha^\dagger(1;\gamma);\gamma)dy\right\} \geq U(k^*(\gamma);\hat{\gamma})\exp\left\{\int_{k^*(\gamma)}^{v} \nu(y,\alpha^\dagger(1;\hat{\gamma});\hat{\gamma})dy\right\}$$

Plugging this into the representation (24) above, $U(\theta,\gamma) \geq U(\theta,\hat{\gamma})$ for all $\theta \in [0,1]$, and strictly so for interior θ's. Since $\gamma \in [\gamma', \gamma'']$ was arbitrary, we conclude that for any $U(\theta,\gamma)$ is a weakly decreasing function of γ, and strictly decreasing for interior θ's. □

Proof of Proposition 1.2. Let $\tau^{CE}(1;\gamma)$ denote the certainty-equivalent delay for $\theta = 1$ under security $S(\cdot,\cdot;\gamma)$; we show that, for all $\gamma \in [\gamma_2,\gamma_1]$, $\tau^{CE}(1;\gamma)$ is strictly increasing in γ, which implies the result.

By Lemma 3, for $\hat{\gamma} > \gamma$ sufficiently close to γ, $k^*(\gamma) > k^{SL}(\hat{\gamma})$. By Proposition 2.1, for small enough $\xi > 0$, buyer $k^{SL}(\hat{\gamma}) + \xi$ must then strictly prefer $S(\cdot,\cdot;\gamma)$ to $S(\cdot,\cdot;\hat{\gamma})$. And yet, by Proposition 5, in expectation $k^{SL}(\hat{\gamma}) + \xi$ pays strictly less under $S(\cdot,\cdot;\gamma)$ than under $S(\cdot,\cdot;\hat{\gamma})$. It follows that $k^{SL}(\hat{\gamma}) + \xi$ must suffer strictly higher discounting costs under $S(\cdot,\cdot;\hat{\gamma})$. Since $k^{SL}(\hat{\gamma}) + \xi$ is in the final trading atom under $S(\cdot,\cdot;\gamma)$ *and* under $S(\cdot,\cdot;\hat{\gamma})$, $\tau^{CE}(1;\hat{\gamma}) > \tau^{CE}(1;\gamma)$, as required. □

Online Appendix

A Necessary Conditions

We begin with a key technical lemma characterizing the direction of skimming as a function of the primitives and the security family. The proof, which adapts classic arguments in Milgrom and Shannon (1994) and Edlin and Shannon (1998), is in Online Appendix C.

Lemma 4 (Skimming)**.** *Fix a security family S. Let ι^S be as in (4), U be given by*

$$U(t,\alpha,\theta) := (1-e^{-rt})A(\theta) + e^{-rt}(V(\theta) - \bar{S}(\alpha,\theta)),$$

and let $\alpha^f(\cdot)$ be as in (3).

Fix an arbitrary (deterministic) sequence of offers $\{\tilde{\alpha}_t\}_{t\geq 0}$, and let $T(\theta) := \arg\max_{t\in\mathbb{R}_+\cup\{+\infty\}} U(t,\tilde{\alpha}_t,\theta)$.

1. *If $\iota^S(\cdot,\alpha)$ is strictly increasing for all α, every selection from $T(\theta)$ is non-decreasing,*[23] *and $\alpha^f(\cdot)$ is strictly decreasing.*
2. *If $\iota^S(\cdot,\alpha)$ is strictly decreasing for all α, every selection from $T(\theta)$ is non-increasing, and $\alpha^f(\cdot)$ is strictly increasing.*

Remark 4 (Relation to Usual Skimming Notions)**.** Lemma 4 is weaker than the usual skimming result invoked in the literature on cash bargaining, so our focus on Markovian skimming equilibria is a stronger restriction than the analogous restriction in models with cash bargaining. To highlight the differences, focus on the favorable selection case. In the literature on Coasean bargaining with cash, if a type θ is indifferent between accepting and rejecting an offer p after a history H_t, then all types $\theta' > \theta$ strictly prefer to accept p at H_t regardless of continuation play after the rejection. In contrast, the present lemma covers only deterministic offer paths, and it allows for offer histories where both θ and θ' are indifferent between accepting and rejecting. Hence, the Lemma does not entirely rule out histories in which θ' accepts strictly earlier—for example, if θ and θ' are both indifferent and randomize over their acceptance decisions.

However, Lemma 4 guides the search for tractable equilibria, since it shows, for example, that when $\iota^S_\theta(\theta,\alpha) > 0$ it is be fruitless to search for skimming equilibria where higher types accept first with certainty. And even though Lemma 4 only covers deterministic offer paths, the stochasticity of equilibrium offers is such that, with some additional arguments, the lemma suffices to verify incentive compatibility for the buyer.

[23] In the usual order on the extended real line.

Given the lemma, and the assumption of a skimming environment, we now derive the necessary conditions behind the uniqueness claims in Theorem 1.

Proof of Theorem 1, Necessary Conditions.

Favorable Selection: We assume the existence of an equilibrium with non-trivial smooth screening and derive a contradiction.

Since higher types trade first, the seller's beliefs are right-truncations of the prior, and the truncation cutoff k is the Markov state controlled by the seller. Let k be a state such that smooth trade is prescribed for all $k' \in (k-\varepsilon, k+\varepsilon), \varepsilon > 0$. Then, by regularity of the equilibrium, the seller's value function at k must satisfy the HJB

$$rJ(k) = \sup_{(|\dot{k}|,\gamma)\in\mathbb{R}^2_+} \Big\{ \big(\overbrace{\bar{S}(\alpha(k),k)}^{\mathbb{E}[S(\alpha(k),\tilde{V})|k]} - J(k) \big) \frac{|\dot{k}|}{k} - J'(k)|\dot{k}| + \gamma \left(\mathbb{E}[\bar{S}(\alpha^f(0),\theta)|\theta \leq k] - J(k) \right) + rc \Big\}. \quad (25)$$

with the right hand side attained by some $|\dot{k}| < \infty, \gamma < \infty$. If $|\dot{k}| < \infty, \gamma < \infty$ is indeed optimal, by the linearity of the objective with respect to $|\dot{k}|, \gamma$, the coefficients on those choice variables must be weakly negative, and (25) must simplify to $J(k) = c$. From the coefficient on γ, we conclude

$$\mathbb{E}[\bar{S}(\alpha^f(0),\theta)|\theta < k] \leq c. \quad (26)$$

Now we have reached a contradiction: $k > 0$, since it is in the interior $(k-\varepsilon, k+\varepsilon)$, so

$$\mathbb{E}[\bar{S}(\alpha^f(0),\theta)|\theta \leq k] > \bar{S}(\alpha^f(0),0) = R(0) \geq c,$$

where the strict inequality follows Assumption 1 and $\alpha^f(0) \neq \underline{\alpha}$.[24]

Hence, the states at which smooth trade is prescribed have zero measure, and the equilibrium path K can have no smooth trade regions of positive duration where it is strictly increasing—if there were such an interval $(\underline{t}, \bar{t})$, then $(K_{\underline{t}}, K_{\bar{t}})$ would be a positive measure interval of states with smooth trade.

Consider now a smooth trade region of positive duration that is *quiet*, that is, K is constant at some level k, The seller's payoffs would be

$$(1-\mathbb{E}[e^{-rT}])c + \mathbb{E}[e^{-rT}]\mathbb{E}[\bar{S}(\alpha^f(0),\theta)|\theta \geq k] < \mathbb{E}[\bar{S}(\alpha^f(0),\theta)|\theta \geq k].$$

The right hand side is achieved with $T = 0$. We again have a contradiction, since the positive-length quiet period is strictly suboptimal.

Hence, the only possible dynamics involve quiet periods at which the pooling offer is

[24] $\alpha^f(0)$ satisfies $\bar{S}(\alpha^f(0),0) = R(0) \geq c$, so it must be greater than $\underline{\alpha}$.

made immediately, or jumps in K. The game must therefore end in the first instant, which can only happen at the pooling offer $\alpha^f(0)$.

<u>Adverse Selection</u>: The seller now controls the *left* truncation of her posterior beliefs as a Markov state. Notice, for $k < k^{SL}$, the seller will never use the pooling offer with positive probability, since she would earn less than c by doing so. With that in mind, we first identify implications of smooth trading. The derivation in the main text then implies that, for any k in the interior of a smooth trade region, $J(k) = c$ and $\bar{S}(\alpha(k), k) \leq c$; if, in addition, $\dot{k} \neq 0$ at such a state, $\bar{S}(\alpha(k), k) = c$.

Second, we show that if there is smooth trade, it happens only on a set of states $[0, k^{smooth})$, i.e., the games starts with smooth trading and ends with a jump. Suppose that, on some continuation game, there were a jump from k to $k' > k$. Since there are countably many jumps, and jumps are isolated, smooth trade must recommence at k'. In particular, $J(k') = c$ and $\mathbb{E}[\bar{S}(\alpha(k'), \theta)|\theta = k'] \leq c$. The seller's payoff from jumping to k' is therefore

$$\left(\frac{k' - k}{1 - k}\right) \mathbb{E}[\bar{S}(\alpha(k'), \theta)|\theta \in [k, k')] + \left(\frac{1 - k'}{1 - k}\right) c.$$

The seller can always freeze trade and ensure a payoff of c, so for such a jump to be optimal,

$$c \leq \mathbb{E}[\bar{S}(\alpha(k'), \theta)|\theta \in [k, k')] \leq \mathbb{E}[\bar{S}(\alpha(k'), \theta)|\theta = k'] \leq c.$$

If the second inequality were an equality, then $\alpha(k') = \underset{\sim}{\alpha}$, by the nondegeneracy condition in Assumption 1. But that would contradict the last inequality, since $\bar{S}(\underset{\sim}{\alpha}, k') < c$. The second inequality must therefore be strict, which is a contradiction. Therefore, the set of smooth trade states must be an interval $[0, k^{smooth})$.

Third, we show that $k^{smooth} = k^{SL}$. By Condition 2 in the equilibrium definition, buyer $\theta = 1$'s equilibrium reservation price $\alpha(1)$ must be *equal* to $\alpha^f(1)$, the highest take-it-or-leave-it offer that he would acccept. Hence, if $k^{smooth} < k^{SL}$, so that continuation play prescribes a jump before the state reaches k^{SL}, then it is weakly optimal for the seller to jump directly to $k = 1$ at k^{smooth}. Her payoffs at k^{smooth} are therefore $\mathbb{E}[\bar{S}(\alpha^f(1), \theta)|\theta \in [k^{smooth}, 1]] < c$, by definition of k^{SL}. This violates the seller's individual rationality, so we must have $k^{smooth} \geq k^{SL}$. However, if $k^{smooth} > k^{SL}$, then for any state $k \in (k^{SL}, k^{smooth})$, the seller can jump the state to $\theta = 1$ with an offer of $\alpha^f(1)$; this gives her a payoff of $\mathbb{E}[\bar{S}(\alpha^f(1), \theta)|\theta \in [k, 1]] > c$, a profitable deviation. In particular, $k^{smooth} = k^{SL}$ implies that, selection is adverse but not severely so, all equilibria have instant trade.

Fourth, we show that, for $k \in (0, k^{SL})$, the optimal $\dot{k}$ for the seller must be strictly positive, i.e., there are no quiet periods. Suppose otherwise. By a typical dynamic programming argument, if starting at state $k \in (0, k^{SL})$ we have $\dot{k} = 0$, then the continuation value of the marginal buyer equals $A(k)$, and $\alpha(k) = \alpha^f(k)$ as defined in Lemma 4. But then $\bar{S}(\alpha(k), k) = R(k) > c$, a contradiction.

Finally, we show that if selection is severely adverse, but there is no gap, trade breaks down. Since selection is severely adverse, $k^{SL} > 0$. By the previous point, the states $[k^{SL}, 1]$ must be reached via smooth trade. As argued in the main text, local incentive compatibility for the marginal buyer, coupled with the fact that K is C^1 under it reaches k^{SL}, implies that K_t must satisfy (10):

$$\dot{K}_t = \frac{r(R(K_t) - c)}{\bar{S}_\theta(\alpha(K_t), K_t)}, \quad K_0 = 0,$$

The right hand side of this IVP is C^1, given the assumptions on primitives and the expression for $\alpha(\cdot)$, so the IVP has a unique local solution $K_t = 0$ on a maximum interval of existence $[0, \varepsilon)$, with $\lim_{t \to \varepsilon} K_t = 0$. (Note that $\alpha(0) > \underline{\alpha}$, since $\bar{s}(\alpha(0), 0) = c$). At $t = \varepsilon$, the situation repeats itself, so K_t never rises above 0. $\square$

B Non-parametric Comparisons

The following is a straightforward consequence of the arguments given for the parametric case, so we present it without proof:

Proposition 6. *Take two securities S^1 and S^2 that both lead to severe adverse selection, and let $\alpha^{f,i}(1)$ and $k^{SL}(S^i), i = 1, 2$ solve $V(1) - \bar{S}^i(\alpha^{f,i}(1), 1) = A(1)$ and $\mathbb{E}[\bar{S}^i(\alpha^{f,i}(1), \theta | \theta \in [k^{SL}(S^i), 1]] = c$. If, for any α_1, α_2 and θ,*

$$\bar{S}^1(\alpha_1, \theta) = \bar{S}^2(\alpha_2, \theta) \Rightarrow \bar{S}^1_\theta(\alpha_1, \theta) > \bar{S}^2_\theta(\alpha_2, \theta)$$

1. *$k^{SL}(S^1) > k^{SL}(S^2)$.*
2. *τ^{CE} is strictly higher for $\theta \in (0, k^{SL}(S^2))$.*
3. *Let k^* solve $\bar{S}^2(\alpha^{f,2}(1), k^*) = c$. Then, for some $k' > \min\{k^*, k^{SL}(S^1)\}$, all types $\theta \in (0, k')$ are worse off under S^1.*

C Auxiliary Lemmas for Sections 3 and 4

Proof of Lemma 2. Equilibrium payoffs are

$$U_i(\theta) = A(\theta) + e^{-r\tau_i^{CE}(\theta)} \left(R(\theta) - \bar{S}^i(\alpha^i(\theta), \theta)\right), \tag{27}$$

and, by the envelope theorem, their derivative is

$$U_i'(\theta) = A'(\theta) + e^{-r\tau_i^{CE}(\theta)} \left(R'(\theta) - \bar{S}^i_\theta(\alpha^i(\theta), \theta)\right) \tag{28}$$

almost everywhere. Using equation (27) to substitute $e^{-r\tau_i^{CE}(\theta)}$ into the envelope condition (28) we get[25]

$$\frac{U_i'(\theta) - A'(\theta)}{U_i(\theta) - A(\theta)} = \frac{\partial}{\partial v} \log \left(U_i(\theta) - A(\theta)\right) = \frac{R'(\theta) - \bar{S}_\theta^i(\alpha^i(\theta), \theta)}{R(\theta) - \bar{S}^i(\alpha^i(\theta), \theta)}$$

almost everywhere. Integrating with respect to θ yields (21). ☐

Proof of Lemma 3. By definition, $\mathbb{E}[\bar{S}(\alpha^f(1;\gamma), \theta; \gamma) | \theta \in [k^{SL}(\gamma), 1]] = c = \bar{S}(\alpha^f(1;\gamma), k^*(\gamma))$, so we must have $k^*(\gamma) > k^{SL}(\gamma)$. The lemma then follows by the continuity of $k^{SL}(\gamma)$, which we now show. Let $\alpha^f(1;\gamma)$ denote the final (pooling) offer under security $S(\cdot, \cdot; \gamma)$. By the continuity of $\bar{S}(\cdot, \theta; \cdot)$, $\alpha^f(1; \cdot)$ is continuous, using the inverse function theorem. The critical type $k^{SL}(\gamma)$ solves

$$\int_k^1 \bar{S}(\alpha^f(1;\gamma), v, \gamma) d\theta - (1-k)c = 0.$$

with respect to k. By the continuity of $\bar{S}$ and $\alpha^f(1; \cdot)$, $k^{SL}(\cdot)$ is also continuous. ☐

Proof of Lemma 4. We prove the case of strictly increasing $\iota^S(\cdot, \alpha)$ (the argument for an decreasing $\iota^S(\cdot, \alpha)$ is symmetric). The statements on α^f follow by implicit differentiation: since α^f solves $R(\theta) = \bar{S}(\alpha^f(\theta), \theta)$,

$$\begin{aligned}\frac{\partial}{\partial \theta}\alpha^f(\theta) = \frac{R'(\theta) - \bar{S}_\theta(\alpha^f(\theta), \theta)}{\bar{S}_\alpha(\alpha^f(\theta), \theta)} - \overbrace{\left[R(\theta) - \bar{S}(\alpha^f(\theta), \theta)\right]}^{=0} \frac{\bar{S}_{\alpha\theta}(\alpha^f(\theta), \theta)}{\bar{S}_\alpha(\alpha^f(\theta), \theta)^2} \\ = -\frac{\partial}{\partial \theta}\iota^S(\theta, \alpha^f(\theta)) < 0 \end{aligned} \quad (29)$$

We separate the statement on $T(\theta)$ into two claims:

1. First, we show that selections from $\arg\max_{t \in \mathbb{R}_+} U(t, \tilde{\alpha}_t, \theta)$ are non-decreasing.
2. Then we show that if $t = +\infty \in T(\underline{\theta})$, then for any $\bar{\theta} > \underline{\theta}$, $T(\bar{\theta}) = \{+\infty\}$. Formally, if $\sup_{t \in R_+} U(t, \tilde{\alpha}, \underline{\theta}) = A(\underline{\theta})$, then $\sup_{t \in R_+} U(t, \tilde{\alpha}, \bar{\theta}) = A(\bar{\theta})$ and $U(t, \tilde{\alpha}_t, \bar{\theta}) < A(\bar{\theta})$ for all $t \in \mathbb{R}_+$.

Claim 1: The key step is an argument by Milgrom and Shannon (1994) and Edlin and Shannon (1998). Edlin and Shannon (1998)'s Theorem 2 has additional conditions that are violated in our setting, but which are only necessary to derive their conclusions on strict comparative statics. For completeness, we reproduce here the part of the argument that suffices for our purposes:

Definition 7. For $U : \mathbb{R}_+ \times [\underline{\alpha}, \bar{\alpha}] \times [0, 1] \to \mathbb{R}$, U satisfies the strict Spence-Mirrlees condition in $((t, \alpha), \theta)$ if if U is C^1, $U_t / |U_\alpha|$ is strictly increasing in t, and $U_\alpha \neq 0$ and has a constant

[25] We know from Theorem 1 that $R(\theta) > \bar{S}^i(\alpha^i(\theta), \theta)$ for all $\theta < 1$.

sign.

Lemma 5 (Adapted from Theorem 2 in Edlin and Shannon (1998))**.** *Assume $U : \mathbb{R}_+ \times [\underline{\alpha}, \bar{\alpha}] \times [0,1]$ satisfies the strict Spence-Mirrlees condition and has path-connected indifference sets. Then every selection from $\arg\max_{t \in \mathbb{R}_+} U(t, \tilde{\alpha}_t, \theta)$ is non-decreasing.*

Proof. By Theorem 3 in Milgrom and Shannon (1994), U is strictly single crossing in $((t, \alpha); \theta)$, where $\mathbb{R}_+ \times [0,1]$ is endowed with the lexicographic order. With that order on $\mathbb{R}_+ \times [\underline{\alpha}, \bar{\alpha}]$, U is quasisupermodular in (t, α) and the set $\{(t, \alpha) : \alpha = \hat{\alpha}_t\}$ is a sublattice of $\mathbb{R}_+ \times [0,1]$. The result then follows by Theorem 4' in Milgrom and Shannon (1994). $\square$

The Spence-Mirrlees condition follows from simple calculus: using $U_\alpha = -e^{-rt}\bar{S}_\alpha < 0$,

$$\frac{U_t}{|U_\alpha|} = -\frac{r(R(\theta) - \bar{S}(\alpha, \theta))}{\bar{S}_\alpha(\alpha, \theta)} = r\iota^S(\alpha, \theta). \tag{30}$$

so U satisfies the strict Spence-Mirrlees condition whenever $\iota^S(\cdot, \theta)$ is strictly increasing.

To show complete regularity of U, fix θ and $\underline{t} < \bar{t}$ and $\underline{\alpha}, \bar{\alpha}$ such that $U(\bar{t}, \bar{\alpha}, \theta) = U(\underline{t}, \underline{\alpha}, \theta) = \bar{u}$. We construct a continuous function $\tilde{\alpha} : [\underline{t}, \bar{t}] \to [0,1]$ satisfying $\tilde{\alpha}(\underline{t}) = \underline{\alpha}$, $\tilde{\alpha}(\bar{t}) = \bar{\alpha}$, and $U(t, \tilde{\alpha}(t), \theta) = \bar{u}$.

If $\bar{u} = A(\theta)$, then $\bar{\alpha} = \underline{\alpha} = \alpha^f(\theta)$ and $U(t, \alpha^f(\theta), \theta)$ is constant in t; setting a constant $\tilde{\alpha}(t) = \alpha^f(\theta)$ trivially suffices. Focus then on $\bar{u} > A(\theta)$; the proof for $\bar{u} < A(\theta)$ is symmetric. For all (t', α') such that $U(t', \alpha', \theta) = \bar{u}$, $R(\theta) - \bar{S}(\alpha', \theta) > 0$ and therefore $U_t(t', \alpha', \theta) = re^{-rt'}(R(\theta) - \bar{S}(\alpha', \theta)) > 0$. By the Implicit Function Theorem, since $U_\alpha < 0$, for any $t_0 \in [\underline{t}, \bar{t}]$, there exists some open neighborhood $\mathcal{O} \subset [\underline{t}, \bar{t}]$ with $t_0 \in \mathcal{O}$ and a $C^1(\mathcal{O})$ function $\tilde{\alpha} : \mathcal{O} \to [0,1]$ satisfying

$$\tilde{\alpha}'(t) = -\frac{U_t(t, \tilde{\alpha}(t), \theta)}{U_\alpha(t, \tilde{\alpha}(t), \theta)}, t \in \mathcal{O}, \quad U(t_0, \tilde{\alpha}(t_0), \theta) = \bar{u},$$

i.e., $\tilde{\alpha}$ is in fact a local solution to an initial value problem.[26]

We extend the solution to the IVP above to yield the desired function $\tilde{\alpha}$. Take the open domain $\mathcal{D} = (\underline{t}, \bar{t}) \times (\underline{\alpha}, \bar{\alpha})$. We only show how to extend $\tilde{\alpha}$ continuously rightward up to $\bar{t}$, since extending it leftward to $\underline{t}$ is done symmetrically. Since U is C^1, U_t is bounded above and below on $\mathcal{D}$, and $U_\alpha < 0$, $g(t, \alpha) := -U_t(t, \alpha, \theta)/U_\alpha(t, \alpha, \theta)$ is continuous and bounded on $\mathcal{D}$. By standard extension theorems (Lemma 2.14 in Teschl (2012) and Theorem 4.1 in Coddington and Levinson (1955)), either $\tilde{\alpha}$ can be extended rightwards inside $\mathcal{D}$ to all of $[t_0, \bar{t})$, or there exists some $t' \in (t_0, \bar{t}]$ such that $\tilde{\alpha}$ extends rightwards up to $[t, t')$ with $\tilde{\alpha}(t') = \bar{\alpha}$.[27] If $\tilde{\alpha}$ can be extended rightwards to all of $[t_0, \bar{t})$, then by the continuity of U, $\tilde{\alpha}(\bar{t}-) = \bar{\alpha}$, and we are done.

[26]Since $U_\alpha < 0$, one can solve for $\tilde{\alpha}(t_0)$ in $U(t_0, \tilde{\alpha}(t_0), \theta) = \bar{u}$.

[27]To be precise, $\tilde{\alpha}$ extends up to $[t, t')$ and $\tilde{\alpha}(t'-) = \bar{\alpha}$.

Suppose, then, that $\tilde{\alpha}$ cannot be extended rightwards to all of $[t_0, \bar{t})$, so there exists some $t' \leq \bar{t}$ with $\tilde{a}(t') = \bar{\alpha}$ as above. If $t' = \bar{t}$, we are done, so focus on the remaining case $t' < \bar{t}$. Since $U(t', \bar{\alpha}, \theta) = \bar{u} = U(\bar{t}, \bar{\alpha}, \theta)$, by Rolle's theorem there exists some $t'' \in (t', \bar{t})$ such that $U_t(t'', \bar{\alpha}, \theta) = 0$. That would require $\bar{\alpha} = \alpha^f(\theta)$, a contradiction to $\bar{u} > A(\theta)$.

Claim 2: If $\sup_{t \in R_+} U(t, \tilde{\alpha}, \underline{\theta}) = A(\underline{\theta})$, so that $t = +\infty$ achieves that supremum, it must be that, for all $t \in \mathbb{R}_+$, $R(\underline{\theta}) \leq \bar{S}(\tilde{\alpha}_t, \underline{\theta})$. But then, using $\bar{S}_\alpha < 0$, it follows that, for all $t \in \mathbb{R}_+$,

$$\tilde{\alpha}_t \geq \alpha^f(\underline{\theta}) > \alpha^f(\bar{\theta}).$$

where the strict inequality was shown in (29). Therefore, using $\bar{S}_\alpha < 0$ and $R(\bar{\theta}) = \bar{S}(\alpha^f(\bar{\theta}), \theta)$, we have that, for all $t \in \mathbb{R}_+$,

$$R(\bar{\theta}) < \bar{S}(\tilde{\alpha}_t, \bar{\theta}) \Rightarrow U(t, \tilde{\alpha}_t, \bar{\theta}) < A(\bar{\theta}) \text{ and } \sup_{t \in R_+} U(t, \tilde{\alpha}, \bar{\theta}) = A(\bar{\theta})$$

We conclude that $\arg\max_{t \in \mathbb{R}_+ \cup \{+\infty\}} U(t, \tilde{\alpha}_t, \bar{\theta}) = \{+\infty\}$, as required.

□

D Proofs for Section 5

Proof of Lemma 1.
The offer in the smooth trading region $\alpha(\theta)$ and the final offer $\alpha^f(1)$, are given by

$$\alpha(\theta) = \frac{c}{\mathbb{E}[(Z-d)^+ | \theta]}$$
$$\alpha^f(1) = \frac{R(1)}{\mathbb{E}[(Z-d)^+ | 1]}.$$

Using integration by parts we can write

$$\mathbb{E}[(Z-d)^+ | \theta] = \int_d^\infty (1 - G(x|\theta)) dx.$$

With some abuse of notation we drop the subscript in $g_V(z|\theta)$. Denote the cumulative density function of V conditional on $\theta \in [k, 1]$ by $\bar{G}(z|k)$, which is given by

$$\bar{G}(z|k) := \frac{1}{1-k} \int_k^1 G(z|\theta) d\theta].$$

We can directly characterize the conditions in terms of the cumulative density function $G(z|\theta)$. The threshold k^{SL} is given by

$$k^{SL} = \inf \left\{ k \leq 1 : \frac{\int_d^\infty \left(1 - \bar{G}(z|k^{SL})\right) dz}{\int_d^\infty \left(1 - G(z|1)\right) dz} \geq \frac{c}{R(1)} \right\}.$$

The environment is upward skimming if

$$-\frac{\int_d^\infty G_\theta(z|\theta)dz}{\int_d^\infty \big(1-G(z|\theta)\big)dz} > \frac{R'(\theta)}{R(\theta)} \tag{31}$$

First, we verify that if the environment is upward skimming for d, it is also upward skimming for any $d' > d$. The left hand side of the upward skimming condition (31) can be written as

$$-\frac{\int_d^\infty G_\theta(x|\theta)dx}{\int_d^\infty (1-G(x|\theta))dx} = \frac{\partial}{\partial v}\log\int_d^\infty (1-G(x|\theta))dx.$$

After changing the order of differentiation, we get

$$-\frac{\partial}{\partial d}\frac{\int_d^\infty G_\theta(x|\theta)dx}{\int_d^\infty (1-G(x|\theta))dx} = -\frac{\partial}{\partial v}\frac{1-G(d|\theta)}{\int_d^\infty (1-G(x|\theta))dx} > 0, \tag{32}$$

where the inequality follows as, for all $z > \theta$, the ratio $\frac{1-G(z|\theta)}{1-G(d|\theta)}$ is increasing in θ by the monotone likelihood ratio property.

Next, we need to verify that if the lemons condition is satisfied for d, then it is also satisfied for $d' > d$. Next, we verify that if the lemon's condition is satisfied for d, then it is also satisfied for $d > d'$. The lemons condition can be written as

$$\frac{\int_d^\infty \big(1-\bar{G}(z|0)\big)dz}{\int_d^\infty \big(1-G(z|1)\big)dz} < \frac{c}{R(1)} \qquad \Longleftrightarrow$$

$$0 < R(1)\int_d^\infty \big(1-\bar{G}(z|0)\big)dz - c\int_d^\infty \big(1-G(z|1)\big)dz.$$

Letting

$$\psi(d) := R(1)\int_d^\infty \big(1-\bar{G}(z|0)\big)dz - c\int_d^\infty \big(1-G(z|1)\big)dz$$

we get

$$\psi'(d) = (1-G(d|1))\left[R(1)\left(\frac{1-\bar{G}(d|0)}{1-G(d|1)}\right) - c\right]$$

Notice that

$$R(1)\left(\frac{1-\bar{G}(\underline{z}|0)}{1-G(\underline{z}|1)}\right) - c = R(1) - c > 0;$$

thus, it is enough to show that $\frac{1-\bar{G}(d|0)}{1-G(d|1)}$ is increasing in d. Differentiating with respect to d we get

$$\frac{\partial}{\partial d}\frac{1-\bar{G}(d|0)}{1-G(d|1)} = \frac{1-\bar{G}(d|0)}{1-G(d|1)}\left[\frac{g(d|1)}{1-G(d|1)} - \frac{\bar{g}(d|0)}{1-\bar{G}(d|0)}\right],$$

which is positive as $G(z|1)$ dominates $\bar{G}(z|0)$ in the hazard rate order.

□

Proof of Proposition 3. The offer in the smooth trading region and the final offer are $\alpha(\theta) = c/\mathbb{E}[(\tilde{V}-d)^+|\theta]$ and $\alpha^f(1) = \mathbb{E}[(\tilde{V}-d)^+|1]$, respectively. The expected value of equity is

$$\hat{V}^d(\theta;\eta) := \mathbb{E}[(\tilde{V}-d)^+|\theta] = \int_d^\infty \Phi\left(\eta(c+\Delta+\zeta\theta - z)\right) dz,$$

where Φ is the CDF of the standard normal distribution and $\zeta := \chi + \beta$. With this notation, $\bar{S}^d_{lev}(\alpha,\theta;\eta) = \alpha\hat{V}^d(\theta;\eta)$. Using the identity $\phi(x)x = -\phi'(x)$, we the following partial derivatives, which we will need below:

$$\hat{V}^d_\eta(\theta;\eta) = -\frac{1}{\eta}\int_d^\infty \phi'(\eta(c+\Delta+\zeta\theta-z))dz = -\frac{1}{\eta^2}\phi(\eta(c+\Delta+\zeta\theta-d))$$

where the second equality uses a change of variables and $\lim_{x\to-\infty}\phi(x) = 0$, and

$$\hat{V}^d_{\eta\theta} = = -\frac{\zeta}{\eta}\phi'(\eta(c+\Delta+\zeta\theta-d)).$$

To prove the first statement in the proposition, we prove the stronger claim

$$\frac{\hat{V}^d_\theta}{\hat{V}^d} \text{ is strictly increasing in } \eta. \tag{$\star$}$$

which implies the result. Indeed, if the claim is true, then whenever $\bar{S}^d_{lev}(\alpha_1,\theta;\eta_1) = \bar{S}^d_{lev}(\alpha_2,\theta;\eta_2)$,

$$\frac{\hat{V}^d_\theta(\theta;\eta_1)}{\hat{V}^d(\theta;\eta_1)}\bar{S}^d_{lev}(\alpha_1,\theta;\eta_1) \quad > \quad \frac{\hat{V}^d_\theta(\theta;\eta_2)}{\hat{V}^d(\theta;\eta_2)}\bar{S}^d_{lev}(\alpha_1,\theta;\eta_1) \quad = \quad \frac{\hat{V}^d_\theta(\theta;\eta_2)}{\hat{V}^d(\theta;\eta_2)}\bar{S}^d_{lev}(\alpha_2,\theta;\eta_2)$$

The left hand side equals $\frac{\partial}{\partial\theta}\bar{S}^d_{lev}(\alpha_1,\theta;\eta_1)$ and the right hand side equals $\frac{\partial}{\partial\theta}\bar{S}^d_{lev}(\alpha_2,\theta;\eta_2)$, so (17) follows.

To prove ($\star$), we sign

$$\hat{V}^d_{\theta\eta}\hat{V}^d - \hat{V}^d_\theta\hat{V}^d_\eta. \tag{33}$$

which is proportional to $\frac{\partial}{\partial\eta}\frac{\hat{V}^d_\theta}{\hat{V}^d}$ by the quotient rule. First, simplify notation by labeling $\Gamma := \eta(c+\Delta+\zeta\theta-d)$ and $\mu := (c+\Delta+\zeta\theta)$. Note that μ is the mean of $\tilde{V}|\theta$. With this notation, $\hat{V}^d_\eta = -\phi(\Gamma)\eta^{-2}$, $\hat{V}^d_{\eta\theta} = \zeta\phi'(\Gamma)\eta^{-1}$. We can directly calculate that

$$\hat{V}^d_\theta(\theta;\eta) = \eta\zeta\int_d^\infty \phi(\eta(c+\Delta+\zeta\theta-z))dz = \zeta\Phi(\Gamma)$$

Next, using standard formulas for the moments of censored random variables and letting $h(\cdot) = \phi(\cdot)/(1-\Phi(\cdot))$ denote the inverse Mills ratio, we calculate $\hat{V}^d$:

$$\begin{aligned}
\hat{V}^d &= -d + \mathbb{E}[\max\{\tilde{V},d\}|\theta] \\
&= -d + [1-\Phi(\eta(d-\mu))]\left[\mu + \frac{1}{\eta}h(\eta(d-\mu))\right] + \Phi(\eta(d-\mu))d \\
&= \frac{1}{\eta}\left[1-\Phi(-\Gamma)\right]\left[\Gamma + h(-\Gamma)\right] \qquad (34)
\end{aligned}$$

Plugging the expressions for $\hat{V}^d_{\eta\theta}$, $\hat{V}^d$, $\hat{V}^d_\eta$ and $\hat{V}^d_\theta$ into (33), we obtain a quantity proportional to

$$-\phi'(\Gamma)\Phi(\Gamma)\left[\Gamma+h(-\Gamma)\right]+\Phi(\Gamma)\phi(\Gamma) \propto -\phi'(\Gamma))\left[\Gamma+h(-\Gamma)\right]+\phi(\Gamma) \tag{35}$$

$$\propto \Gamma\left[\Gamma+h(-\Gamma)\right]+1, \tag{36}$$

where the last line uses the identify $\phi'(x)=-x\phi(x)$. If $\Gamma \geq 0$, we are done. Otherwise, if $\Gamma < 0$, we apply the following classic bound on the inverse Mills ratio (see Gordon (1941)):

$$h(x) < x + \frac{1}{x}, x > 0$$

to obtain

$$\Gamma\left[\Gamma+h(-\Gamma)\right]+1 > \Gamma\left[\Gamma+\left(-\Gamma+\frac{1}{-\Gamma}\right)\right]+1=0,$$

which proves that (33) is strictly positive.

Moving on to the second statement, to show that adverse selection is preserved as η rises, notice that, similar to an environment with unlevered equity, there is adverse selection if

$$\frac{\hat{V}^d_\theta(\theta;\eta)}{\hat{V}^d(\theta;\eta)} > \frac{R'(\theta)}{R(\theta)}.$$

By ($\star$), it follows that if selection is adverse for some η, it will be adverse for any $\eta' > \eta$.

The preservation of *severe* adverse selection as η rises also follows from ($\star$). Indeed, adverse selection is severe if

$$R(1)\int_0^1 \frac{\hat{V}^d(\theta;\eta)}{\hat{V}^d(1;\eta)}d\theta < c \tag{37}$$

where we have used the fact that the final offer is $R(1)/\hat{V}^d(1;\eta)$. From ($\star$) one has

$$0 < \frac{\partial}{\partial\eta}\frac{\hat{V}^d_\theta(\theta;\eta)}{\hat{V}^d(\theta;\eta)} = \frac{\partial^2}{\partial\eta\partial\theta}\log\hat{V}^d(\theta;\eta) = \frac{\partial}{\partial\theta}\frac{\hat{V}^d_\eta(\theta;\eta)}{\hat{V}^d(\theta;\eta)}.$$

Hence, for all $\theta \in [0,1]$,

$$\frac{\hat{V}^d_\eta(\theta;\eta)}{\hat{V}^d(\theta;\eta)} \leq \frac{\hat{V}^d_\eta(1;\eta)}{\hat{V}^d(1;\eta)} \Rightarrow \frac{\partial}{\partial\eta}\frac{\hat{V}^d(\theta;\eta)}{\hat{V}^d(1;\eta)} \leq 0.$$

The integrand in (37) therefore decreases as η increases, which concludes the proof.

Finally, for the third statement, selection is adverse if (omitting some arguments to reduce clutter)

$$\frac{\beta}{c+\Delta+\beta\theta} < \frac{\hat{V}^d_\theta(\theta)}{\hat{V}^d(\theta)} = \frac{\zeta\eta}{\eta(c+\Delta+\zeta\theta-d)+h(-\eta(c+\Delta+\zeta\theta-d))} \quad \forall\theta. \tag{38}$$

where we have substituted our previous expressions for $\hat{V}^d_\theta$ and $\hat{V}^d$. From Lemma 1, it suffices to satisfy (38) for $d=0$, so we will have favorable selection if

$$\frac{\beta}{\beta+\chi} < \frac{\eta(c+\Delta+\beta\theta)}{\eta(c+\Delta+\zeta\theta)+h(-\eta(c+\Delta+\zeta\theta))}$$

Using the monotone hazard rate property of the normal distribution, a crude lower bound for the right hand side is

$$\frac{\eta(c+\Delta)}{\eta(c+\Delta)+\eta(\beta+\chi)+h(-\eta(c+\Delta))}.$$

Using, $h(0)=2/\sqrt{2\pi}$, an even cruder bound is

$$\frac{\eta(c+\Delta)}{\eta(c+\Delta)+\eta(\beta+\chi)+2/\sqrt{2\pi}}.$$

□

E Equilibrium Verification

Proof of Theorem 1: Equilibrium Verification. We present details for the case with non-trivial delayed trade, $S \in \mathcal{D}_{A,\tilde{V},c}$; the remaining cases are similar, but much simpler.

Verification: Seller's On-Path Strategies: Now, we verify that the seller's choice of $\{K^k\}_{k\in[0,1]}$ and F are optimal, given the buyer's strategy

$$\alpha(k) = \begin{cases} \alpha^f(1) & \text{if } k \in (k^{SL},1] \\ \bar{S}^{-1}(c,k) & \text{if } k \in [0,k^{SL}), \end{cases} \tag{39}$$

where the inverse $\bar{S}^{-1}(c,k)$ is defined by $\bar{S}\big(\bar{S}^{-1}(c,k),k\big)=c$. From the previous, given $\alpha(k)$ in equation (39), seller's the continuation value is

$$J(k) = \begin{cases} c & \text{if } k \in [0,k^{SL}] \\ \mathbb{E}[S(\alpha^f(1),\tilde{V})|\theta \in [k,1]] & \text{if } k \in (k^{SL},1] \end{cases} \tag{40}$$

Notice that $J(\cdot)$ has a kink at k^{SL} as

$$J'(k^{SL}-)=0<J'(k^{SL}+)=\frac{\partial}{\partial k}\mathbb{E}[S(\alpha^f(1),\tilde{V})|\theta\in[k,1]]\big|_{k=k^{SL}}$$

The value function fails to be differentiable at at k^{SL} due to the discontinuity in $\alpha(\cdot)$, Moreover, this implies that the HJB equation is discontinuous at this point. To avoid the technical complications associated to working with discontinuous HJB equations, and the theory of viscosity solutions, we take advantage that admissible cutoff polices are non-decreasing, and we split the verification of the optimal policies in two steps: First starting at $k_0 \in (k^{SL},1]$, and then starting $k_0 \in [0,k^{SL}]$.

Verification for $k_0 \in [k^{SL}, 1]$.: Let's ignore the fact that for $\alpha(K_t) = \alpha^f(1)$, all types accept the offer, and consider a relaxed formulation in which the seller is allowed to smoothly screen on $k_0 \in [k^{SL}, 1]$. To simplify notation, we consider F which are absolutely continuous and let $\Lambda_t = \int_0^t \lambda_s ds$, where λ_s denotes the hazard rate of F at s. This is without loss of generality as the payoff from a distribution F with atoms can be approximated by a sequence of absolutely continuous functions (in other words, we can take a sequence of absolutely continuous distributions F^n that converges to F in the weak* topology). The seller's value function is

$$J(k_0) = \sup_{Q,\lambda} \int_0^\infty e^{-rt-\Lambda_t} \Big(\mathbb{E}\left[\bar{S}\left(\alpha(Q_t), v\right) \Big| \theta \in [Q_{t-}, Q_t] \right] \frac{dQ_t}{1-k_0} \\ + \lambda_t \mathbb{E}\left[\bar{S}\left(\alpha^f(1), v\right) \Big| \theta \in [Q_{t-}, 1] \right] \Big) + \left(1 - \int_0^\infty e^{-rt-\Lambda_t} \left(\frac{1-Q_t}{1-k_0} \lambda_t dt + \frac{dQ_t}{1-k_0} \right) \right) c.$$

Rather than working with the value function $J(\cdot)$, it is convenient to work with the equivalent value function $\bar{J}(k) \equiv (1-k)J(k)$, so

$$\bar{J}(k_0) = \sup_{Q,\lambda} \int_0^\infty e^{-rt-\Lambda_t} \Big(\mathbb{E}\left[\bar{S}\left(\alpha(Q_t), v\right) \Big| \theta \in [Q_{t-}, Q_t] \right] dQ_t \\ + \lambda_t (1-k_0) \mathbb{E}\left[\bar{S}\left(\alpha^f(1), v\right) \Big| \theta \in [Q_{t-}, 1] \right] \Big) + \left(1 - k_0 - \int_0^\infty e^{-rt-\Lambda_t} \big(\lambda_t (1-Q_t) dt + dQ_t \big) \right) c.$$

We conjecture, and then verify, that the value function $\bar{J}(\cdot)$ satisfies the quasi-variational inequality

$$0 = \max \Big\{ \sup_{\dot{k} \geq 0, \lambda \geq 0} \big(\bar{S}(\alpha(k), k) + \bar{J}'(k) \big) \dot{k} + \lambda \left(\int_k^1 \bar{S}\left(\alpha^f(1), v\right) d\theta] - \bar{J}(k) \right) \\ + r(1-k)c - r\bar{J}(k) \,,\, \mathcal{M}\bar{J}(k) - \bar{J}(k) \Big\}, \quad (41)$$

where the operator $\mathcal{M}$ is defined by

$$\mathcal{M}\bar{J}(k) = \max_{k' \in [k,1]} \left\{ (k'-k) \, \mathbb{E}[\bar{S}(\alpha(k'), \theta) | \theta \in [k, k']] + \bar{J}(k') \right\}$$

First we verify that $\bar{J}(k) = (1-k)J(k)$, where $J(k)$ is as in (40) satisfies this quasi-variational inequality. First, it is immediate to verify that $\bar{J}(k) = \mathcal{M}\bar{J}(k)$, so the second term of (41) is satisfied. For the first term, notice that

$$\big(\bar{S}(\alpha(k), k) + \bar{J}'(k) \big) \dot{k} + \lambda \left(\int_k^1 \bar{S}\left(\alpha^f(1), v\right) d\theta] - \bar{J}(k) \right) + r(1-k)c - r\bar{J}(k) \leq \\ \big(\bar{S}(\alpha(k), k) + \bar{J}'(k) \big) \dot{k} = 0,$$

where we have used that $\bar{J}'(k) = -\bar{S}(\alpha^f(1), k)$. From here on, the verification is standard. Consider an arbitrary admissible policy Q_t. Using the change of value formula, we get that

$$e^{-rt-\Lambda_t}\bar{J}(Q_t) = J(k_0) + \int_0^t e^{-rs-\Lambda_s}\left(\dot{q}_s\bar{J}'(Q_{s-}) + \lambda_s\left(\int_k^1 \bar{S}\left(\alpha^f(1), v\right) d\theta] - \bar{J}(Q_{s-})\right)\right.$$
$$\left. -r\bar{J}(Q_{s-})\right) ds + \sum_{s<t} e^{-rs-\Lambda_s}\left(\bar{J}(Q_{s-} + \Delta Q^d_{s-}) - \bar{J}(Q_{s-})\right)$$

From the quasi-variational inequality (41) we get that

$$\bar{J}(Q_s) - \bar{J}(Q_{s-}) \leq (Q_s - Q_{s-})\,\mathbb{E}[\bar{S}(\alpha(Q_s),\theta)|\theta \in [Q_{s-}, Q_s]]$$

and the term in the integral is less than

$$-r(1-Q_s)c - \dot{q}_s\bar{S}(\alpha(Q_{s-}), Q_{s-}) - \lambda_s\int_k^1 \bar{S}\left(\alpha^f(1), v\right) d\theta.$$

It follows that

$$\begin{aligned}
\bar{J}(k_0) \geq & \int_0^t e^{-rs-\Lambda_s}\left(r(1-Q_{s-})c + \dot{q}_s\bar{S}(\alpha(Q_{s-}), Q_{s-})\right. \\
& \quad \left. +\lambda_s\int_k^1 \bar{S}\left(\alpha^f(1), v\right) d\theta]\right) ds \\
& \quad \sum_{s<t} e^{-rs-\Lambda_s}\left(Q^d_s - Q^d_{s-}\right)\mathbb{E}[\bar{S}(\alpha(Q_s),\theta)|\theta \in [Q_{s-}, Q_s]] + e^{-rt-\Lambda_t}\bar{J}(Q_t) \\
= & \int_0^t e^{-rt-\Lambda_t}\mathbb{E}\left[S\left(\alpha(Q_s), \tilde{V}\right)\middle|\theta \in [Q_{s-}, Q_s]\right] dQ_s \\
& \quad + \left(1 - k_0 - e^{-rt-\Lambda_t}(1-Q_t) - \int_0^t e^{-rs-\Lambda_s}\left((1-Q_s)\lambda_s ds + dQ_s\right)\right) c \\
& \quad + e^{-rt-\Lambda_t}\left(\bar{J}(Q_t) - (1-Q_t)c\right),
\end{aligned}$$

where the equality

$$1 - k_0 - e^{-rt-\Lambda_t}(1-Q_t) - \int_0^t e^{-rs-\Lambda_s}\left((1-Q_s)\lambda_s ds + dQ_s\right) = \int_0^t e^{-rs-\Lambda_s} r(1-Q_{s-})ds,$$

follows by integration by parts. Taking the limit when $t \to \infty$, we get that $\bar{J}(k_0)$ is an upper bound on the payoff that the seller can attain starting at any $k_0 \geq k^{SL}$. Finally, because all the inequalities hold with equality in the case of equation for our conjecture policy K, it follows that K is optimal starting at $k_0 \in [k^{SL}, 1]$.

Verification for $k_0 \in [0, k^{SL})$.: Using the previous characterization of the value function $\bar{J}(\cdot)$ on $[k^{SL}, 1]$, by the principle of dynamic programming, we can state the optimization problem on $[0, k^{SL})$, as

$$\bar{J}(k_0) = \sup_{Q} \int_0^{\tau(Q)} e^{-rt-\Lambda_t} \mathbb{E}\left[\bar{S}\left(\alpha(Q_t), v\right) \middle| \theta \in [Q_{t-}, Q_t]\right] dQ_t$$
$$+ \left(1 - k_0 - \int_0^{\tau(Q)} e^{-rt-\Lambda_t}\left(\lambda_t(1-Q_t)dt + dQ_t\right)\right) c + e^{-r\tau(Q)}\left(\bar{J}(Q_{\tau(Q)}) - (1-Q_{\tau(Q)})c\right).$$

where $\tau(Q) = \inf\{t > 0 : Q_t \geq k^{SL}\}$. Notice that the factor $(1-Q_{\tau(Q)})c$ is added to account for the constant $(1-k)c$ in the expected payoff. Once again, we conjecture that the value function $\bar{J}(\cdot)$ satisfies the quasi-variational inequality (41).

First, we can verify that $\bar{J}(\cdot)$ defined by (40) (multiplied by $1-k$) satisfies equation (41) on $[0, k^{SL})$. By construction, $\bar{S}(\alpha(k), k) = \bar{J}'(k) = -c$. Also,

$$\mathcal{M}\bar{J}(k) - \bar{J}(k) = \max_{k' \in [k,1]} \left\{(k'-k)\,\mathbb{E}[S(\alpha(k'), \tilde{V})|\theta \in [k, k']] + \bar{J}(k')\right\} - (1-k)c < 0,$$

so $\max_{\lambda \geq 0}\{\lambda(\int_k^1 \bar{S}\left(\alpha^f(1), v\right) d\theta] - \bar{J}(k))\} = 0$. Thus, the first term of the variational inequality is equal to zero, and because $\mathcal{M}\bar{J}(k) - \bar{J}(k)) \leq 0$, the second term also satisfies the required inequality. It follows then that $\bar{J}(k) = (1-k)c$ is a solution of (41) on $[0, k^{SL})$. Consider an arbitrary policy Q, so, once again, using the change of value formula we get that

$$\mathbb{E}^Q\left[e^{-rt\wedge\tau(Q)}\bar{J}(Q_{t\wedge\tau(Q)})\right] = J(k_0) + \int_0^{t\wedge\tau(Q)} e^{-rs}\left(\dot{q}_s\bar{J}'(Q_{s-}) + \lambda_s\left(\bar{J}(Q_{s-} + \Delta Q^s_{s-})\right.\right.$$
$$\left.\left.-\bar{J}(Q_{s-})\right) - r\bar{J}(Q_{s-})\right) ds + \sum_{s<t\wedge\tau(Q)} e^{-rs}\left(\bar{J}(Q_{s-} + \Delta Q^d_{s-}) - \bar{J}(Q_{s-})\right)$$

Following the same steps that we did before, we get

$$\bar{J}(k_0) \geq \int_0^{t\wedge\tau(Q)} e^{-rs-\Lambda_s}\left(r(1-Q_{s-})c + \dot{q}_s\bar{S}(\alpha(Q_{s-}), Q_{s-})\right.$$
$$\left.+\lambda_s\left(Q^s_s - Q^s_{s-}\right)\mathbb{E}[\bar{S}(\alpha(Q_s), \theta)|\theta \in [Q_{s-}, Q_s]]\right) ds$$
$$\sum_{s<t\wedge\tau(Q)} e^{-rs-\Lambda_s}\left(Q^d_s - Q^d_{s-}\right)\mathbb{E}[\bar{S}(\alpha(Q_s), \theta)|\theta \in [Q_{s-}, Q_s]] + e^{-rt\wedge\tau(Q)-\Lambda_{t\wedge\tau(Q)}}\bar{J}(Q_{t\wedge\tau(Q)})$$
$$= \int_0^{t\wedge\tau(Q)} e^{-rs-\Lambda_s}\mathbb{E}\left[\bar{S}\left(\alpha(Q_s), v\right)\middle|\theta \in [Q_{s-}, Q_s]\right] dQ_s + (1-k_0)c$$
$$-\int_0^{t\wedge\tau(Q)} e^{-rs-\Lambda_s}\left((1-Q_s)\lambda_s ds + dQ_s\right)c + e^{-rt\wedge\tau(Q)-\Lambda_{t\wedge\tau(Q)}}\left(\bar{J}(Q_{t\wedge\tau(Q)}) - (1-Q_{t\wedge\tau(Q)})c\right).$$

Taking the limit as $t \to \infty$ we get that $t \wedge \tau(Q) \to \tau(Q)$. It follows that $\bar{J}(k_0)$ is and upper bound on the seller's expected payoff. Finally, because in the case of the policy K all the inequalities hold with equality, we get that the value of the policy K is given by $\bar{J}(k_0)$, so K is optimal on $[0, k^{SL})$.

Verification: Seller's Off-Path Strategy: Finally, we characterize the off-equilibrium seller's offer $\sigma(\cdot|k',\alpha')$, where $\sigma(\cdot|k',\alpha')$ has to maximize

$$\int_0^1 \left\{(\alpha^{-1}(\tilde{\alpha})-k')^{+}\mathbb{E}\left[S\left(\tilde{\alpha},\tilde{V}\right)\middle|\theta\in[k',\alpha^{-1}(\tilde{\alpha})\wedge k']\right]\right. \\ \left. +(1-\alpha^{-1}(\tilde{\alpha}))J\left(\alpha^{-1}(\tilde{\alpha})\right)\right\} d\sigma(\tilde{\alpha}|k',\alpha').$$

We consider an off-equilibrium offer with two mass points, given by

$$\sigma(\alpha|k',\alpha')=\begin{cases}\alpha(k') & \text{w.p. } p(k',\alpha')\\ \alpha^f(1) & \text{w.p. } 1-p(k',\alpha'),\end{cases}$$

If $k'<k^{SL}$, then, conditional on rejection of α', the cut-off is $\alpha^{-1}(\alpha')=k^{SL}$. In this case, $\bar{S}(\alpha(k^{SL}),k^{SL})=\mathbb{E}\left[S(\alpha^f(1),\tilde{V})|\theta\in[k^{SL},1]\right]=c=J(k^{SL})$, and this payoff is higher than any other serious offer. Thus, any probability $p(k',\alpha')\in[0,1]$ is optimal, and in particular $p(k',\alpha')$ solving

$$\bar{S}(\alpha',k^{SL})=p(k^{SL},\alpha')\bar{S}(\alpha^f(1),k^{SL})+(1-p(k^{SL},\alpha'))\bar{S}(\alpha(k^{SL}),k^{SL}).$$

If $k>k^{SL}$, then the optimal offer is $p(k',\alpha')=1$, as any other offer that is accepted with positive probability yields $\mathbb{E}\left[S(\alpha^f(1),\tilde{V})|\theta\in[k',k]\right]<\mathbb{E}\left[S(\alpha^f(1),\tilde{V})|\theta\in[k',1]\right]=J(k')$.

Verification: Buyer's On-Path Strategy: The proof use a direct mechanism representation of the continuation play together with the characterization in Lemma 4. We cannot apply Lemma 4 directly because the characterization only applies to a deterministic path of cut-offs, and the path cut-off is stochastic in our equilibrium construction (it jumps to $K_T=1$ at time T). The first step then is to establish that, given the seller strategy, the buyer acceptance strategy is incentive compatible only if it incentive compatible for a (suitably defined) deterministic path with the same delay costs for the pooling offer $\alpha^f(1)$. Let $\tau(k)=\inf\{t>0: K_t\geq k\}$, let $\alpha(k)\equiv\alpha(K_{\tau(k)})$, and $y(k)=1-\mathbb{E}[e^{-r\tau(k)}]$. Notice that, given the seller strategy K, we have that $\alpha(k)$ is a deterministic function of k, so the only random variable is $\tau(k)$. Thus, we can write the buyer's problem as

$$\begin{aligned}B(\theta,k)&=\max_{k'\in[k,1]}\mathbb{E}^{K^k}[(1-e^{-r\tau(k')})A(\theta)+e^{-r\tau(k')}\left(V(\theta)-\bar{S}(\alpha(K_{\tau(k')}),\theta)\right)]\\&=\max_{k'\in[k,1]}y(k')A(\theta)+(1-y(k'))\left(V(\theta)-\bar{S}(\alpha(k'),\theta)\right)\\&=\max_{k'\in[k,1]}\tilde{U}(y(k'),\alpha(k'),\theta).\end{aligned}$$

where $\tilde{U}(y,\alpha,\theta):=yA(\theta)+(1-y)\left(V(\theta)-\bar{S}(\alpha,\theta)\right)$.

It follows that it is without loss of generality to consider the incentive compatibility conditions for a deterministic mechanism inducing the same $y(k)$ as K^k. By the arguments in Lemma 4, we know, for increasing $\iota^S(\alpha,\theta)$, $\tilde{U}(y,\alpha,\theta)$ satisfies strict single crossing differences in $((y, \alpha), \theta)$, where (y, α) is ordered lexicographically. Hence, for any $y \mapsto \breve{\alpha}(y)$, $U(y, \breve{\alpha}(y), \theta)$ has strict single-crossing differences in (y, θ).

We have shown that $y(\cdot)$ is non-decreasing. Let $\tilde{\alpha}(z)$ be the candidate equilibrium locus that maps (1 - expected delay costs) $y^\dagger$ to equilibrium offer α, i.e., $\tilde{\alpha}(z) := \alpha(y^{-1}(z))$.[28] If we prove that $\tilde{U}(y, \tilde{\alpha}(y), \theta)$ satisfies *smooth* single crossing differences, and if the following envelope condition is satisfied

$$\tilde{U}(y(\theta), \alpha(\theta), \theta) = \tilde{U}(y(0), \alpha(0), 0) + \int_0^v \tilde{U}_\theta(y(s), \alpha(s), s) ds, \tag{42}$$

then by Theorem 4.2 in Milgrom (2004), the buyer acceptance strategy $\alpha(\theta)$ will be incentive compatible. To check smooth single-crossing differences, take (y, θ) such that $\frac{d}{dy}\tilde{U}(y, \tilde{\alpha}(y), \theta) = 0$. Taking the derivative, we have

$$\bar{S}_\alpha(\tilde{\alpha}(y), \theta) \left[\iota^S(\tilde{\alpha}(y), \theta) - \tilde{\alpha}'(y)\right] = 0. \tag{43}$$

By assumption, $\tilde{S}_\alpha > 0$, so if the above display is 0, $\iota^S(\tilde{\alpha}(y), \theta) = \tilde{\alpha}'(y)$. Then whenever the derivative exists,

$$\frac{\partial}{\partial \theta} \frac{d}{dy} \tilde{U}(y, \tilde{\alpha}(y), \theta) = \bar{S}_\alpha(\tilde{\alpha}(y), \theta) \left[\frac{\partial}{\partial v} \iota^S(\tilde{\alpha}(y), \theta)\right] > 0,$$

since the environment has adverse selection.

Now we show the relevant envelope condition. By definition, we have that for any θ and any (y, α)

$$\tilde{U}_\theta(y, \alpha, \theta) = yX'(\theta) + (1 - y)\left(V'(\theta) - \bar{S}_\theta(\alpha, \theta)\right)$$

For any $\theta \in [0, k^{SL}]$ we have

$$\begin{aligned}\tilde{U}(y(\theta), \alpha(\theta), \theta) = \tilde{U}(y(0), \alpha(0), 0) + \int_0^\theta \Big(&\tilde{U}_\theta(y(s), \alpha(s), s) \\ &+ \tilde{U}_y(y(s), \alpha(s), s)y'(s) + \tilde{U}_\alpha(y(s), \alpha(s), s)\alpha'(s)\Big) ds,\end{aligned}$$

where

$$\begin{aligned}\tilde{U}_y(\cdot)y'(s) + \tilde{U}_\alpha(\cdot)\alpha'(s) &= \left(A(s) - V(s) + \bar{S}(\alpha(s), s)\right) y'(s) - (1 - y(s))\bar{S}_\alpha(\alpha(s), s)\alpha'(s) \\ &= -\left(R(s) - \bar{S}(\alpha(s), s)\right) y'(s) - (1 - y(s))\bar{S}_\alpha(\alpha(s), s)\alpha'(s).\end{aligned}$$

From the local IC constraint way have that

$$r\left(R(K_t) - \bar{S}(\alpha(K_t), K_t)\right) = -\dot{K}_t \alpha'(K_t)\bar{S}_\alpha(\alpha(K_t), K_t).$$

By definition, on $[0, k^{SL})$, $y'(k) = re^{-r\tau(k)}\tau'(k)$ and $\alpha'(k) = \alpha'(K_{\tau(k)})\dot{K}_{\tau(k)}\tau'(k)$. Hence, multiplying both sides of the local incentive compatibility constraint by $e^{-r\tau(k)}\tau'(k)$, and

[28] $y^{-1}(z)$ is the generalized inverse $y^{-1}(z) = \sup\{x : y(x) \geq z\}$.

using the definition $K_{\tau(k)} = k$, we get

$$\left(R(k) - \bar{S}(\alpha(k), k)\right) y'(k) = -(1 - y(k))\alpha'(k)\bar{S}_\alpha(\alpha(k), k),$$

so $\tilde{U}_y(\cdot)y'(s) + \tilde{U}_\alpha(\cdot)\alpha'(s) = 0$, and we obtain equation (42). Next, we verify the envelope representation (42) for $k \in (k^{SL}, 1]$. Because $\alpha(k)$ and $y(k)$ are constant on $(k^{SL}, 1]$ and $\tilde{U}_y(\cdot)y'(s) + \tilde{U}_\alpha(\cdot)\alpha'(s) = 0$ on $\theta \in [0, k^{SL}]$ we have that

$$\begin{aligned}\tilde{U}(y(\theta), \alpha(\theta), \theta) = \tilde{U}(y(0), \alpha(0), 0) + \int_0^v \tilde{U}_\theta(y(s), \alpha(s), s)ds \\ + \tilde{U}(y(k^{SL}+), \alpha(k^{SL}+), k^{SL}) - \tilde{U}(y(k^{SL}), \alpha(k^{SL}), k^{SL}).\end{aligned}$$

By construction, the delay D in equation (11) is such

$$\tilde{U}(y(k^{SL}+), \alpha(k^{SL}+), k^{SL}) = \tilde{U}(y(k^{SL}), \alpha(k^{SL}), k^{SL}),$$

so the expected payoff $\tilde{U}(y(\theta), \alpha(\theta), \theta)$ satisfies the envelope condition (42).

Verification: Buyer's Off-Path Strategy: The only step left is to verify the optimality of the reservation price strategy $\alpha(k)$ following an off-equilibrium offer $\alpha' \notin \alpha([0, 1])$. By construction, the $\sigma(\alpha|k', \alpha')$ is such the type k^{SL} buyer is indifferent between accepting α' and reject it. Thus, we only need to verify that types above k^{SL} are better off rejecting it. By construction

$$\bar{S}(\alpha', k^{SL}) = p(k^{SL}, \alpha')\bar{S}(\alpha^f(1), k^{SL}) + (1 - p(k^{SL}, \alpha'))\bar{S}(\alpha(k^{SL}), k^{SL}).$$

Let $p' \equiv p(k^{SL}, \alpha')$, because $\bar{S}(\alpha', \theta)$ is increasing in θ, we have that

$$\begin{aligned}V(\theta) - \bar{S}(\alpha', \theta) &< V(\theta) - \bar{S}(\alpha', k^{SL}) \\ &= p'\left(V(\theta) - \bar{S}(\alpha^f(1), k^{SL})\right) + (1 - p')\left(V(\theta) - \bar{S}(\alpha(k^{SL}), k^{SL})\right) \\ &< p'\left(V(\theta) - \bar{S}(\alpha^f(1), k^{SL})\right) + (1 - p')B(\theta, k^{SL}),\end{aligned}$$

which means that types $\theta > k^{SL}$ are strictly better off rejecting α'. A similar calculation shows that types $\theta < k^{SL}$ are strictly better off accepting α'.

□

F Connection to Deneckere and Liang (2006)

In their classic analysis of interdependent values cash bargaining, Deneckere and Liang (2006) posit a privately informed seller with type $\theta \sim U[0, 1]$ and preferences

$$(1 - e^{-rt})c(\theta) + e^{-rt}\alpha$$

bargaining with an uninformed buyer with preferences

$$e^{-rt}(v(\theta) - \alpha).$$

where α is the cash offer. To highlight the tractability of our continuous-time analysis, we revisit their setup.

As in our model, their leading case assumes the uninformed party makes all the offers, there is a gap $(v(\theta) - c(\theta) > 0)$ and first-best efficiency is unattainable $(\mathbb{E}[v(\theta) - c(1)] < 0)$. Unlike our setup, t is on a discrete grid $\{0, \Delta, 2\Delta, 3\Delta, \dots\}$, and they assume v and c are step functions, i.e., there are finitely many payoff types. c is weakly increasing. They study the limit of Markovian equilibria as $\Delta \to 0$. A subtle proof shows that, in the limit, the sequence of offers is a step function: there are quiet periods of *deterministic* length, punctuated by discontinuous drops in price.

If one assumes instead that v and c are both smooth and strictly increasing, a direct consequence of Theorem 1 is that there is a unique (regular, weak Markov) outcome path.[29] Let $k^* = \inf\{k : \mathbb{E}[v(\theta) - c(1) | \theta \geq k] = 0\}$. Then, for $k < k^*$, trade is smooth according

$$\dot{K}_t = \frac{r(v(K_t) - c(K_t))}{v'(K_t)},\ K_0 = k.$$

with offers $\alpha(k) = v(k)$. At k^*, there is a (stochastic) quiet period that causes expected discounting costs δ given by

$$v(k^*) = (1-\delta)c(k^*) + \delta c(1) \Rightarrow \delta = \frac{v(k^*) - c(k^*)}{c(1) - c(k^*)},$$

after which the pooling offer $c(1)$ is made. Comparing this to their Theorem 3 and equation (15) in their paper, we see three effects of a continuous-type, continuous-time formulation. First, there is a large tractability gain, with the dynamics essentially pinned down by "first order conditions" (9) and (10). Second, the equilibrium acquires a smooth region with gradual concessions. Third, the certainty equivalent delay after reaching k^* is *half* the amount implied by their discrete-type model— *mutatis mutandi*, our expression for δ exactly matches their expression for ρ_1.

[29] The MRS between α and t for the privately informed seller is $\iota(\theta, \alpha) = -r(\alpha - c(\theta))$, which is strictly increasing in θ.