Strategic Trading and Blockholder Dynamics*

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Abstract

We study strategic trading by a privately informed blockholder who monitors a company and trades its shares. Private information results in larger block sizes in good states, but by increasing the speed of the blockholder's selling, it can result in lower block sizes in bad states. Despite the heterogeneous impact on expected block size, we show that asymmetric information leads to Pareto improvements: it raises stock prices, benefits small uninformed shareholders, and benefits the block owner, despite the negative impact on liquidity.

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1 Introduction

Blockholders play a prominent role in capital markets (Holderness (2007)). They monitor firms and promote changes that affect firm productivity through various channels (e.g., negotiations with management, proxy fights, etc.). These endeavors are costly to the blockholder, and small shareholders may free-ride on them. A blockholder thus faces a trade-off: he can

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mitigate free-riding by owning a large block, thereby enhancing his own incentive to monitor the firm. However, by doing so, he compromises his own portfolio (diversification) needs.[1]

We study strategic trading when a large blockholder is privately informed about his (time-varying) ability/cost to monitor the firm and investigate the impact of asymmetric information on the dynamics of blockholder ownership, firm productivity, and stock prices.

Empirically, the monitoring of the blockholder varies over time in ways that are difficult to anticipate for the market participants (see Hadlock and Schwartz-Ziv (2019)) as these decisions depend on the incentives and circumstances of the blockholder, which are private information. For example, sometimes, a blockholder's involvement may be boosted by the arrival of ideas and opportunities to improve firm management or be hampered by a deterioration of the relationship between the blockholder and the firm's manager. Sometimes, the blockholder's involvement may change as a consequence of variation in the blockholder's liquidity needs. This uncertain blockholder involvement is not only empirically relevant but also has important consequences that have not been theoretically studied.

Building on Admati et al. (1994) and DeMarzo and Urošević (2006), we consider a dynamic trading model between a large investor (or blockholder) and a continuum of competitive investors but consider an environment with private information.[2] In each period, the blockholder can both trade and work/monitor the firm to increase the firm's cash flows. Crucially, the blockholder cannot commit to holding a large block; thus, he trades over time based on his private information and portfolio preferences. The productivity of the blockholder's effort is private information and varies over time. Thus, we depart from previous literature by considering a setting that combines moral hazard and trading under asymmetric information. We also contribute to the literature studying the signaling role of ownership retention and extend it to a dynamic environment (Leland and Pyle, 1977; Gale and Stiglitz, 1989; DeMarzo and Duffie, 1999).[3]

[1]These trade-offs have been long identified by corporate governance scholars and practitioners, at least going back to work by Berle and Means (1932), Alchian and Demsetz (1972), and Jensen and Meckling (1976).

[2]The seminal papers on large shareholder monitoring are Huddart (1993) and Admati et al. (1994). The closest paper is DeMarzo and Urošević (2006), which extends the static models to a fully dynamic environment where blockholders can monitor and trade over time. Unlike DeMarzo and Urošević (2006), we consider a setting with asymmetric information.

[3]Gomes (2000) also studies a reputation game with two types of managers/owners who differ in terms of their cost of effort. In Gomes (2000), the manager's effort is observable. Unlike in Gomes, we allow for hidden effort and time-varying private information.

As in settings without asymmetric information, the blockholder sells his entire block over time – regardless of his ability to increase firm value via monitoring effort – due to lack of commitment (DeMarzo and Urošević, 2006).

We assume that the blockholder bears holding costs to capture the blockholder's liquidity and diversification needs. Two cases need to be distinguished depending on the magnitude of holding costs. First, when the blockholder's holding costs are low, the blockholder's continuation payoff is convex in his stake, giving rise to increasing returns to scale. This convexity arises because the monitoring effort is proportional to the blockholder's stake. In this case, the blockholder refrains from selling in the high-productivity state and sells smoothly in the low state.

However, when the cost of holding a large position is high, the blockholder's continuation payoff becomes concave in the position. In this case, the blockholder sells smoothly under high productivity, but as soon as the productivity drops, the blockholder liquidates all his holdings, consistent with the Coase conjecture.

We study the welfare impact of asymmetric information. Contrary to static models, where signaling entails deadweight costs to the seller, in our dynamic setting, the presence of private information yields a Pareto improvement relative to the symmetric information case. On the one hand, asymmetric information benefits small, uninformed shareholders because it leads to a larger block, thereby boosting blockholder monitoring and, ultimately, the firm's cash flows. As mentioned above, by reducing liquidity, asymmetric information reduces the speed of selling in high-productivity states, thus extending the length of the blockholder's monitoring, particularly when it is most effective. This leads to a higher stock price. In turn, this increases the dividends earned by small shareholders.

On the other hand, the blockholder's payoff weakly increases when he has access to private information, unlike in a static setting where private information would typically force the blockholder to signal his type through inefficient retention (Leland and Pyle, 1977; Vanasco, 2017). Again, the size of this effect depends on the blockholder's cost of holding a large position. When this cost is small, asymmetric information has no impact on the blockholder's payoff. In other words, the blockholder obtains the same payoff regardless of the information structure. The reason is that, due to lack of commitment, the blockholder can neither extract rents from trading nor bear signaling costs as in a static setting but trades in a competitive fashion, regardless of whether he has access to private information or not.

Surprisingly, when the blockholder's holding cost is high, the blockholder's payoff is higher under asymmetric information, particularly in the high-productivity state. In this context, it becomes too costly for the low type to sell slowly, and as mentioned above, the low type sells immediately. Since the low type exits the market immediately, the high type faces a more liquid market thereafter, which allows him to sell the stock without triggering a large price drop. Thus, when holding costs are large, the blockholder is able to extract some of the gains from trade. Indeed, under asymmetric information, the high type refrains from selling too fast because of its price impact, which acts as a commitment device that mitigates Coasian forces, allowing the blockholder to extract rents from trade.

So, contrary to conventional wisdom, the blockholder's access to private information improves the welfare of uninformed shareholders without having an adverse effect on the blockholder's payoff, in contrast with a setting without monitoring[4]

We consider two extensions to our analysis. First, we extend the analysis of our two-type model to consider the case when there is a finite number of types. We find that the main insights of the two-type model remain when there are multiple types. Interestingly, we find that when the lowest type payoff is convex in his stake, there is a breakdown in trade. Only the lowest type trade, and all the remaining types refrain from selling. However, when the lowest type payoff is concave in his stake, this type exits immediately, which allows the rest of the types to trade at positive rates that are decreasing in their types.

Second, we also consider pooling equilibrium. Throughout the paper, we focus on the least costly separating equilibrium. In this equilibrium, reputation concerns provide some level of commitment to the high type. In his attempt to separate from the low type, the high type refrains from quickly selling his shares, which ameliorates the commitment problem. On the other hand, in the pooling equilibrium, it is the low type's commitment problem that is ameliorated by reputation concerns. In an attempt to pool with the high type, the low type trades at a rate that is lower than if his type were known. In this way, reputational concerns allow an increase in the low type equilibrium payoff.

[4]This result is reminiscent of the theory of the second best (Lipsey and Lancaster, 1956), whereby two frictions combined (lack of commitment and asymmetric information) lead to a more efficient outcome than a single friction (lack of commitment).

Related Literature

Our paper speaks to the literature on the role of liquidity in corporate governance. A key issue in this literature is that a blockholder may have incentives to sell his shares ("cut and run") instead of bearing the cost of monitoring, particularly when the firm is underperforming. This has led some authors to conclude that market liquidity might be detrimental to corporate governance (Coffee, 1991; Kahn and Winton, 1998; Noe, 2002; Faure-Grimaud and Gromb, 2004).[5]

One counterargument is that liquidity might reduce the free-riding problem in takeovers (Grossman and Hart, 1980; Shleifer and Vishny, 1986). By facilitating the creation of a large block in the first place, liquidity can actually strengthen the firm's corporate governance and improve performance (Kyle and Vila, 1991; Maug, 1998; Back et al., 2018).[6] Another counterargument is that liquidity facilitates the use of "voice" as a governance mechanism (Hirschman, 1970).[7] Indeed, if the manager's compensation is tied to the price of the company, so the manager is hurt by selling forces that would bring the price down, investors can discipline the firm by threatening to sell their shares (Admati and Pfleiderer, 2009; Edmans, 2009). Our results support the notion that illiquidity/adverse selection can have a positive impact as it mitigates blockholders' lack of commitment to keeping their shares and monitoring the firm, especially when this is most useful, i.e., when productivity is high.[8]

To conclude, we note that our paper is related to the literature on durable good monopoly with incomplete information and to the literature on bargaining with two-sided asymmetric information (Cho, 1990; Ausubel and Deneckere, 1992). The closest paper in this literature is Ortner (2020), which considers a bargaining model with time-varying costs. Due to the different focus and applications, our model differs in a number of ways. First, unlike this previous literature, we consider a setting with common values in which the seller's private

[5]This idea has been behind policy proposals attempting to reduce trading. For example, the European Union agreed to implement a transaction tax in September 2016.

[6]We depart from this literature by considering a dynamic signaling model – in which blockholdings are observable – rather than a microstructure model, where trading is unobservable and trading by the blockholder is obscured by the presence of noise traders.

[7]This literature is surveyed in Becht, Bolton, and Röell (2003) and Edmans and Holderness (2017).

[8]A related literature on loan sales and security design considers the impact that liquidity in secondary markets has on ex-ante screening of project quality. For example, Vanasco (2017) studies the role that adverse selection may play in fostering ex-ante screening to originate projects of high quality.

information affects the buyer's valuation. Second, the buyer's valuation is directly affected by the blockholder trading strategy. These differences have a significant effect on the nature of the equilibrium. In contrast to the literature looking at durable goods monopolies with two-sided incomplete information, we derive conditions under which the equilibrium entails trade by all types and positive rents for blockholders. We show that the Coase conjecture holds (in the sense that the monopolist is unable to extract rents) only when the blockholder's cost of holding large positions is small. However, we show that the Coase conjecture fails if this cost is sufficiently high. In durable good monopoly and bargaining models, incomplete information about costs generates an extreme form of inefficiency by completely eliminating trade. On the contrary, we show that asymmetric information increases welfare in our setting, where blockholding has an effect on productivity. Not only does overall welfare increase, but we also show that the equilibrium with asymmetric information Pareto dominates the equilibrium with symmetric information.

2 Setting

Drawing from the works of Admati et al. (1994) and DeMarzo and Urošević (2006) (2006), we delve into an examination of the actions undertaken by a large investor, referred to as a blockholder, who possesses the ability to engage in stock trading and incur expenses for activities such as monitoring in order to enhance a firm's overall productivity. Our analysis encompasses not only the influence of a blockholder but also considers a vast array of small investors who act as price-takers and lack the capacity to impact the firm's cash flows.

Asset Time t is continuous, and the horizon is infinite. There is a single firm in unit supply with cumulative cash flows δ_t

$$d\delta_t = (\mu + \theta_t a_t)dt + dM_t, \tag{1}$$

where $a_t \geq 0$ is the blockholder's effort, $\theta_t \in \{\underline{\theta}, \bar{\theta}\}$ where $0 \leq \underline{\theta} < \bar{\theta}$, is the blockholder's productivity, and M_t is a martingale. By assuming that the blockholder effort increases the firm's dividends, we capture the idea that the blockholder's incentives to exert effort are proportional to the blockholder's ownership. In turn, this assumption will drive a connection between the stake of the owner and the value of the firm, as documented in Larrain et al. (2020). We assume a multiplicative technology for a_t and θ_t to capture the fact that empirically there is wide variation in blockholder involvement, which suggests that blockholders differ in terms of productivity (see, e.g., Hadlock and Schwartz-Ziv (2019)).

To be specific, variation in θ_t may capture the fact that the arrival of ideas and projects to improve firm management is random. Alternatively, this may capture the idea that the relationship between a blockholder and the firm management is subject to shocks that affect the ability or willingness of the blockholder to engage the firm and boost its performance.

By blockholder productivity, we mean the quality of the match between a firm and the blockholder broadly. This quality is uncertain and varies over time, insofar as the blockholder's incentives and ability to monitor –as well as the intensity of agency frictions– also vary randomly (an alternative interpretation is that θ_t captures the blockholder's opportunity cost of monitoring the firm, which depends on how busy the blockholder is at a given point in time).

We assume that the cash flows δ_t are publicly observable, but the blockholder's effort a_t and productivity θ_t are not, so there is a moral hazard problem. Moreover, due to the shock M_t, the blockholder's output $a_t\theta_t$ cannot be perfectly inferred from the realized cash flows δ_t.[9] The realized cash flows are distributed as dividends to shareholders in each period. Therefore, we interpret δ_t as the company's dividend payout. Conditional on θ_t, the firm's dividend δ_t is random. This assumption is important because otherwise, the market would infer the state θ_t from the dividend, and the blockholder's trading would not be informative about the fundamentals θ_t.

We refer to a_t as blockholder effort but interpret it broadly as any costly action undertaken by the blockholder affecting the firm's cash flows. We are agnostic as to the source of this effect. In the case of an external investor, one can think of a_t as the blockholder's monitoring of the firm's management — which disciplines managers and mitigates agency conflicts— or as the influence the blockholder exerts on the firm's management choices (as in Admati et al. (1994); Stoughton and Zechner (1998); DeMarzo and Urošević (2006)). Examples of a_t include public criticism of management or launching a proxy fight, advising management on strategy, figuring out how to vote on proxy contests launched by others, or abstaining from

[9] For example, this is the case if M_t is a Brownian motion or a compensated Poisson process $M_t = N_t^{\theta,a} - \theta_t a_t$, where $N_t^{\theta,a}$ is a Poisson process with intensity $\theta_t a_t$.

extracting private benefits at the expense of minority shareholders. In the case of a CEO or founder of a company, a_t can represent effort or a reduction of private benefits that increase the productivity of the firm.

Productivity follows a two-state Markov-chain with switching intensity $\{\lambda_H, \lambda_L\}$, where λ_H is the switching intensity from $\underline{\theta}$ to $\bar{\theta}$, and λ_L is the switching intensity from $\bar{\theta}$ to $\underline{\theta}$. We generalize this aspect of the model in section 5.1. The probability of the high state in the stationary distribution is $\lim_{t\to\infty} \Pr(\theta_t = \bar{\theta}) = \bar{\pi} \equiv \lambda_H/(\lambda_L + \lambda_H)$, and the mean reversion of the process is $\lambda \equiv \lambda_L + \lambda_H$. Whenever convenient, we parameterize the stochastic process θ_t in terms of $(\lambda, \bar{\pi})$.

Information: The owner of the company observes the productivity θ_t privately in addition to the public dividend δ_t. The market observes the dividend δ_t and the blockholder's block x_t. We denote by $\hat{\mathbb{E}}_t[\cdot]$ the expected value at time t given the market's information and let $\mathbb{E}_t[\cdot]$ denote the expected value given the blockholder's information.

Strategies: The blockholder selects effort a_t and holds a stake or block $x_t \in [0, 1]$. The blockholder chooses effort continuously but is restricted to trade at dates $t \in \{0, t_1, t_2, \dots, t_N\}$ where $t_n = n\Delta$ for $n \le N$ and $t_{N+1} = \infty$. The stake x_t is constant between trading dates, so $x_t = x_{t_n}$ for all $t \in [t_n, t_{n+1})$. We denote the quantity traded by the blockholder at t_n by $\Delta x_n = x_{t_n} - x_{t_n-}$.

Although our main focus will be the continuous time (infinite horizon) case, we assume here that there are a finite number of trading periods so we can use the concept of Perfect Bayesian Equilibrium, and we then take the limit as $N \to \infty$ and $\Delta \to 0$.

Preferences: Small competitive investors (hereafter the market) are risk neutral and discount future cash flows at a rate r. Because competitive investors are risk neutral, in equilibrium, the price of the stock is given by

$$p_t = \hat{\mathbb{E}}_t\left[\int_t^\infty e^{-r(s-t)}(\mu + \theta_s \hat{a}_s)ds\right],$$

where $\hat{a}_t$ is the effort of the blockholder conjectured by the market. The blockholder expected payoff is given by

$$V_0 = \mathbb{E}\left[\int_0^\infty e^{-rs}u(x_s, a_s, \theta_s)ds - \sum_{t_n \geq 0} e^{-rt_n} p_{t_n} \Delta x_n\right],$$

where the flow payoff $u(x, a, \theta)$ is given by

$$u(x, a, \theta) = (\mu + \theta a)x - \frac{1}{2}\left(\phi^{-1}a^2 + \gamma x^2\right).$$

The first term, $(\mu+\theta a)x$ corresponds to the expected cash flows $x_t\,\mathbb{E}_t[d\delta_t]$. The second term, $a^2/2\phi$, is the private cost of effort, where $1/\phi$ captures the severity of moral hazard and the degree to which the blockholder will become involved. In practice, this varies between types of blockholders. Indeed, the empirical literature has documented that blockholder involvement varies across blockholder types, for example, being relatively weaker for financial blockholders.[10] In our model, this would be consistent with financial blockholders having a high cost of monitoring, or a small ϕ.

The final term $\gamma x^2/2$ captures the costs of holding a stake x. Although the holding cost cannot be directly linked to risk aversion, it can represent the financing cost of holding a large position in the firm[11] The quadratic holding cost is popular among financial institutions practitioners (Almgren and Chriss, 2001), and has also been used extensively in the dynamic trading literature (Vives, 2011; Du and Zhu, 2017; Duffie and Zhu, 2017).[12]

Equilibrium definition We study the Perfect Bayesian Equilibrium of the game. On the one hand, because the effort is not observable and the cash flows are noisy (with full support

[10]For example, Hadlock and Schwartz-Ziv (2019) argue that "many of the data patterns can be interpreted as consistent with a governance role through monitoring by nonfinancial blocks and through trading for financial blocks."

[11]The one exception where the holding cost can be directly linked to risk aversion is the case in which the cash flow process is Gaussian, and traders have CARA preferences.

[12]As Duffie and Zhu (2017) point out holding costs may be related to regulatory capital requirements, collateral requirements, financing costs, agency costs related to the lack of transparency of the position to higher-level firm managers or clients regarding true asset quality, as well as the expected cost of being forced to raise liquidity by quickly disposing the remaining inventory into an illiquid market. This assumption is common in models of trading, as well as in models of divisible auctions. See e.g. Vives (2011); Rostek and Weretka (2012); Du and Zhu (2017).

for all effort levels), there is no need to consider the beliefs about deviations in effort a_t.

If $\alpha_{t_{n+1}}$ denotes the market's belief that the blockholder type is $\bar{\theta}$ at the beginning of period t_{n+1}, and $V_{n+1}(x, \theta, \alpha)$ denotes the blockholder's continuation value at the beginning of period t_{n+1}, then the effort policy $a = (a_t)_{t \geq 0}$ must solve

$$\max_{(a_s)_{s \in [t, t_{n+1})}} \mathbb{E}_t \left[\int_t^{t_{n+1}} e^{-r(s-t)} u(x_{t_n}, a_s, \theta_s) ds + e^{-r(t_{n+1}-t)} V_{n+1}(x_{t_n}, \theta_{t_{n+1}}, \alpha_{t_{n+1}}) \right]$$

Since the block x_t is observable and θ_t is private information, we need to consider the direct impact of trading on market beliefs. As is well known, signaling models have multiple equilibria. Therefore, we need a refinement criterion. We focus on equilibria that survive repeated application of the Intuitive Criterion of Cho and Kreps (1987) starting from the final trading date t_N. In particular, at each trading date, t_n, we consider equilibria satisfying the Intuitive Criterion in the stage game with blockholder's payoff

$$\hat{V}_n \left(x + \Delta x, \theta, \alpha \right) + \hat{p}_n \left(x + \Delta x, \alpha \right) \Delta x,$$

where α is the market belief, $\hat{p}_n$ is the price, and $\hat{V}_n$ is the post-trade continuation payoff, given by

$$\hat{V}_n(x, \theta, \alpha) = \max_{(a_s)_{s \in [t_n, t_{n+1})}} \mathbb{E}_{t_n} \left[\int_{t_n}^{t_{n+1}} e^{-r(s-t_n)} u(x, a_s, \theta_s) ds + e^{-r(t_{n+1}-t_n)} V_{n+1}(x, \theta_{t_{n+1}}, \alpha_{t_{n+1}}) \right].$$

This approach will allow us to characterize the least costly separating equilibrium of the trading game. In this context, we can drop the dependence of the continuation payoff on the market belief α and denote the continuation payoffs before and after trading at t_n by $V_n(x, \theta)$ and $\hat{V}_n(x, \theta)$, respectively.

Our model is closely related to models of durable goods monopoly with incomplete information about costs and models of bargaining with one-sided offers and two-sided incomplete information (Ausubel and Deneckere (1992); Cho (1990); Ortner (2020)). In particular, the existing literature highlights that a broad range of equilibria can be sustained by incorporating pessimistic off-equilibrium beliefs. These beliefs assign a substantial probability to the weakest type in the event of a deviation. This approach has been well-documented in the literature (Cho (1990) and Ortner (2020)) and aligns with our emphasis on exploring separating equilibria.

3 Equilibrium Characterization

3.1 Final Trading Date

We solve the equilibrium by backward induction. The first step is to find the stake of the blockholder on the last trading date. This corresponds to a static signaling model in which the blockholder starts the period with $x_N = x_{t_N-}$ and chooses the final stake $x_{t_N} = x_{N+1}$.

Once trading is no longer possible, the blockholder effort a_t is chosen to maximize the flow payoff $u(x_{N+1}, a_t, \theta_t)$, which yields that $a_t = \phi\theta_t x_{N+1}$. Conditional on $\theta_{t_N} = \theta$ the buyers' valuation boils down to

$$\hat{p}_N(x_{N+1}, \theta) = \frac{\mu}{r} + \left(\hat{C}_N(\theta) + \frac{\gamma}{r}\right) x_{N+1}$$

and the blockholder payoff is

$$\hat{V}_N(x_{N+1}, \theta) = \frac{\mu}{r} x_{N+1} + \frac{1}{2}\hat{C}_N(\theta) x_{N+1}^2$$

where $\hat{C}_N(\theta) = C^{\text{nt}}(\theta)$ is the coefficient corresponding to the expected payoff without trade,[13] which is given by

$$\begin{aligned} rC^{\text{nt}}(\underline{\theta}) &= \phi\underline{\theta}^2 - \gamma + \phi\frac{\lambda_H}{r + \lambda_L + \lambda_H}(\bar{\theta}^2 - \underline{\theta}^2) \\ rC^{\text{nt}}(\bar{\theta}) &= \phi\bar{\theta}^2 - \gamma - \phi\frac{\lambda_L}{r + \lambda_L + \lambda_H}(\bar{\theta}^2 - \underline{\theta}^2). \end{aligned} \tag{2}$$

If the blockholder trades Δx_N and the market beliefs are α, then the trading price at t_N is

$$\hat{p}_N(x_N + \Delta x_N, \alpha) = \alpha\hat{p}_N(x_N + \Delta x_N, \bar{\theta}) + (1 - \alpha)\hat{p}_N(x_N + \Delta x_N, \underline{\theta}).$$

Thus, if the market beliefs are $\alpha_N(\Delta x_N)$ then the blockholder chooses the trading quantity

[13] The payoff without trade is

$$V^{\text{nt}}(x, \theta) = \mathbb{E}\left[\int_0^\infty e^{-rt} u(x, \phi\theta_t x, \theta_t) dt \middle| \theta_0 = \theta\right] = \frac{\mu}{r} x + \frac{1}{2} C^{\text{nt}}(\theta) x^2.$$

Δx_N by solving

$$\Delta x_N(x,\theta) = \arg\max_{\Delta x}\ \hat{V}_N(x+\Delta x,\theta) + \hat{p}_N(x+\Delta x,\alpha_N(\Delta x))\Delta x.$$

By standard arguments, the unique equilibrium satisfying the Intuitive Criterion is the least costly separating equilibrium. Thus the trade of the low type $\Delta x_N(x,\underline{\theta})$ solves

$$V_N(x,\underline{\theta}) = \max_{\Delta x}\ \hat{V}_N(x+\Delta x,\underline{\theta}) + \hat{p}_N(x+\Delta x,\underline{\theta})\Delta x, \tag{3}$$

whereas the high-type trading $\Delta x_N(x,\bar{\theta})$ solves

$$\begin{aligned} V_N(x,\bar{\theta}) = \max_{\Delta x}\ & \hat{V}_N(x+\Delta x,\bar{\theta}) + \hat{p}_N(x+\Delta x,\bar{\theta})\Delta x \\ & \text{s.t} \\ & \hat{V}_N(x+\Delta x,\underline{\theta}) + \hat{p}_N(x+\Delta x,\bar{\theta})\Delta x \leq V_N(x,\underline{\theta}), \end{aligned} \tag{4}$$

For signaling reasons, the high type cannot sell as much as he would in a symmetric information environment: he has to retain a sufficiently large number of shares to convey the firm's value credibly. Put differently, to sustain a separating equilibrium; the high type has to sell relatively little to ensure that the low type does not have the incentive to deviate and mimic him.

Proposition 1 provides the solution to this problem.

Proposition 1. *On the last trading date t_N, the traded quantity of the blockholder, under the least costly separating equilibrium, is $\Delta x_N(x,\theta) = -\beta_N(\theta)x$, where*

$$\beta_N(\underline{\theta}) = \frac{\gamma}{r\hat{C}_N(\underline{\theta}) + 2\gamma}$$

$$\beta_N(\bar{\theta}) = \frac{\left(r\hat{C}_N(\bar{\theta})+\gamma\right) - r\hat{C}_N(\underline{\theta}) - \sqrt{\left(r\hat{C}_N(\bar{\theta})+\gamma\right)^2 - 2rC_N(\underline{\theta})\left(r\hat{C}_N(\bar{\theta})+\gamma-\frac{1}{2}r\hat{C}_N(\underline{\theta})\right)}}{2\left(r\hat{C}_N(\bar{\theta})+\gamma\right) - r\hat{C}_N(\underline{\theta})}.$$

and the market beliefs are

$$\alpha_N(\Delta x) = \begin{cases} 1 & \textit{if } \Delta x \geq -\beta_N(\bar{\theta})x \\ 0 & \textit{if } \Delta x < -\beta_N(\bar{\theta})x. \end{cases}$$

The blockholder payoff is

$$V_N(x, \theta) = \frac{\mu}{r}x + \frac{1}{2}C_N(\theta)x^2,$$

where

$$\begin{aligned} C_N(\underline{\theta}) &= \frac{1}{r}\frac{\left(r\hat{C}_N(\underline{\theta}) + \gamma\right)^2}{r\hat{C}_N(\underline{\theta}) + 2\gamma} \\ C_N(\bar{\theta}) &= C_N(\underline{\theta}) + \left(1 - \beta_N(\bar{\theta})\right)^2 \Gamma_N \\ \Gamma_N &= \hat{C}_N(\bar{\theta}) - \hat{C}_N(\underline{\theta}). \end{aligned}$$

The equilibrium in the last trading date is equivalent to a static version of the game. The blockholder sells only a fraction of his holdings. Because of the signaling incentives, the high type retains a bigger stake than he would under symmetric information. This inefficiency reduces the payoff of the high type relative to the symmetric information case. However, asymmetric information favors the small initial shareholders, who benefit from the blockholder's stronger incentives to retain his holdings and exert effort, thereby increasing the firm value. As we shall see, the welfare properties of the equilibrium change dramatically in the fully dynamic game.

3.2 Equilibrium with Discrete Trading

The next step is to characterize the equilibrium for trading dates preceding the last one, $t_n < t_N$. If the blockholders' continuation payoff at time t_{n+1} is independent of the market's prior belief, then so is the post-trade value function $\hat{V}_n(t, x, \bar{\theta})$ at time t, which satisfies the following HJB equation on $t \in (t_n, t_{n+1})$

$$r\hat{V}_n(t,x,\bar{\theta}) = \max_a \; (\mu + \bar{\theta}a)x - \frac{1}{2}\left(\phi^{-1}a^2 + \gamma x^2\right) + \dot{\hat{V}}_n(t,x,\bar{\theta}) + \lambda_L\left(\hat{V}_n(t,x,\underline{\theta}) - \hat{V}_n(t,x,\bar{\theta})\right)$$
$$r\hat{V}_n(t,x,\underline{\theta}) = \max_a \; (\mu + \underline{\theta}a)x - \frac{1}{2}\left(\phi^{-1}a^2 + \gamma x^2\right) + \dot{\hat{V}}_n(t,x,\underline{\theta}) + \lambda_H\left(\hat{V}_n(t,x,\bar{\theta}) - \hat{V}_n(t,x,\underline{\theta})\right),$$

with terminal condition $\hat{V}_n(t_{n+1},x,\theta) = V_{n+1}(x,\theta)$. It follows that the optimal effort is

$$a(x,\theta) = \phi\theta x. \tag{5}$$

By standard results, the transition probability of θ_t is $\Pr(\theta_{t_{n+1}} = \bar{\theta}|\theta_{t_n} = \theta) = \pi(\Delta|\theta)$ where

$$\pi(\Delta|\bar{\theta}) = \bar{\pi} + (1-\bar{\pi})e^{-\lambda\Delta}$$
$$\pi(\Delta|\underline{\theta}) = \bar{\pi}\left(1 - e^{-\lambda\Delta}\right).$$

Substituting the optimal effort policy in the HJB equation and integrating backward, we get that the post-trade continuation value $\hat{V}_n(x,\theta) \equiv \hat{V}_n(t_n,x,\theta)$ is given by

$$\hat{V}_n(x,\theta) = \frac{1}{r}\left(1-e^{-r\Delta}\right)\left(\mu x + \frac{1}{2}(\nu(\theta)-\gamma)x^2\right)$$
$$+ e^{-r\Delta}\left[\pi(\Delta|\theta)V_{n+1}(x,\bar{\theta}) + (1-\pi(\Delta|\theta))V_{n+1}(x,\underline{\theta})\right],$$

where

$$\nu(\bar{\theta}) = \phi\underline{\theta}^2 + \phi\left(\bar{\theta}^2 - \underline{\theta}^2\right)\left[\bar{\pi} + (1-\bar{\pi})\frac{r}{r+\lambda}\left(\frac{1-e^{-(r+\lambda)\Delta}}{1-e^{-r\Delta}}\right)\right]$$
$$\nu(\underline{\theta}) = \phi\underline{\theta}^2 + \phi\left(\bar{\theta}^2 - \underline{\theta}^2\right)\left[\bar{\pi} - \bar{\pi}\frac{r}{r+\lambda}\left(\frac{1-e^{-(r+\lambda)\Delta}}{1-e^{-r\Delta}}\right)\right],$$

Similar calculations lead to the market's post-trade valuation at time t_n conditional on θ_{t_n}. Letting $p_{n+1}(x,\theta) \equiv \hat{p}_{n+1}\left(x + \Delta x_{n+1}(x,\theta),\theta\right)$ be the price at the beginning of the trading date $n+1$, we can write the price at the end of the trading date n as

$$\hat{p}_n(x,\theta) = \frac{1}{r}\left(1-e^{-r\Delta}\right)(\mu + \nu(\theta)x) + e^{-r\Delta}\left[\pi(\Delta|\theta)p_{n+1}(x,\bar{\theta}) + (1-\pi(\Delta|\bar{\theta}))p_{n+1}(x,\theta)\right].$$

We can proceed inductively at t_n, as in the last trading date t_N. We consider the trading policy

$$\Delta x_n(x,\theta) = \arg\max_{\Delta x} \; \hat{V}_n(x+\Delta x,\theta) + \hat{p}_n(x+\Delta x, \alpha_n(\Delta x))\Delta x.$$

where $\hat{p}_n(x+\Delta x,\alpha)$ is the price at t_n given market beliefs α, which is given by

$$\hat{p}_n(x_n+\Delta x_n,\alpha) = \alpha\hat{p}_n(x_n+\Delta x_n,\bar{\theta}) + (1-\alpha)\hat{p}_n(x_n+\Delta x_n,\underline{\theta}).$$

Once again, by standard arguments, the Intuitive Criterion selects the least costly separating equilibrium. The trading strategy $\Delta x_n(x,\theta)$ is the solution to optimization problems identical to (3) and (4) in which the incentive constraint of the weaker type is binding. The next proposition provides a characterization of the equilibrium.

Proposition 2 (Discrete Trading Asymmetric Information)**.** *Suppose that $\beta_n(\bar{\theta})$ in equation (6) satisfies*

$$\beta_n(\bar{\theta}) \leq \frac{\hat{p}_n(\bar{\theta}) - \hat{C}_n(\bar{\theta})}{2\hat{p}_n(\bar{\theta}) - \hat{C}_n(\bar{\theta})}.$$

Then, in the least costly separating equilibrium, the blockholder trading strategy at time t_n is $\Delta x_n(x,\theta) = -\beta_n(\theta)x$, where

$$\begin{aligned}\beta_n(\underline{\theta}) &= \frac{\hat{p}_n(\underline{\theta}) - \hat{C}_n(\underline{\theta})}{2\hat{p}_n(\underline{\theta}) - \hat{C}_n(\underline{\theta})}\\ \beta_n(\bar{\theta}) &= \frac{\hat{p}_n(\bar{\theta}) - \hat{C}_n(\underline{\theta}) - \sqrt{\hat{p}_n(\bar{\theta})^2 - 2\hat{p}_n(\bar{\theta})C_n(\underline{\theta}) + C_n(\underline{\theta})\hat{C}_n(\underline{\theta})}}{2\hat{p}_n(\bar{\theta}) - \hat{C}_n(\underline{\theta})}.\end{aligned} \tag{6}$$

and the market's belief is

$$\alpha_n(\Delta x) = \begin{cases} 1 & \text{if } \Delta x \geq -\beta_n(\bar{\theta})x \\ 0 & \text{if } \Delta x < -\beta_n(\bar{\theta})x. \end{cases} \tag{7}$$

The post-trade price $\hat{p}_n(x,\theta)$ and the blockholder payoffs $V_n(x,\theta)$ are

$$\begin{aligned}\hat{p}_n(x,\theta) &= \frac{\mu}{r} + \hat{p}_n(\theta)x\\ V_n(x,\theta) &= \frac{\mu}{r}x + \frac{1}{2}C_n(\theta)x^2\end{aligned} \tag{8}$$

where

$$C_n(\underline{\theta}) = \frac{\hat{p}_n(\underline{\theta})^2}{2\hat{p}_n(\underline{\theta}) - \hat{C}_n(\underline{\theta})}$$
$$C_n(\bar{\theta}) = C_n(\underline{\theta}) + \left(1 - \beta_n(\bar{\theta})\right)^2 \Gamma_n,$$

and the coefficients $\hat{C}_n(\underline{\theta}), \hat{p}_n(\theta), \Gamma_n$ *satisfy the difference equation.*

$$\hat{C}_n(\underline{\theta}) = \frac{1}{r}\left(1 - e^{-r\Delta}\right)\left(\nu(\underline{\theta}) - \gamma\right) + e^{-r\Delta}\left[C_{n+1}(\underline{\theta}) + \pi(\Delta|\underline{\theta})\left(1 - \beta_{n+1}(\bar{\theta})\right)^2 \Gamma_{n+1}\right]$$
$$\Gamma_n = \frac{1}{r}\left(1 - e^{-r\Delta}\right)\left(\nu(\bar{\theta}) - \nu(\underline{\theta})\right) + e^{-(r+\lambda)\Delta}\left(1 - \beta_{n+1}(\bar{\theta})\right)^2 \Gamma_{n+1}$$
$$\hat{p}_n(\theta) = \frac{1}{r}\left(1 - e^{-r\Delta}\right)\nu(\theta) + e^{-r\Delta}\left[(1 - \pi(\Delta|\theta))C_{n+1}(\underline{\theta}) + \pi(\Delta|\theta)\left(1 - \beta_{n+1}(\bar{\theta})\right)\hat{p}_{n+1}(\bar{\theta})\right].$$

Two features are worth noting. In each period, both blockholder types sell a positive amount –which, as we shall see, does not hold in the continuous-time limit. The high type sells less than the low type and less than under symmetric information due to a stronger price impact. The high type faces a relatively illiquid market, which forces him to retain a larger stake than he would under symmetric information. Although the stake of the blockholder decreases over time, it is always strictly positive.[14]. The relatively low-frequency of trading (compared to the continuous-time case) gives the blockholder some commitment power and mitigates Coasian forces. Indeed, as we shall see, increasing the frequency of trading will exacerbate Coasian forces, leading the blockholder to eventually sell his entire stake.

We are now equipped to derive the limit as $N \to \infty$. In the infinite horizon limit when $N \to \infty$, the equilibrium in Proposition 2 becomes stationary and converges to a linear equilibrium with coefficients $(\beta_\Delta(\theta), \hat{p}_\Delta(\theta), \hat{C}_\Delta(\theta))$ where $\beta_\Delta(\theta)$ is given by (6) and $(\hat{p}_\Delta(\theta), \hat{C}_\Delta(\theta))$ solve the system of equations

Corollary 1. *In the limit, as the number of trading periods grows large $N \to \infty$ the valuation*

[14] The evidence in the USA is broadly consistent with these dynamics: on average, insiders decrease their ownership by 1% per year after IPO. A majority of these firms have insider ownership below 20% after 10 years. See Helwege et al. (2007). Internationally, a similar pattern is found in countries with strong investor protection. See Foley and Greenwood (2009)

coefficients satisfy

$$\hat{C}_\Delta(\underline{\theta}) = \frac{1}{r}\left(1-e^{-r\Delta}\right)\left(\nu(\underline{\theta})-\gamma\right) + e^{-r\Delta}\left[\frac{\hat{p}_\Delta(\underline{\theta})^2}{2\hat{p}_\Delta(\underline{\theta})-\hat{C}_\Delta(\underline{\theta})} + \pi(\Delta|\underline{\theta})\left(1-\beta_\Delta(\bar{\theta})\right)^2\Gamma_\Delta\right]$$

$$\hat{p}_\Delta(\theta) = \frac{1}{r}\left(1-e^{-r\Delta}\right)\nu(\theta) + e^{-r\Delta}\left[(1-\pi(\Delta|\theta))\frac{\hat{p}_\Delta(\underline{\theta})^2}{2\hat{p}_\Delta(\underline{\theta})-\hat{C}_\Delta(\underline{\theta})} + \pi(\Delta|\theta)\left(1-\beta_\Delta(\bar{\theta})\right)\hat{p}_\Delta(\bar{\theta})\right]$$

$$\Gamma_\Delta = \frac{1}{r}\frac{\left(1-e^{-r\Delta}\right)\left(\nu(\bar{\theta})-\nu(\underline{\theta})\right)}{1-e^{-(r+\lambda)\Delta}\left(1-\beta_\Delta(\bar{\theta})\right)^2}.$$

Unlike in the static model, here, the blockholder eventually sells all his holdings over time, so all the benefits arising from the blockholder effort eventually vanish.

Figure 1 illustrates the impact of Δ on the equilibrium payoffs. The price coefficient $\hat{p}_\Delta(\theta)$ is decreasing in Δ due to the higher cash flows associated with larger blockholdings. The non-monotonicity of $\hat{C}_\Delta(\theta)$ reflects the Blockholder's cost of infrequent trading. However, the pre-trade value $C_\Delta(\theta)$ is increasing in Δ because the higher price obtained by the blockholder in the current period compensates for the future cost of less frequent trading.

3.3 Continuous Time Limit

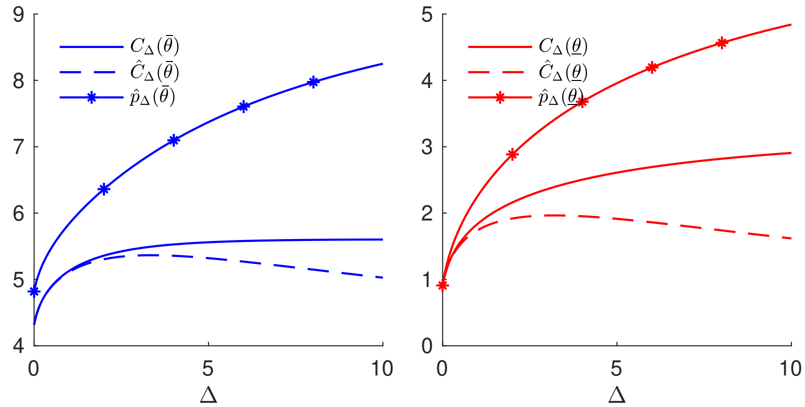
In this section, we consider the limit with continuous-time trading. Unfortunately, the system of equilibrium coefficients cannot be solved in closed form in the infinite-horizon limit. However, we can still find a clear characterization of the equilibrium. For a fixed $\Delta = t_{n+1} - t_n$, we let N go to infinity, so $t_N \to \infty$, and then take the limit when $\Delta \to 0$. The next proposition characterizes the equilibrium.

Proposition 3 (Continuous Time Limit Asymmetric Information)**.** *Let*

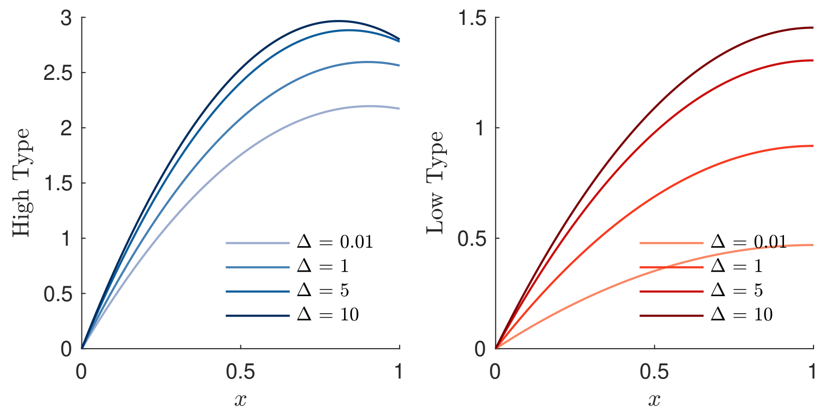
$$\kappa_* \equiv \underline{\theta}^2 + \frac{\lambda\bar{\pi}}{r+\lambda}(\bar{\theta}^2 - \underline{\theta}^2)$$

$$\kappa_\dagger \equiv \underline{\theta}^2 + 2\bar{\theta}^2.$$

In the limit when $t_N \to \infty$ and $\Delta \to 0$, the equilibrium payoff $V_\Delta(x,\theta)$, price $\hat{p}_\Delta(x,\theta)$, and stake x_t^Δ converge to the following limit:



(a) Payoff Coefficients



(b) Aggregate Welfare $W_\Delta(x, \theta)$

Figure 1: Impact of Δ on payoffs. Parameters: $r = 0.1$, $\lambda = 1$, $\bar\pi = 0.1$, $\bar\theta = 2$, $\underline\theta = 0.5$, $\gamma = 0.5$ $\phi = 1$. The total welfare is defined as $W_\Delta(x, \theta) = V_\Delta(x, \theta) + (1 - x)p_\Delta(x, \theta)$, we normalize $\mu = 0$.

- *If* $\frac{\gamma}{\phi} < \kappa_*$*, the limit is*

$$V(x, \theta) = \frac{\mu}{r}x + \frac{1}{2}C^{nt}(\theta)x^2$$
$$\hat p(x, \theta) = \frac{\mu}{r} + \hat p(\theta)x$$
$$x_t = x_0 e^{-\beta(\underline\theta)\int_0^t \mathbf{1}_{\{\theta_s = \underline\theta\}} ds},$$

where $C^{nt}(\theta)$ *is the no-trade valuation in equation* (2)*, and the coefficients* $\hat p(\theta), \beta(\underline\theta)$

are

$$
\begin{aligned}
\hat{p}(\underline{\theta}) &= C^{nt}(\underline{\theta}) \\
\hat{p}(\bar{\theta}) &= \frac{\phi\bar{\theta}^2 + \lambda(1-\bar{\pi})C^{nt}(\underline{\theta})}{r + \lambda(1-\bar{\pi})} \\
\beta(\underline{\theta}) &= \frac{\gamma + \lambda\bar{\pi}\left[\hat{p}(\bar{\theta}) - C^{nt}(\bar{\theta})\right]}{C^{nt}(\underline{\theta})}
\end{aligned}
$$

- *If* $\frac{\gamma}{\phi} \in [\kappa_*, \kappa_\dagger)$*, the limit is*

$$
\begin{aligned}
V(x,\theta) &= \frac{\mu}{r}x + \frac{1}{2}C(\theta)x^2 \\
\hat{p}(x,\theta) &= \frac{\mu}{r} + \hat{p}(\theta)x \\
x_t &= x_0 e^{-\beta(\bar{\theta})t}\mathbf{1}_{\{t<\underline{\tau}\}}
\end{aligned}
$$

where $\underline{\tau} = \inf\{t > 0 : \theta_t = \underline{\theta}\}$ *is the first transition to the low state,* $C(\underline{\theta}) = \hat{p}(\underline{\theta}) = 0$*, and the coefficients* $(\beta(\bar{\theta}), C(\bar{\theta}), \hat{p}(\bar{\theta}))$ *are the unique positive solution to the system of equations*

$$
\begin{aligned}
\beta(\bar{\theta}) &= \frac{\gamma - \phi\underline{\theta}^2 - \lambda\bar{\pi}C(\bar{\theta})}{2\hat{p}(\bar{\theta})} \\
\hat{p}(\bar{\theta}) &= \frac{\phi\bar{\theta}^2}{r + \lambda(1-\bar{\pi}) + \beta(\bar{\theta})} \\
C(\bar{\theta}) &= \frac{\phi(\bar{\theta}^2 - \underline{\theta}^2)}{r + \lambda + 2\beta(\bar{\theta})}.
\end{aligned}
$$

- *If* $\frac{\gamma}{\phi} \geq \kappa_\dagger$*, there is an immediate atom of trade, so* $x_t = 0$*. The price is* $\hat{p}(x,\theta) = \frac{\mu}{r}$ *and the payoff of the blockholder is* $V(x,\theta) = \frac{\mu}{r}x$*.*

Due to a lack of commitment, the blockholder sells his entire stake over time until x_t reaches zero. Compared with the first-best benchmark, this is socially inefficient because, without holdings, the blockholder loses his incentive to increase the firm's cash flows. The blockholder's selling behavior causes an externality to small shareholders: the firm's productivity gradually deteriorates over time.

The size of the holding cost γ can have a major effect on the equilibrium, influencing the trading patterns and the payoffs. Three cases must be considered. First, when $\gamma/\phi < \kappa_*$, the blockholder does not trade when $\theta_t = \bar{\theta}$, even though his marginal valuation is lower than that of small investors ($\hat{p}(x, \bar{\theta}) > V_x(x, \bar{\theta})$). The blockholder refrains from selling because of his price impact; if the blockholder were to sell, the market would interpret this as a negative signal of productivity, and the stock price would drop significantly below the blockholder's marginal valuation; the blockholder would thus experience a capital loss. In contrast, in the low state, the blockholder sells gradually until his holdings are fully depleted or until a positive shock stops the sale.[15]

To understand the properties of the equilibrium, note that the low type's payoff is the same as that arising under symmetric information and equal to the payoff without trade. As in the previous literature on the monopoly of durable goods, the lack of commitment prevents the low-type blockholder from extracting rents from trade, which yields the continuation payoff of no trade $V(x, \underline{\theta}) = V^{\text{nt}}(x, \underline{\theta})$. However, contrary to the standard prediction of the Coase conjecture, the blockholder does not trade immediately toward his long-term target of zero but does it smoothly.

Why does this happen? Recall that in a competitive setting, the price equals the marginal cost. In our setting, the marginal cost is represented by $V_x(x, \underline{\theta})$. Figure 2(a) illustrates that a competitive equilibrium would imply payoffs for the blockholder that are below the value of not trading at all. This is a version of the well-known result that the price cannot equal the marginal cost in the presence of increasing returns to scale because this would generate losses to the seller. However, the equilibrium must involve some trade. If there were no trade, the price would be above μ/r, generating incentives for trade because the blockholder does not internalize the impact of selling on other shareholders' payoffs. Thus, in equilibrium, the blockholder moves smoothly along the curve $V(x, \underline{\theta})$ trading at a price $V_x(x, \underline{\theta})$.

The high-type behavior is different. The incentive compatibility constraint determines the high-type trading. Since the low-type does not extract gains from trade due to Coasian forces, the high-type cannot be willing to trade in equilibrium, or else the low-type would be

[15] The long-run holdings would be positive if the blockholder enjoyed private benefits, in which case the order flow would sometimes be positive, for instance, when the initial holdings are smaller than the long-run target.

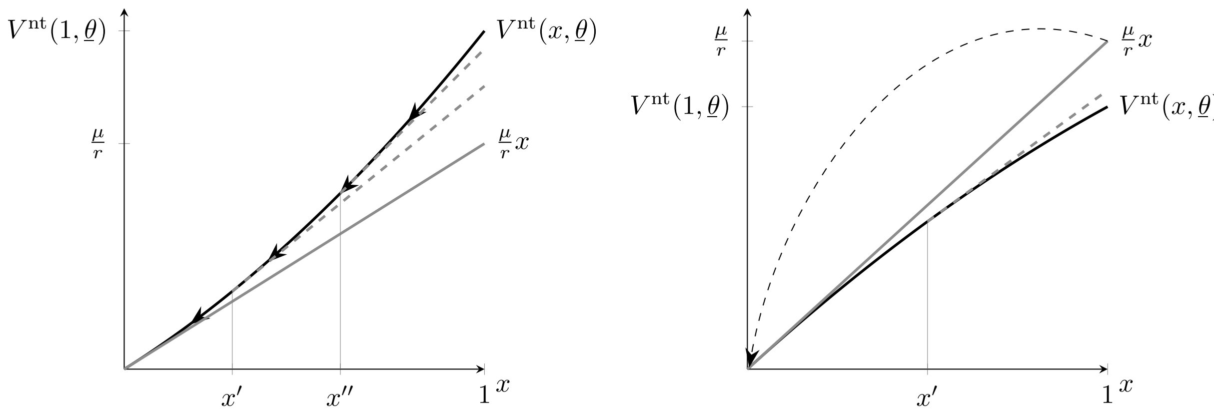
able to deviate for a profit by mimicking the high-type. Thus, the only possibility consistent with the equilibrium is that either the high-type buys or does not trade. Due to holding costs, the high type has no incentive to buy; thus, in equilibrium, the high type refrains from trade.

The previous argument does not apply when $\gamma/\phi > \kappa_*$ because the coefficient $C^{\text{nt}}(\underline{\theta})$ is negative, so the no-trade valuation of the low type $\underline{\theta}$ is concave in x. In this case, the standard prediction of the Coase conjecture applies for the low type, so he trades immediately towards his long-term target $x = 0$. Figure 2(b) shows that the low type prefers to liquidate his holdings immediately, regardless of price impact, selling his entire block at a μ/r. As the low type quickly exits the market, the high type can sell smoothly. In this case, the high type trading speed is the highest, satisfying the low-type's incentive compatibility constraint.

Finally, for high holding costs $\frac{\gamma}{\phi} \geq \kappa_\dagger$, both types sell their stake immediately, leading to low productivity and low prices. Any productivity gain potentially arising from the blockholder effort instantly disappears due to the blockholder's lack of commitment to hold on to his holdings.

As we will show in section 5.1, the properties of the equilibrium seem to generalize when there are more than two types. We show that if the holding cost is low enough so that the lowest type sells smoothly, then there is an unraveling in which all higher types abstain from trade. By contrast, when the holding cost is large, the lowest type sells immediately, and all higher types sell simultaneously and smoothly at a positive rate.

Productivity Shocks vs. Liquidity Shocks In our model, there is asymmetric information about blockholder productivity. In practice, the blockholder might also be informed about other things. For example, it is natural to consider the case where the blockholder's information is the holding cost γ. In this case, shocks to γ could be interpreted as privately observed liquidity shocks. Then, the relevant source of asymmetric information is captured by the reduced-form parameter $\phi\theta^2 - \gamma$. Thus, we can reinterpret a negative productivity shock in our model as a liquidity shock that increases the holding cost γ. This specification of liquidity shocks differs from the linear specification commonly used in dynamic trading models. Unlike in linear models, the mean reversion of holdings is state-dependent. As we will see in Section 4, such non-linearity has important implications for the dynamics of blockholdings.

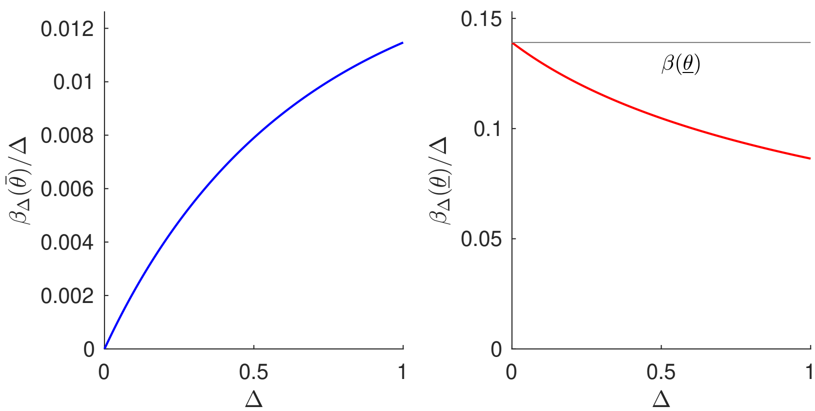


(a) Convex value function and smooth trading

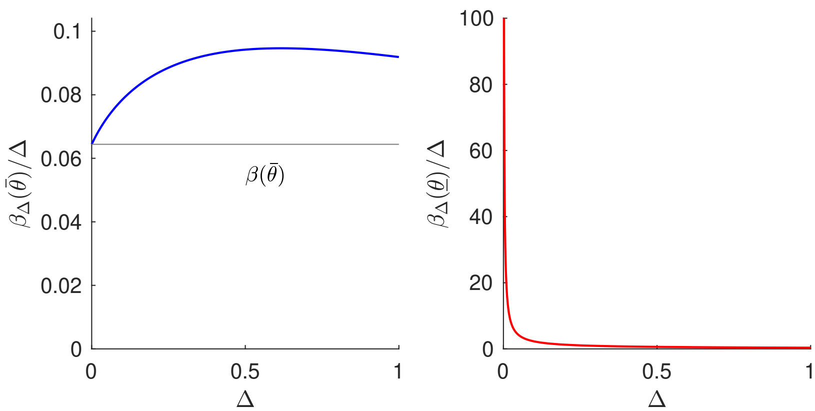
(b) Concave value function and atomic trading

Figure 2: Value function with and without smooth trading. In this figure, $V^{\text{nt}}(x,\underline{\theta})$ indicates the value of holding to x forever. In (a), the value of staying put at x is above the liquidating value. In equilibrium, the blockholder trades moving around the curve $V^{\text{nt}}(x,\underline{\theta})$. In (b), the blockholder is better off liquidating everything at the lowest possible price (μ/r), so the equilibrium entails immediate liquidation (there is an immediate jump to $x = 0$).

Private Benefits of Control The model predicts that the blockholder only sells shares but never buys. This can be easily generalized by assuming that the blockholder enjoys a private benefit of control. We can capture this by adding a private benefit term bx to the payoffs of the blockholder, so his preferences are given by $bx + u(x, a, \theta)$. The derivation of the equilibrium closely follows that without private benefits. The only difference is that the block no longer converges to zero but to $x^{\dagger} = b/\gamma$. This is the ownership level that balances the holding cost and the private benefit of the controls.



(a) Example of convergence with $\frac{\gamma}{\phi} < \kappa_*$



(b) Example of convergence with $\frac{\gamma}{\phi} > \kappa_*$

Figure 3: Convergence of trading rate for $\frac{\gamma}{\phi}$ below and above κ_*. Parameters: $r = 0.1$, $\lambda = 1$, $\bar{\pi} = 0.5$, $\bar{\theta} = 1.5$, $\underline{\theta} = 0.5$, $\phi = 1$. For these parameters, $\kappa_* = 1.1591$. In the top figure, $\gamma/\phi = 0.5$, while in the bottom figure, $\gamma/\phi = 1.5$.

3.4 Equilibrium with Observable Shocks

To understand the consequences of information asymmetry on the efficiency of the equilibrium, we now study a benchmark where productivity shocks are publicly observable. This case only differs from DeMarzo and Urošević (2006) in the time-varying nature of the effort productivity, but the equilibrium is qualitatively similar. The equilibrium resembles that for the low-type characterized in Proposition 2, so we omit the details. The main difference is that we can eliminate the incentive compatibility constraint in the optimization problem of the high type in (4).

The solution at time t_N is

$$\beta_N^{\mathrm{o}}(\theta) = \frac{\gamma}{rC^{\mathrm{nt}}(\theta) + 2\gamma},$$

and the equilibrium payoff is

$$\begin{aligned} V_N^{\mathrm{o}}(x,\theta) &= \frac{\mu}{r}x + \frac{1}{2}C_N^{\mathrm{o}}(\theta)x^2 \\ C_N^{\mathrm{o}}(\theta) &= \frac{1}{r}\frac{\left(rC^{\mathrm{nt}}(\theta) + \gamma\right)^2}{rC^{\mathrm{nt}}(\theta) + 2\gamma}, \end{aligned}$$

For time t_n, $n < N$, the equilibrium can be solved recursively as in Proposition 2 which lead us to the following proposition.

Proposition 4 (Discrete Trading Symmetric Information)**.** *If θ_t is observable, then in the unique subgame perfect equilibrium, the blockholder trading strategy at time t_n is $\Delta x_n^o(x,\theta) = -\beta_n^o(\theta)x$, where*

$$\beta_n^o(\theta) = \frac{\hat{p}_n^o(\theta) - \hat{C}_n^o(\theta)}{2\hat{p}_n^o(\theta) - \hat{C}_n^o(\theta)} \tag{9}$$

The post-trade price $\hat{p}_n^o(x,\theta)$ and the blockholder equilibrium payoff $V_n^o(x,\theta)$ are

$$\begin{aligned} \hat{p}_n^o(x,\theta) &= \frac{\mu}{r} + \hat{p}_n^o(\theta)x \\ V_n^o(x,\theta) &= \frac{\mu}{r}x + \frac{1}{2}C_n^o(\theta)x^2 \end{aligned} \tag{10}$$

where

$$C^o_n(\theta) = \frac{\hat{p}^o_n(\theta)^2}{2\hat{p}^o_n(\theta) - \hat{C}^o_n(\theta)}$$

and the coefficients $\hat{C}^o_n(\theta), \hat{p}^o_n(\theta)$ *satisfy the difference equation*

$$\begin{aligned}\hat{C}^o_n(\theta) &= \frac{1}{r}\left(1-e^{-r\Delta}\right)(\nu(\theta)-\gamma) + e^{-r\Delta}\left[\left(1-\pi(\Delta|\theta)\right)C^o_{n+1}(\underline{\theta}) + \pi(\Delta|\theta)C^o_{n+1}(\bar{\theta})\right] \\ \hat{p}^o_n(\theta) &= \frac{1}{r}\left(1-e^{-r\Delta}\right)\nu(\theta) + e^{-r\Delta}\left[\left(1-\pi(\Delta|\theta)\right)C^o_{n+1}(\underline{\theta}) + \pi(\Delta|\theta)C^o_{n+1}(\bar{\theta})\right].\end{aligned}$$

Intuitively, in this case the blockholder sells faster than under asymmetric information, particularly in the high productivity state, because selling entails a lower price impact under symmetric information. We can now characterize the limit.

Proposition 5 (Continuous Time Limit Symmetric Information)**.** *Let*

$$\kappa_{**} \equiv \bar{\theta}^2 - \frac{\lambda(1-\bar{\pi})}{r+\lambda}(\bar{\theta}^2 - \underline{\theta}^2)$$

In the limit when $t_N \to \infty$ *and* $\Delta \to 0$*, the equilibrium payoff* $V^o_\Delta(x,\theta)$*, price* $\hat{p}^o_\Delta(x,\theta)$*, and path of blockholdings* x^Δ_t *converge to the following limit:*

- *If* $\frac{\gamma}{\phi} < \kappa_*$*, the limit is*

$$\begin{aligned}V^o(x,\theta) &= \frac{\mu}{r}x + \frac{1}{2}C^{nt}(\theta)x^2 \\ \hat{p}^o(x,\theta) &= \frac{\mu}{r} + \hat{p}(\theta)x \\ x^o_t &= x_0 e^{-\int_0^t \beta^o(\theta_s)ds},\end{aligned}$$

  where $C^{nt}(\theta)$ *is the no-trade valuation in equation* (2)*,* $\hat{p}^o(\theta) = C^{nt}(\theta)$*, and* $\beta^o(\theta) = -\frac{\gamma}{\hat{p}^o(\theta)}$.

- *If* $\frac{\gamma}{\phi} \in [\kappa_*, \kappa_{**})$, *the limit is*

$$\begin{aligned} V^o(x,\theta) &= \frac{\mu}{r}x + \frac{1}{2}C^o(\theta)x^2 \\ \hat{p}^o(x,\theta) &= \frac{\mu}{r} + \hat{p}^o(\theta)x \\ x_t &= x_0 e^{-\beta^o(\bar{\theta})t}\mathbf{1}_{\{t<\underline{\tau}\}}, \end{aligned}$$

  where $\underline{\tau} = \inf\{t > 0 : \theta_t = \underline{\theta}\}$ *is the first transition to the low state,* $C^o(\underline{\theta}) = \hat{p}^o(\underline{\theta}) = 0$, $p^o(\bar{\theta}) = C^o(\bar{\theta}) = \frac{\phi\bar{\theta}^2 - \gamma}{r+\lambda(1-\bar{\pi})}$, *and* $\beta^o(\bar{\theta}) = -\frac{\gamma}{\hat{p}^o(\bar{\theta})}$.

- *If* $\frac{\gamma}{\phi} \geq \kappa_{**}$, *there is an immediate atom of trade so* $x_t^o = 0$. *The price is* $\hat{p}^o(x,\theta) = \frac{\mu}{r}$ *and the blockholder payoff is* $V^o(x,\theta) = \frac{\mu}{r}x$.

Again, the blockholder sells his stake over time until fully depleted. Higher productivity (θ) does not change the zero long-run target, but it does slow down the blockholder's selling toward that target. When holding costs are high, the blockholder sells immediately. In general, Coasian dynamics prevents the blockholder from extracting gains from trade, and his payoff is thus identical to that under no trade.

4 The Impact of Asymmetric Information

Having solved for the public information benchmark, we can analyze the impact of asymmetric information on the dynamics of trading, prices, and welfare.

Small investors' welfare depends on the stock price. The higher the price, the higher their payoffs. The stock price in turn depends on blockholder effort, which in turn depends on the speed at which the blockholder sells his stock.

As mentioned above, under asymmetric information the blockholder's trading has price impact because of signaling effects. Thus, it is natural to think that asymmetric information slows selling, and this would be consistent with previous work on dynamic signaling models (Daley and Green, 2012; Admati and Perry, 1987). However, our analysis shows that this is not necessarily the case. In the low state, the blockholder selling speed is higher under asymmetric information. The key behind this result is that types are not fully persistent in our model, so changes in the incentives of the high-type also affect the incentives of the low type (let us stress that while the previous observation is immediate in a pooling equilibrium, it is far from obvious in a separating equilibrium). Suppose that the low-type trading strategy is not affected by asymmetric information. If this were the case, the price in the low state would necessarily be higher under asymmetric information (because the blockholder stake and his effort would be larger for all states), and this would generate incentives for the low type to sell even faster, contradicting the premise that the trading strategy of the low type is unaffected by the information structure. Thus, in equilibrium, the low type must sell faster to offset the reduction in the selling speed by the high type, and keep the price constant across information structures for the low-type.

Given the ambiguity of the effect of asymmetric information on the blockholder's selling speed, one might expect the effect on price to be ambiguous as well. However, we can show that asymmetric information increases the price in all states. The fact that the blockholder slows down his selling in the high-state leads to higher cash flows and ultimately to a higher stock price because the blockholder effort increases precisely when it is most effective, namely in the high-state.

Having characterized the effect of asymmetric information on the dynamics of trading and prices, we turn our attention to the impact of asymmetric information on blockholder payoffs. We mentioned that asymmetric information leads to higher stock prices by boosting the effort in high-productivity states. This benefits small shareholders. Now, because excessive retention is costly to the blockholder, it is not clear that information asymmetry also benefits the blockholder. In fact, the opposite is true in a static setting.

When holding costs are sufficiently low, the blockholder payoffs with and without asymmetric information are the same. On the other hand, when holding costs are relatively high, asymmetric information increases the payoffs of the blockholder in the high state. Since the low-type sells his stake immediately, the high-type faces a greater liquidity which allows him to sell his holdings over time, rather than forcing him to hold on to them until a negative shock arrives. Asymmetric information thus mitigates Coasian forces, allowing the blockholder to extract gains from trade. We summarize these results in the following corollary.

Proposition 6 (Blockholder's Payoff)**.** *Asymmetric information leads to a Pareto improvement relative to the case with symmetric information. Small investors are always better-off when the blockholder is privately informed about* θ_t*. Similarly, the blockholder's payoff is* *higher under asymmetric information.*

- *If $\gamma/\phi < \kappa_*$, then the blockholder payoff is the same with and without asymmetric information.*

- *On the other hand, if condition $\gamma/\phi \in [\kappa_*, \kappa_\dagger)$, the blockholder payoff in the high state is higher with asymmetric information, while the equilibrium payoff in the low state is not affected by asymmetric information.*

This result shows that information asymmetry not only increases the stock price, benefitting small shareholders, but also boosts the blockholder's payoff, in stark contrast with static settings or settings without effort. Private information thus has welfare-enhancing properties in our setting. This leads to the following corollary.

Again, two cases must be distinguished here. First, when holding costs are low, the payoff of the blockholder is invariant with the information environment (that is, symmetric vs. asymmetric information about θ). However, when holding costs are high, the payoff to the blockholder is greater with asymmetric information. Asymmetric information mitigates Coasian forces: The blockholder holds on to his stake for longer, which increases the firm productivity and the stock price. At the same time, the blockholder faces a relatively liquid market, which allows him to sell his shares over time, and thus extract gains from trade. Thus, the blockholder payoff increases.

This contrasts with a static setting Leland and Pyle (1977) where signaling incentives would impose a deadweight cost on the (high-type) blockholder, forcing him to retain a large fraction of his stake to signal his productivity, thus ultimately reducing his payoffs. In a static setting, the blockholder is unambiguously worse off under asymmetric information. In a static setting, the signaling friction operates in conjunction with the sender's market power. But in a dynamic setting, the blockholder does not necessarily exert monopoly power, due to Coase's conjecture. The blockholder's lack of commitment implies that, no matter the information environment, he always trades at a competitive price that equals his own marginal valuation.

5 Extensions

5.1 Multiple Types

Next, we consider the case with more than two types. In particular, suppose that there are K possible types, and that the blockholder's type $\theta_t \in \{\theta_1, \theta_2, \ldots, \theta_K\}$ follows a continuous time Markov chain with transition rate matrix $\mathbf{\Lambda}$. The matrix of transition probabilities $\pi_{ij}(\Delta) = \Pr(\theta_{t+\Delta} = \theta_j | \theta_t = \theta_i)$ is given by $\mathbf{\Pi}(\Delta) = e^{\mathbf{\Lambda}\Delta}$.

Throughout this section, we use the following notation: For any function $f(\theta)$, we denote the column vector $(f(\theta_1), \ldots, f(\theta_k))^\intercal$ by $\mathbf{f}(\theta)$; we define the vector $\boldsymbol{\theta}^2 \equiv (\theta_1^2, \ldots, \theta_K^2)^\intercal$, and we denote the $K \times K$ identity matrix by $\mathbf{I}$. The no-trade valuation can be written as[16]

$$\mathbf{V}^{\mathrm{nt}}(x, \theta) = \frac{\mu}{r} x + \frac{1}{2} \mathbf{C}^{\mathrm{nt}}(\theta) x^2,$$

where

$$\mathbf{C}^{\mathrm{nt}}(\theta) = \phi \left(r\mathbf{I} - \mathbf{\Lambda}\right)^{-1} \boldsymbol{\theta}^2 - \frac{\gamma}{r}.$$

Following calculations similar to the ones with two types, the post-trade continuation value $\hat{V}_n(x, \theta_k)$ is given by

$$\hat{\mathbf{V}}_n(x, \theta) = \frac{1}{r}\left(1 - e^{-r\Delta}\right)\left(\mu x + \frac{1}{2}(\boldsymbol{\nu}(\theta) - \gamma) x^2\right) + e^{-r\Delta} \mathbf{\Pi}(\Delta) \mathbf{V}_{n+1}(x, \theta),$$

where

$$\boldsymbol{\nu}(\theta) = \frac{r\phi\left(\mathbf{I} - e^{-(r\mathbf{I} - \mathbf{\Lambda})\Delta}\right)\left(r\mathbf{I} - \mathbf{\Lambda}\right)^{-1} \boldsymbol{\theta}^2}{1 - e^{-r\Delta}},$$

while the price satisfies the recursive relation

$$\hat{\mathbf{p}}_n(x, \theta) = \frac{1}{r}\left(1 - e^{-r\Delta}\right)\left(\mu + \boldsymbol{\nu}(\theta) x\right) + e^{-r\Delta} \mathbf{\Pi}(\Delta) \mathbf{p}_{n+1}(x, \theta).$$

[16]For the calculations of expected payoffs, we use the following two properties of the matrix exponential: $\int_0^\infty e^{-rt} e^{\mathbf{\Lambda} t} = (r\mathbf{I} - \mathbf{\Lambda})^{-1}$ and

$$\int_0^T e^{-(r\mathbf{I} - \mathbf{\Lambda})t} = \left(\mathbf{I} - e^{-(r\mathbf{I} - \mathbf{\Lambda})T}\right)\left(r\mathbf{I} - \mathbf{\Lambda}\right)^{-1}$$

We assume that valuations are increasing in type, that is $C^{\text{nt}}(\theta_{k+1}) > C^{\text{nt}}(\theta_k)$ and $\nu(\theta_{k+1}) > \nu(\theta_k)$, which is the case if the Markov chain $\{\theta_t\}$ is monotone.[17]

As in the two-type case, the lowest type trading strategy, $\Delta x_n(x, \theta_1)$, is the solution to

$$V_n(x, \theta_1) = \max_{\Delta x} \ \hat{V}_n(x + \Delta x, \theta_1) + \hat{p}_n(x + \Delta x, \theta_1)\Delta x, \tag{11}$$

whereas for the remaining types trading $\Delta x_n(x, \theta_k)$ can be found recursively by solving the sequence of problems

$$\begin{aligned} V_n(x, \theta_k) = \max_{\Delta x} \ & \hat{V}_n(x + \Delta x, \theta_k) + \hat{p}_n(x + \Delta x, \theta_k)\Delta x \\ & \text{s.t} \\ & \hat{V}_n(x + \Delta x, \theta_{k-1}) + \hat{p}_n(x + \Delta x, \theta_k)\Delta x \leq V_n(x, \theta_{k-1}). \end{aligned} \tag{12}$$

As in the two-type case, the equilibrium is linear and can be solved in closed form. The next proposition provides a characterization of the equilibrium:

Proposition 7 (Discrete Trading Asymmetric Information)**.** *In the least costly separating equilibrium, the blockholder trading strategy at time t_n is $\Delta x_n(x, \theta) = -\beta_n(\theta)x$, where*

$$\begin{aligned} \beta_n(\theta_1) &= \frac{\hat{p}_n(\theta_1) - \hat{C}_n(\theta_1)}{2\hat{p}_n(\theta_1) - \hat{C}_n(\theta_1)} \\ \beta_n(\theta_k) &= \frac{\hat{p}_n(\theta_k) - \hat{C}_n(\theta_{k-1}) - \sqrt{\hat{p}_n(\theta_k)^2 - 2\hat{p}_n(\theta_k)C_n(\theta_{k-1}) + C_n(\theta_{k-1})\hat{C}_n(\theta_{k-1})}}{2\hat{p}_n(\theta_k) - \hat{C}_n(\theta_{k-1})}, \ 2 \leq k \leq K. \end{aligned}$$

The post-trade price $\hat{p}_n(x, \theta)$ and the blockholder equilibrium payoff $V_n(x, \theta)$ are

$$\begin{aligned} \hat{p}_n(x, \theta) &= \frac{\mu}{r} + \hat{p}_n(\theta)x \\ V_n(x, \theta) &= \frac{\mu}{r}x + \frac{1}{2}C_n(\theta)x^2 \end{aligned} \tag{13}$$

[17] A Markov chain $(\theta_t)_{t \geq 0}$ is stochastically monotone if for any $t > 0$ and $\theta_0'' > \theta_0'$, the distribution of θ_t conditional in θ_0'' first-order stochastically dominates the distribution conditional on θ_0'.

where

$$C_n(\theta_1) = \frac{\hat{p}_n(\theta_1)^2}{2\hat{p}_n(\theta_1) - \hat{C}_n(\theta_1)}$$
$$C_n(\theta_k) = C_n(\theta_1) + \sum_{2\leq i\leq k} (1-\beta_n(\theta_i))^2 \left(\hat{C}_n(\theta_i) - \hat{C}_n(\theta_{i-1})\right), \ 2 \leq k \leq K.$$

The coefficients $\hat{C}_n(\theta), \hat{p}_n(\theta)$ satisfy the recursion

$$\hat{\mathbf{C}}_n(\theta) = \frac{1}{r}\left(1-e^{-r\Delta}\right)(\boldsymbol{\nu}(\theta)-\gamma) + e^{-r\Delta}\mathbf{\Pi}(\Delta)\mathbf{C}_{n+1}(\theta)$$
$$\hat{\mathbf{p}}_n(\theta) = \frac{1}{r}\left(1-e^{-r\Delta}\right)\boldsymbol{\nu}(\theta) + e^{-r\Delta}\mathbf{\Pi}(\Delta)\left(1-\boldsymbol{\beta}_{n+1}(\theta)\right)^\intercal \mathbf{I}\hat{\mathbf{p}}_{n+1}(\theta),$$

with terminal condition

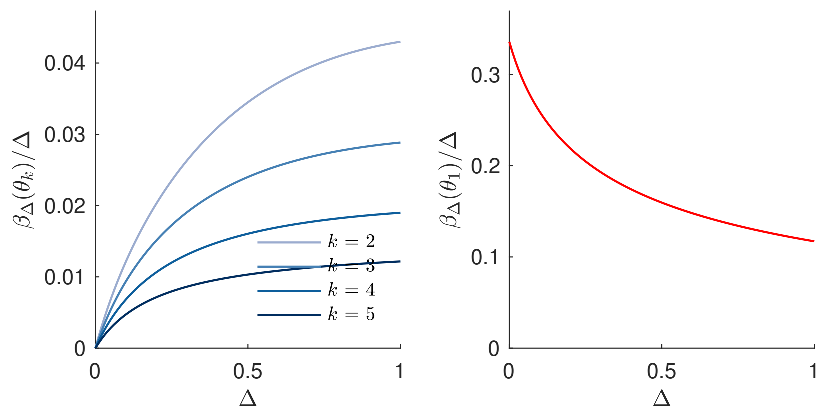
$$\hat{\mathbf{C}}_N(\theta) = \mathbf{C}^{nt}(\theta)$$
$$\hat{\mathbf{p}}_N(\theta) = \mathbf{C}^{nt}(\theta) + \frac{\gamma}{r}.$$

As in Section 3.3, we are interested in the double limit when $t_N \to \infty$ and $\Delta \to 0$. As $N \to \infty$, the solution converges the fixed point of the recursion in Proposition 7. Our main objective is to analyze the behavior of the trading rate $\beta_\Delta(\theta)/\Delta$ as Δ goes to zero. A full analytical characterization of the limit as the one in Proposition 3 is beyond the scope of this section, but we explore numerically what happens with the equilibrium in the continuous-time limit. The objective of the exercise is to illustrate how the qualitative behavior of the limit is determined by the sign of $C^{\mathrm{nt}}(\underline{\theta})$ as in the two type case. With two types, the condition $\gamma/\phi < \kappa_*$ corresponds to the case when $C^{\mathrm{nt}}(\underline{\theta}) > 0$ whereas the conditions $\gamma/\phi > \kappa_*$ corresponds to the case when $C^{\mathrm{nt}}(\underline{\theta}) < 0$. The next example illustrates how this same condition determines the limit behavior when there are more than two types. For the purpose of the numerical example, we consider the following simple parametrization of the transition $\mathbf{\Lambda}$

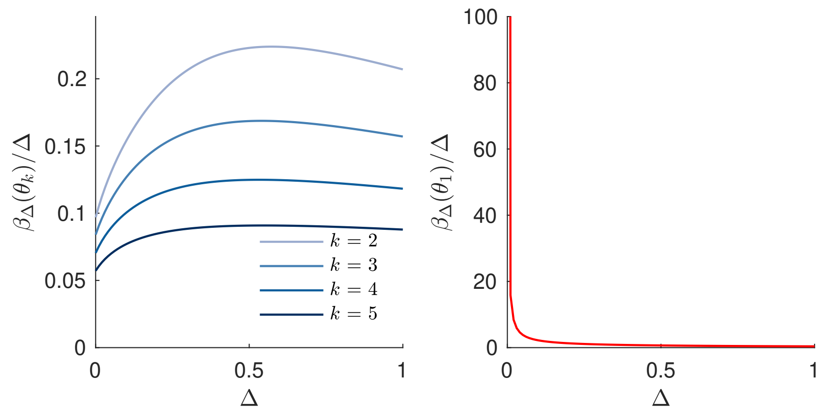
$$\Lambda_{ij} = \begin{cases} -\frac{(K-1)\lambda}{K} & \text{if } i = j \\ \frac{\lambda}{K} & \text{if } i \neq j. \end{cases}$$

This specification of $\mathbf{\Lambda}$ corresponds to the case where the blockholder suffers shocks at a rate λ, and conditionally on a shock, his new type θ' is chosen from a uniform distribution. When $K = 2$, this corresponds to setting $\bar{\pi} = 1/2$.

Figure 4 provides a numerical example showing the convergence when $\Delta \to 0$. The qualitative properties of the equilibrium generalize to the case with multiple types. In panel (a) we depict the case when γ is small. For $\Delta > 0$ then all types trade at a rate that is decreasing in types, and the presence of price impact slows down the trading rate of higher types. However, in the limit when Δ converges to zero, there is a market breakdown where only the lowest type trade and all the remaining types refrain trading. Because the lowest type gets no rents from trading in equilibrium, any price that would induce a higher type to trade would generate a deviation by the lowest type. So, in equilibrium there is no trade by the remaining types. However, when γ is large, the lowest type sells immediately. Hence, imitating the trading strategy of higher types becomes too costly and there is no market breakdown. This case is depicted in panel (b). In the continuous trade limit, the lowest type sells immediately, and all higher types sell smoothly over time.



(a) Example of convergence with $C^{\text{nt}}(\theta_1) > 0$



(b) Example of convergence with $C^{\text{nt}}(\theta_1) < 0$

Figure 4: Convergence of trading rate. Parameters: $r = 0.1$, $\lambda = 1$, $K = 5$, $\theta_1 = 1$, $\theta_{k+1} = \theta_k + 0.25$, $\phi = 1$. In the top panel, $\gamma = 0.5$ which implies $C^{\text{nt}}(\theta_1) = 5.45$; in the bottom panel $\gamma = 1.5$ which implies $C^{\text{nt}}(\theta_1) = -4.54$

5.2 Pooling Equilibrium

In addition to a separating equilibrium, multiple pooling equilibria can be sustained using off-equilibrium beliefs to punish deviations. In this section, we construct an example of such a pooling equilibrium. In particular, we consider a pooling equilibrium where the trading rate is chosen to maximize the high type's payoff, this equilibrium selection criterion is similar to the one in Gomes (2000), and it is inspired by the notion of undefeated equilibrium developed by Mailath et al. (1993).

To simplify the analysis, we concentrate on the case with $\bar{\pi} = 0$ (this means that $\lambda_H = 0$), so negative shocks are permanent. Moreover, we impose the belief restriction that once the market assigns probability one to the low type, their beliefs remain forever at that level.

This means that the equilibrium in the continuation game that follows a deviation by the low type is equivalent to the equilibrium with symmetric information in section 3.4.

A pooling equilibrium is technically complex. In a separate equilibrium, the choice of effort is only affected by the direct incentives of ownership x_t. However, this is not necessarily the case in a pooling equilibrium, as cash flows are now informative about the blockholder's type; the market uses cash flows to infer the blockholder's type. Hence, the blockholder is incentivized to exert more effort to improve his reputation, thereby influencing the stock price. For example, suppose that the martingale in the cash flow process in equation (1) is the Brownian motion $M_t = \sigma B_t$. If the market assesses that the blockholder effort strategy is given by a function $a_t(x, \theta_t)$, but the blockholder follows the strategy $\tilde{a}_t$, then the belief of the market $\alpha_t = \Pr_t(\theta_t = \bar{\theta})$ evolves between trading dates according to

$$d\alpha_t = -\lambda \alpha_t dt + \frac{\bar{\theta} a_t(x, \bar{\theta}) - \underline{\theta} a_t(x, \underline{\theta})}{\sigma} \alpha_t (1 - \alpha_t) \left[\frac{\theta_t \tilde{a}_t - E_t[\theta_t a_t(x, \theta_t)]}{\sigma} dt + dB_t \right]$$

In principle, one could try to consider the Markov perfect equilibrium using α_t as a state variable and find the value function $\hat{V}_n(x, \theta, \alpha)$ at the beginning of the interval (t_n, t_{n+1}) using techniques like those in Faingold and Sannikov (2011). This is a difficult problem that is beyond the scope of this paper. Instead, we consider the simpler case where cash flows are an extremely noisy signal of effort, so the signal-to-noise ratio is zero, corresponding to the limit when $\sigma \to \infty$.[18] In this case, only the direct incentive of ownership affects the

[18] Alternatively, we could assume that the impact of effort on cash flows is only observed in the very long term.

effort, so the effort policy is the same as that in the separation equilibrium, allowing us to isolate the impact of pooling on trading. Under these assumptions, the second term in the evolution of beliefs disappears, so the belief at the beginning of the trading date t_{n+1} is $\alpha_{t_{n+1}-} = e^{-\lambda\Delta}\alpha_{t_n+}$.

To simplify the exposition, we take $N = \infty$ and consider a linear equilibrium (corresponding to the limit when $N \to \infty$). Let $\nu(\alpha) \equiv (1-\alpha)\nu(\underline{\theta}) + \alpha\nu(\bar{\theta})$ be the expected flow payoff given beliefs α. If the market expects the blockholder to trade according to the pooling strategy $\Delta x = -\bar{\beta}_\Delta(\alpha)x$, then the price given stake x and beliefs α is given by a function

$$\hat{p}(x,\alpha) = \frac{\mu}{r} + \hat{p}_\Delta(\alpha)x,$$

where

$$\hat{p}_\Delta(\alpha) = \frac{1}{r}\left(1-e^{-r\Delta}\right)\nu(\alpha) + e^{-r\Delta}\left(1-\bar{\beta}_\Delta(\alpha')\right)\hat{p}_\Delta(\alpha'),$$

and $\alpha' = e^{-\lambda\Delta}\alpha$. We can guess and verify that the blockholder post-trade continuation value is a quadratic function

$$\hat{V}_\Delta(x,\theta,\alpha) = \frac{\mu}{r} + \frac{1}{2}\hat{C}_\Delta(\theta,\alpha)x^2.$$

At each trading date, the high-type equilibrium payoff is

$$V_\Delta(x,\bar{\theta},\alpha) = \max_x \hat{V}_\Delta(x',\bar{\theta},\alpha) + (x-x')\hat{p}_\Delta(x',\alpha).$$

Taking first-order conditions and solving for $x' = x + \Delta x$, we get that $\Delta x = -\bar{\beta}_\Delta(\alpha)$, where

$$\bar{\beta}_\Delta(\alpha) = \frac{\hat{p}_\Delta(\alpha) - \hat{C}_\Delta(\bar{\theta},\alpha)}{2\hat{p}_\Delta(\alpha) - \hat{C}_\Delta(\bar{\theta},\alpha)}. \tag{14}$$

Hence, after substituting the $\bar{\beta}_\Delta(\alpha)$ in the objective function we get

$$V_\Delta(x,\bar{\theta},\alpha) = \frac{\mu}{r}x + \frac{1}{2}C_\Delta(\bar{\theta},\alpha)x^2$$

where

$$C_\Delta(\bar{\theta},\alpha) = \frac{\hat{p}_\Delta(\alpha)^2}{2\hat{p}_\Delta(\alpha) - \hat{C}_\Delta(\bar{\theta},\alpha)}.$$

Similarly, if the low type pools with the high type, his payoff is

$$V_\Delta(x, \underline{\theta}, \alpha) = \frac{\mu}{r}x + \frac{1}{2}C_\Delta(\underline{\theta}, \alpha)x^2$$

where

$$C_\Delta(\underline{\theta}, \alpha) = \left(1 - \frac{\hat{C}_\Delta(\bar{\theta}, \alpha) - \hat{C}_\Delta(\underline{\theta}, \alpha)}{2\hat{p}_\Delta(\alpha) - \hat{C}_\Delta(\bar{\theta}, \alpha)}\right) C_\Delta(\bar{\theta}, \alpha).$$

It follows that the coefficients $\hat{C}_\Delta(\theta, \alpha)$ solve the recursion

$$\hat{C}_\Delta(\bar{\theta}, \alpha) = \frac{1}{r}\left(1 - e^{-r\Delta}\right)\left(\nu(\bar{\theta}) - \gamma\right) + e^{-r\Delta}\left[\pi(\Delta|\bar{\theta})C_\Delta(\bar{\theta}, e^{-\lambda\Delta}\alpha) + (1 - \pi(\Delta|\bar{\theta}))C_\Delta(\underline{\theta}, e^{-\lambda\Delta}\alpha)\right]$$
$$\hat{C}_\Delta(\underline{\theta}, \alpha) = \frac{1}{r}\left(1 - e^{-r\Delta}\right)\left(\nu(\underline{\theta}) - \gamma\right) + e^{-r\Delta}C_\Delta(\underline{\theta}, e^{-\lambda\Delta}\alpha),$$

and that the price coefficient satisfies

$$\hat{p}_\Delta(\alpha) = \frac{1}{r}\left(1 - e^{-r\Delta}\right)\nu(\alpha) + e^{-r\Delta}\frac{\hat{p}_\Delta(e^{-\lambda\Delta}\alpha)^2}{2\hat{p}_\Delta(e^{-\lambda\Delta}\alpha) - \hat{C}_\Delta(\bar{\theta}, e^{-\lambda\Delta}\alpha)}$$

The strategy $\Delta x = -\bar{\beta}(\alpha)x$ and the price $\hat{p}(x, \alpha)$ constitute an equilibrium as long as the low type finds it optimal to pool with the high type rather than deviating and revealing his type. After a deviation, the low-type continuation payoff is given by the value function $V^o_\Delta(x, \underline{\theta})$ in section 3.4. Therefore, the low type prefers to imitate the high type only if $V_\Delta(x, \underline{\theta}, \alpha) \geq V^o_\Delta(x, \underline{\theta})$, which means that the coefficient $\hat{C}_\Delta(\underline{\theta}, \alpha)$ must satisfy the condition $\hat{C}_\Delta(\underline{\theta}, \alpha) \geq C^o_\Delta(\underline{\theta})$, where $C^o_\Delta(\underline{\theta})$ is provided in Proposition 4. When this condition is not satisfied, the full pooling in the high-type trading quantity is not an equilibrium for all beliefs, so there is only partial pooling, and the construction of the equilibrium involves mixed strategies. In the continuous-time limit when $\Delta \to 0$, the equilibrium is characterized by a simple system of ordinary differential equations.

Proposition 8 (Pure Strategy Pooling Equilibrium)**.** *Let*

$$\bar{\beta}(\alpha) = \frac{\gamma - (1 - \alpha)\phi(\bar{\theta}^2 - \underline{\theta}^2) + \lambda\Gamma(\alpha)}{\hat{p}(\alpha)},$$

and let $\hat{p}(\alpha), \Gamma(\alpha)$ be a non-negative solution to the ordinary differential equation

$$\begin{aligned} r\hat{p}(\alpha) &= \phi\bar{\theta}^2 - \gamma - \lambda\Gamma(\alpha) - \lambda\alpha\hat{p}'(\alpha) \\ (r+\lambda+2\bar{\beta}(\alpha))\Gamma(\alpha) &= \phi\left(\bar{\theta}^2 - \underline{\theta}^2\right) - \lambda\alpha\Gamma'(\alpha) \end{aligned} \tag{15}$$

with boundary condition $\lim_{\alpha\to 0}\alpha\hat{p}'(\alpha) = \lim_{\alpha\to 0}\alpha\Gamma'(\alpha) = 0$. *If* $\hat{p}(\alpha) \geq \Gamma(\alpha) + \max\{C^{nt}(\underline{\theta}), 0\}$, *then, in the limit as* $\Delta \to 0$, *the equilibrium payoff* $V_\Delta(x,\theta,\alpha)$, *price* $\hat{p}_\Delta(x,\alpha)$, *and path of blockholdings* x_t^Δ *in the pooling equilibrium converge to*

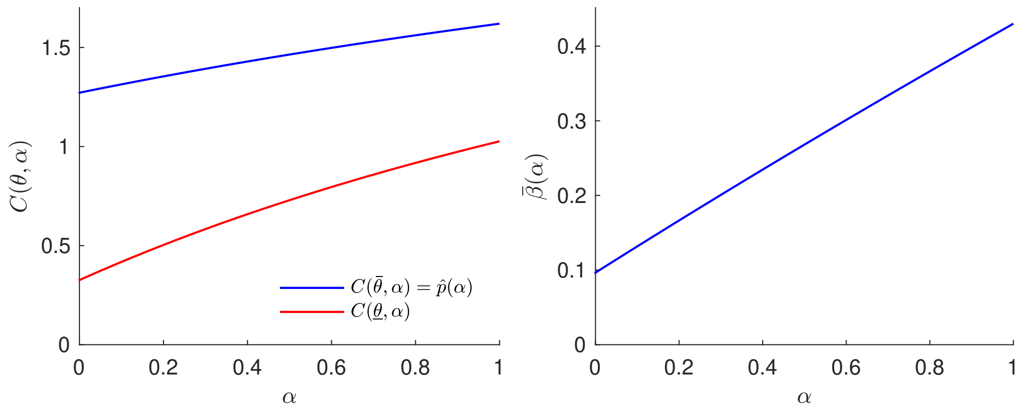
$$\begin{aligned} V(x,\theta,\alpha) &= \frac{\mu}{r}x + \frac{1}{2}C(\theta,\alpha)x^2 \\ \hat{p}(x,\alpha) &= \frac{\mu}{r} + \hat{p}(\alpha)x \\ x_t &= x_0 e^{-\int_0^t \bar{\beta}(\alpha_s)ds}, \end{aligned}$$

where $C(\bar{\theta},\alpha) = \hat{p}(\alpha)$ *and* $C(\underline{\theta},\alpha) = \hat{p}(\alpha) - \Gamma(\alpha)$.

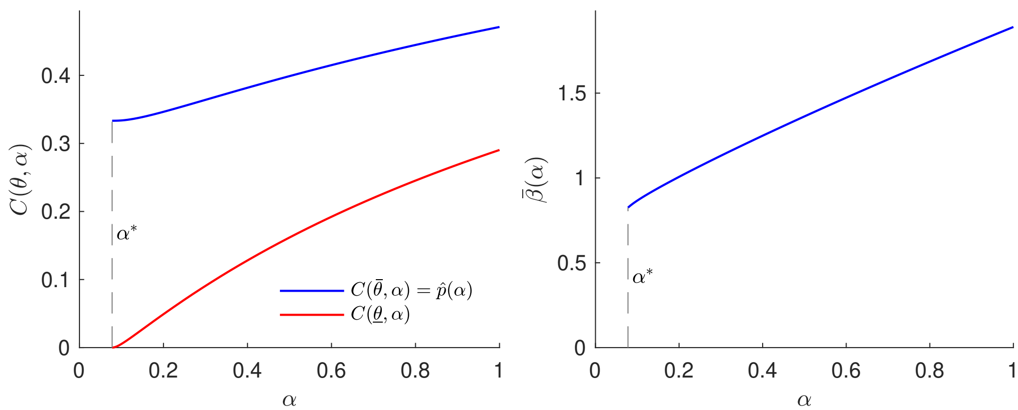
Mixed Strategy Equilibrium For some parameters, the incentive compatibility condition $\hat{C}_\Delta(\underline{\theta},\alpha) \geq C^o_\Delta(\underline{\theta})$ is violated. In this case, the equilibrium entails mixed strategies for low beliefs. Let the threshold α^* be determined by the indifference condition $\hat{C}_\Delta(\underline{\theta},\alpha^*) = C^o_\Delta(\underline{\theta})$. For $\alpha \geq \alpha^*$, the high type trading strategy is given by equation (14), while for $\alpha < \alpha^*$ his trading strategy is $\bar{\beta}_\Delta(\alpha^*)$. For $\alpha \geq \alpha^*$, the low type pools with the high type. However, when $\alpha < \alpha^*$, the low type mixes between pooling at $\bar{\beta}_\Delta(\alpha^*)$ and separating at $\beta^o_\Delta(\underline{\theta})$. The probability of pooling with the high type $m(\alpha)$ satisfies

$$\alpha^* = \frac{\alpha}{\alpha + (1-\alpha)m(\alpha)}$$

In the limit when $\Delta \to 0$, for beliefs $\alpha \in (\alpha^*, 1)$ the equilibrium is characterized by $\bar{\beta}(\alpha)$ in Proposition (8) where $\hat{p}(\alpha), \Gamma(\alpha)$ satisfy equation (15). For $\alpha < \alpha^*$, the low type pools with probability $m(\alpha)$, while for $\alpha = \alpha^*$, the low type separates at an exponential time with mean arrival rate $\lambda/(1-\alpha^*)$. Given this strategy, market beliefs remain constant at α^* once that threshold is reached, at which point trade continues at a rate $\bar{\beta}(\alpha^*)$ until the time the low type separates to follow the symmetric information strategy $\beta^o(\underline{\theta})$. The boundary condition



(a) Pooling equilibrium



(b) Semi-pooling equilibrium with mixed strategy below threshold α^*

Figure 5: Examples of Pooling Equilibrium. Parameters: $r = 0.1$, $\lambda = 0.5$, $\bar{\pi} = 0$, $\bar{\theta} = 1$, $\underline{\theta} = 0.5$, $\phi = 1$. In the top figure, $\gamma = 0.4$, while in the bottom figure, $\gamma = 0.8$. In the bottom figure, the mixing threshold belief is $\alpha^* = 0.07$

at α^* is

$$r\hat{p}(\alpha^*) = \phi\bar{\theta}^2 - \gamma - \lambda\Gamma(\alpha^*)$$
$$\Gamma(\alpha^*) = \frac{\phi\left(\bar{\theta}^2 - \underline{\theta}^2\right)}{r + \lambda + 2\bar{\beta}(\alpha^*)},$$

and the threshold α^* satisfies the indifference condition $\hat{p}(\alpha^*) = \Gamma(\alpha^*) + \max\{C^{\text{nt}}(\underline{\theta}), 0\}$.

Figure 5 presents a numerical example of the pooling equilibrium constructed above. In the first panel, we present an example of full pooling for all beliefs α. In contrast, in the second panel, we present an example where the equilibrium involves some separation for low beliefs and entails using mixed strategies. One interesting feature in this example is that even as α approaches zero, the pooling trading rate remains below the full information for the low type. For the parameters in panel (a), the limit pooling rate when α goes to zero is around $\bar{\beta}(0) \approx 0.1$ while the full information rate for the low type is $\beta^{\mathrm{o}}(\underline{\theta}) = \infty$ (in this example $C^{\mathrm{nt}}(\underline{\theta}) < 0$). By pooling with the high type, the low type can ameliorate the commitment problem and escape the Coasian dynamics that prevent him from extracting any trade gains. Such a commitment device remains valuable even when reputation is very low. This is not the case in panel (b), where for low beliefs, the benefit of the implicit commitment device is not high enough to compensate for the cost of pooling. Hence, the equilibrium entails some separation for low reputation levels.

6 Conclusions

This paper studies the monitoring and trading behavior of a blockholder with access to private information.

Our analysis shows that when a blockholder has access to private information (and can trade based on it) welfare improves. Asymmetric information leads the blockholder to hold their shares for longer, and monitor the firm more intensively, thus increasing the stock price. The lack of liquidity arising under asymmetric information, mitigates the blockholder's commitment problem, and allows him to sell slowly and extract some monopoly rents, contrary to the symmetric information case.

This paper has implications for the long-standing debate on the role of liquidity for blockholder activism (see e.g., Norli et al. (2014); Edmans (2009)). Our results suggest that information asymmetry reduces the liquidity facing the blockholder, but has desirable social effects, insofar as it leads to larger blocks and stronger monitoring (a similar point is made by Vanasco (2017)).

Our model has a number of limitations. For example, we assume that the blockholder's order flow is perfectly observable and we focus on separating equilibria. Perfect observability of the order flow is a simplification, because in practice these data is available to investors with some delay. This assumption also leads to an equilibrium where cash flows are uninformative, conditional on the order flow. Extending our model to allow for noise trading, to obscure the blockholder's order flow, is an interesting (and challenging) extension that we hope future research will address.

The observability of blockholder trading is also a policy question. The SEC requires that a blockholder discloses their stakes within 10 days of the purchase of more than 5% of the shares of a public company. This regulation, and the socially optimal level of the disclosure threshold is subject of an intense policy debate.[19] Evaluating the effects of this regulation is also an open theory question.

We have ruled out the possibility that the blockholder takes value-destroying actions, as is often argued in the popular press. Indeed, critics often warn that blockholder activists exacerbate firms' short-termist tendencies (See e.g., "Let's do it my Way", The Economist, May 13, 2013). This possibility could be incorporated in our model by allowing the blockholder effort to have, at the same time, a negative impact on the firm's cash flows and a positive effect on the blockholder's payoff.

Finally, as another interesting extension one could consider the possibility of competition (or cooperation) among multiple blockholders with heterogenous beliefs to gain control of the firm and influence its corporate strategy (see e.g, Hadlock and Schwartz-Ziv (2019)).

[19] For example, The Economist notes that "Wachtell, Lipton, Rosen & Katz, the law firm that invented the poison pill, has also been seeking to make things harder still for activists by proposing a rule that anyone building a stake of 5% or more in a firm must disclose it within one day, not ten as now. So far the Securities and Exchange Commission is showing little interest. Indeed, its chairman, Mary Jo White, has argued that activists attempts to jog boards are not always a bad thing." See Nasty Medicine, The Economist, Jul 5th, 2014.

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Appendix

A Trading with Asymmetric Information

Proof of Proposition 1

Proof. Letting $x_L = x + \Delta x(x, \underline{\theta})$, the first-order condition for the low-type problem becomes

$$\hat{C}_N x_L + \frac{\mu}{r} + (x - x_L)\left(\hat{C}_N(\underline{\theta}) + \frac{\gamma}{r}\right) - \frac{\mu}{r} - \left(\hat{C}_N(\underline{\theta}) + \frac{\gamma}{r}\right) x_L = 0$$

so we get that

$$x_L = \frac{r\hat{C}_N(\underline{\theta}) + \gamma}{r\hat{C}_N(\underline{\theta}) + 2\gamma} x.$$

Moreover, the second-order condition is satisfied as $r\hat{C}_N(\underline{\theta}) + 2\gamma > 0$. Substituting x_L above in $\hat{p}_N(x, \underline{\theta})$ we get that

$$\hat{p}_N(x_L, \underline{\theta}) = \frac{\mu}{r} + \frac{1}{r} \frac{\left(r\hat{C}_N(\underline{\theta}) + \gamma\right)^2}{r\hat{C}_N(\underline{\theta}) + 2\gamma} x$$

while substituting x_L in the blockholders objective function, we get

$$\begin{aligned} V_N(x, \underline{\theta}) &= \hat{V}_N(x_L, \underline{\theta}) + (x - x_L)\hat{p}_N(x_L, \underline{\theta}) \\ &= \frac{\mu}{r} x_L + \frac{1}{2}\hat{C}_N(\underline{\theta}) x_L^2 + (x - x_L)\left[\frac{\mu}{r} + \frac{1}{r}\frac{\left(r\hat{C}_N(\underline{\theta}) + \gamma\right)^2}{r\hat{C}_N(\underline{\theta}) + 2\gamma} x\right] \\ &= \frac{\mu}{r} x + \frac{1}{2r}\frac{\left(r\hat{C}_N(\underline{\theta}) + \gamma\right)^2}{r\hat{C}_N(\underline{\theta}) + 2\gamma} x^2 \end{aligned}$$

The next step is to solve the high-type problem. We start verifying that the incentive compatibility constraint for the low type must be binding. If the constraint were slack, then the solution to the high-type problem would be to trade

$$x + \Delta x(x, \bar{\theta}) = x_H = \frac{rC(\bar{\theta}) + \gamma}{rC(\bar{\theta}) + 2\gamma} x,$$

at a price

$$\hat{p}_N(x_H, \bar{\theta}) = \frac{\mu}{r} + \frac{1}{r}\frac{\left(r\hat{C}_N(\bar{\theta}) + \gamma\right)^2}{r\hat{C}_N(\bar{\theta}) + 2\gamma}x.$$

The constraint is slack only if

$$\frac{\mu}{r}x + \frac{1}{2r}\frac{r\hat{C}_N(\underline{\theta}) + 2\gamma}{r\hat{C}_N(\bar{\theta}) + 2\gamma}\frac{\left(r\hat{C}_N(\bar{\theta}) + \gamma\right)^2}{r\hat{C}_N(\bar{\theta}) + 2\gamma}x^2 \leq \frac{\mu}{r}x + \frac{1}{2r}\frac{\left(r\hat{C}_N(\underline{\theta}) + \gamma\right)^2}{r\hat{C}_N(\underline{\theta}) + 2\gamma}x^2,$$

which cannot be the case if $x > 0$ as $\hat{C}_N(\bar{\theta}) > \hat{C}_N(\underline{\theta})$. From here, we get that $x + \Delta x(x, \bar{\theta}) = x_H$ solves

$$\hat{V}_N(x_H, \underline{\theta}) + (x - x_H)\hat{p}_N(x_H, \bar{\theta}) = V_N(x, \underline{\theta}),$$

which means that

$$\left[\frac{1}{2}r\hat{C}_N(\underline{\theta}) - r\hat{C}_N(\bar{\theta}) - \gamma\right] x_H^2 + \left(r\hat{C}_N(\bar{\theta}) + \gamma\right) x x_H = \frac{1}{2}rC_N(\underline{\theta})x^2.$$

Let $x_H = \rho x$, then ρ solves the quadratic equation

$$\left[r\hat{C}_N(\bar{\theta}) + \gamma - \frac{1}{2}r\hat{C}_N(\underline{\theta})\right]\rho^2 - \left(r\hat{C}_N(\bar{\theta}) + \gamma\right)\rho + \frac{1}{2}rC_N(\underline{\theta}) = 0.$$

This equation has two roots,

$$\rho_1 = \frac{r\hat{C}_N(\bar{\theta}) + \gamma - \sqrt{\left(r\hat{C}_N(\bar{\theta}) + \gamma\right)^2 - 2rC_N(\underline{\theta})\left[r\hat{C}_N(\bar{\theta}) + \gamma - \frac{1}{2}r\hat{C}_N(\underline{\theta})\right]}}{2\left[r\hat{C}_N(\bar{\theta}) + \gamma - \frac{1}{2}r\hat{C}_N(\underline{\theta})\right]}$$

$$\rho_2 = \frac{r\hat{C}_N(\bar{\theta}) + \gamma + \sqrt{\left(r\hat{C}_N(\bar{\theta}) + \gamma\right)^2 - 2rC_N(\underline{\theta})\left[r\hat{C}_N(\bar{\theta}) + \gamma - \frac{1}{2}r\hat{C}_N(\underline{\theta})\right]}}{2\left[r\hat{C}_N(\bar{\theta}) + \gamma - \frac{1}{2}r\hat{C}_N(\underline{\theta})\right]},$$

only one of these roots satisfies $x_H > x_L$, which is ρ_2. It follows that $\beta_N(\bar{\theta}) = 1 - \rho_2$. □

Proof of Proposition 2

Proof. Letting $V_{n+1}(x, \theta)$ be the continuation value at date t_{n+1} in state θ, and $\pi(t|\theta) = \Pr(\theta_t = \bar{\theta}|\theta_{t_n} = \theta)$, we get

$$V_n(x,\theta) = \frac{1}{r}\left(1 - e^{-r\Delta}\right)\left(\mu x + \frac{1}{2}(\phi\underline{\theta}^2 - \gamma)x^2\right) + \frac{1}{2}\phi\left(\bar{\theta}^2 - \underline{\theta}^2\right)x^2 \int_{t_n}^{t_{n+1}} e^{-rt}\pi(t_{n+1}|\theta)dt$$
$$+ e^{-r\Delta}\left[\pi(t_{n+1}|\theta)V_{n+1}(x,\bar{\theta}) + (1 - \pi(t_{n+1}|\theta))V_{n+1}(x,\underline{\theta}))\right]$$

It follows from

$$\int_{t_n}^{t_{n+1}} e^{-rt}\pi(t_{n+1}|\bar{\theta})dt = \frac{1}{r}\left(1 - e^{-r\Delta}\right)\bar{\pi} + (1 - \bar{\pi})\frac{1}{r+\lambda}\left(1 - e^{-(r+\lambda)\Delta}\right)$$
$$\int_{t_n}^{t_{n+1}} e^{-rt}\pi(t_{n+1}|\underline{\theta})dt = \frac{1}{r}\left(1 - e^{-r\Delta}\right)\bar{\pi} - \bar{\pi}\frac{1}{r+\lambda}\left(1 - e^{-(r+\lambda)\Delta}\right)$$

that we can write the payoff as

$$\hat{V}_n(x,\bar{\theta}) = \frac{1}{r}\left(1 - e^{-r\Delta}\right)\left(\mu x + \frac{1}{2}(\nu(\bar{\theta}) - \gamma)x^2\right) + e^{-r\Delta}\left[\pi(\Delta|\bar{\theta})V_{n+1}(x,\bar{\theta}) + (1 - \pi(\Delta|\bar{\theta}))V_{n+1}(x,\underline{\theta})\right]$$
$$\hat{V}_n(x,\underline{\theta}) = \frac{1}{r}\left(1 - e^{-r\Delta}\right)\left(\mu x + \frac{1}{2}(\nu(\underline{\theta}) - \gamma)x^2\right) + e^{-r\Delta}\left[\pi(\Delta|\underline{\theta})V_{n+1}(x,\bar{\theta}) + (1 - \pi(\Delta|\underline{\theta}))V_{n+1}(x,\underline{\theta})\right]$$

Similar calculations yield the buyer valuation, which is given by

$$\hat{p}_n(x,\bar{\theta}) = \frac{1}{r}\left(1 - e^{-r\Delta}\right)\left(\mu + \nu(\bar{\theta})x\right) + e^{-r\Delta}\left[\pi(\Delta|\bar{\theta})p_{n+1}(x,\bar{\theta}) + (1 - \pi(\Delta|\bar{\theta}))p_{n+1}(x,\underline{\theta})\right]$$
$$\hat{p}_n(x,\underline{\theta}) = \frac{1}{r}\left(1 - e^{-r\Delta}\right)\left(\mu + \nu(\underline{\theta})x\right) + e^{-r\Delta}\left[\pi(\Delta|\underline{\theta})p_{n+1}(x,\bar{\theta}) + (1 - \pi(\Delta|\underline{\theta}))p_{n+1}(x,\underline{\theta})\right],$$

where $p_{n+1}(x,\theta) = \hat{p}_{n+1}\left(x + \Delta x_{n+1}(x,\theta), \theta\right)$. By induction, if the blockholder and market valuation at t_{n+1} are $V_{n+1}(x,\theta) = \frac{\mu}{r}x + \frac{1}{2}C_{n+1}(\theta)x$ and $p_{n+1}(x,\theta) = \frac{\mu}{r} + p_{n+1}(\theta)x$, then we can write the valuations at t_n as

$$\hat{V}_n(x,\theta) = \frac{\mu}{r}x + \frac{1}{2}\hat{C}_n(\theta)x^2$$
$$\hat{p}_n(x,\theta) = \frac{\mu}{r} + \hat{p}_n(\theta)x.$$

where the coefficients $\hat{C}_n(\theta)$ and $\hat{p}_n(\theta)$ are given by

$$\hat{C}_n(\theta) = \frac{1}{r}\left(1 - e^{-r\Delta}\right)(\nu(\theta) - \gamma) + e^{-r\Delta}\left[\pi(\Delta|\theta)C_{n+1}(\bar{\theta}) + (1 - \pi(\Delta|\theta))C_{n+1}(\underline{\theta})\right]$$

$$\hat{p}_n(\theta) = \frac{1}{r}\left(1 - e^{-r\Delta}\right)\nu(\theta) + e^{-r\Delta}\left[\pi(\Delta|\theta)p_{n+1}(\bar{\theta}) + (1 - \pi(\Delta|\theta))p_{n+1}(\underline{\theta})\right]$$

and $p_{n+1}(\theta) \equiv (1 - \beta_{n+1}(\theta))\hat{p}_{n+1}(\theta)$.

In the least costly separating equilibrium, at $t = t_n$, the low type chooses x' to maximize

$$V_n(x, \underline{\theta}) = \max_{x'} \hat{V}_n(x', \underline{\theta}) + (x - x')\hat{p}_n(x', \underline{\theta})$$

whereas the high type solves

$$V_n(x, \bar{\theta}) = \max_{x'} \hat{V}_n(x', \bar{\theta}) + (x - x')\hat{p}_n(x', \bar{\theta})$$

$$\text{s.t.}$$

$$\hat{V}_n(x', \underline{\theta}) + (x - x')\hat{p}_n(x', \bar{\theta}) \leq V_n(x, \underline{\theta})$$

The first-order condition for the low-type problem is

$$\hat{C}_n(\underline{\theta})x_L + \hat{p}_n(\underline{\theta})x - 2\hat{p}_n(\underline{\theta})x_L = 0$$

so

$$x_L = \frac{\hat{p}_n(\underline{\theta})}{2\hat{p}_n(\underline{\theta}) - \hat{C}_n(\underline{\theta})}x$$

The second-order condition is satisfied if $2\hat{p}_n(\underline{\theta}) > \hat{C}_n(\underline{\theta})$. If this is the case, the solution to the low-type problem is x_L and

$$p_n(x, \underline{\theta}) = \frac{\mu}{r} + p_n(\underline{\theta})x$$

$$V_n(x, \underline{\theta}) = \frac{\mu}{r}x + \frac{1}{2}C_n(\underline{\theta})x^2$$

where

$$p_n(\underline{\theta}) = \frac{\hat{p}_n(\underline{\theta})^2}{2\hat{p}_n(\underline{\theta}) - \hat{C}_n(\underline{\theta})}$$
$$C_n(\underline{\theta}) = \frac{\hat{p}_n(\underline{\theta})^2}{2\hat{p}_n(\underline{\theta}) - \hat{C}_n(\underline{\theta})}.$$

The next step is to check the incentives for the high type. If the low type incentive compatibility constraint were not binding, then we would have that

$$x_H = \frac{\hat{p}_n(\bar{\theta})}{2\hat{p}_n(\bar{\theta}) - \hat{C}_n(\bar{\theta})} x.$$

The IC constraint is satisfied only if $x_H = \rho x$, where

$$\left(2\hat{p}_n(\bar{\theta}) - \hat{C}_n(\underline{\theta})\right) \rho^2 - 2\hat{p}_n(\bar{\theta})\rho + C_n(\underline{\theta}) \geq 0.$$

Hence, the unconstrained solution violates the low-type incentive compatibility constraint if

$$\frac{\hat{p}_n(\bar{\theta}) - \sqrt{\hat{p}_n(\bar{\theta})^2 - 2\hat{p}_n(\bar{\theta})C_n(\underline{\theta}) + C_n(\underline{\theta})\hat{C}_n(\underline{\theta})}}{2\hat{p}_n(\bar{\theta}) - \hat{C}_n(\underline{\theta})} \leq \frac{\hat{p}_n(\bar{\theta})}{2\hat{p}_n(\bar{\theta}) - \hat{C}_n(\bar{\theta})}$$
$$\leq \frac{\hat{p}_n(\bar{\theta}) + \sqrt{\hat{p}_n(\bar{\theta})^2 - 2\hat{p}_n(\bar{\theta})C_n(\underline{\theta}) + C_n(\underline{\theta})\hat{C}_n(\underline{\theta})}}{2\hat{p}_n(\bar{\theta}) - \hat{C}_n(\underline{\theta})}.$$

The first inequality is necessarily satisfied as $\hat{C}_n(\bar{\theta}) > \hat{C}_n(\underline{\theta})$, the second inequality is satisfied if

$$\frac{\hat{p}_n(\bar{\theta})}{2\hat{p}_n(\bar{\theta}) - \hat{C}_n(\bar{\theta})} \leq \frac{\hat{p}_n(\bar{\theta}) + \sqrt{\hat{p}_n(\bar{\theta})^2 - 2\hat{p}_n(\bar{\theta})C_n(\underline{\theta}) + C_n(\underline{\theta})\hat{C}_n(\underline{\theta})}}{2\hat{p}_n(\bar{\theta}) - \hat{C}_n(\underline{\theta})},$$

which means that

$$\beta_n(\bar{\theta}) \leq \frac{\hat{p}_n(\bar{\theta}) - \hat{C}_n(\bar{\theta})}{2\hat{p}_n(\bar{\theta}) - \hat{C}_n(\bar{\theta})}.$$

If this condition is satisfied, the low-type incentive compatibility constraint binds, so

$$\hat{V}_n(x_H, \underline{\theta}) + (x - x_H)\hat{p}_n(x_H, \bar{\theta}) = V_n(x, \underline{\theta}).$$

From here, we get

$$\left(\hat{C}_n(\underline{\theta}) - 2\hat{p}_n(\bar{\theta})\right) x_H^2 + 2\hat{p}_n(\bar{\theta})x_H x - C_n(\underline{\theta})x^2 = 0.$$

Letting $x_H = \rho_n x$ we get the quadratic equation

$$\left(2\hat{p}_n(\bar{\theta}) - \hat{C}_n(\underline{\theta})\right) \rho_n^2 - 2\hat{p}_n(\bar{\theta})\rho_n + C_n(\underline{\theta}) = 0.$$

So we get that

$$\rho_n = \frac{\hat{p}_n(\bar{\theta}) + \sqrt{\hat{p}_n(\bar{\theta})^2 - 2\hat{p}_n(\bar{\theta})C_n(\underline{\theta}) + C_n(\underline{\theta})\hat{C}_n(\underline{\theta})}}{2\hat{p}_n(\bar{\theta}) - \hat{C}_n(\underline{\theta})}.$$

From here, we get that the equilibrium payoff are

$$\begin{aligned} p_n(x, \theta) &= \frac{\mu}{r} + p_n(\theta)x \\ V_n(x, \theta) &= \frac{\mu}{r}x + \frac{1}{2}C_n(\theta)x^2 \end{aligned}$$

where

$$\begin{aligned} p_n(\underline{\theta}) &= \frac{\hat{p}_n(\underline{\theta})^2}{2\hat{p}_n(\underline{\theta}) - \hat{C}_n(\underline{\theta})} \\ p_n(\bar{\theta}) &= \rho_n \hat{p}_n(\bar{\theta}) \\ C_n(\underline{\theta}) &= \frac{\hat{p}_n(\underline{\theta})^2}{2\hat{p}_n(\underline{\theta}) - \hat{C}_n(\underline{\theta})} \\ C_n(\bar{\theta}) &= C_n(\underline{\theta}) + \rho_n^2 \left(\hat{C}_n(\bar{\theta}) - \hat{C}_n(\underline{\theta})\right) \\ \rho_n &= \frac{\hat{p}_n(\bar{\theta}) + \sqrt{\hat{p}_n(\bar{\theta})^2 - 2\hat{p}_n(\bar{\theta})C_n(\underline{\theta}) + C_n(\underline{\theta})\hat{C}_n(\underline{\theta})}}{2\hat{p}_n(\bar{\theta}) - \hat{C}_n(\underline{\theta})} \end{aligned}$$

Letting $\Gamma_n \equiv \hat{C}_n(\bar{\theta}) - \hat{C}_n(\underline{\theta})$ we can write the recursion for the coefficients $\hat{C}_n(\theta)$ and $\hat{p}_n(\theta)$

as

$$\hat{C}_n(\underline{\theta}) = \frac{1}{r}\left(1-e^{-r\Delta}\right)(\nu(\underline{\theta})-\gamma) + e^{-r\Delta}\frac{\hat{p}_{n+1}(\underline{\theta})^2}{2\hat{p}_{n+1}(\underline{\theta})-\hat{C}_{n+1}(\underline{\theta})} + e^{-r\Delta}\pi(\Delta|\underline{\theta})\rho_{n+1}^2\Gamma_{n+1}$$

$$\Gamma_n = \frac{1}{r}\left(1-e^{-r\Delta}\right)\left(\nu(\bar{\theta})-\nu(\underline{\theta})\right) + e^{-r\Delta}\left(\pi(\Delta|\bar{\theta})-\pi(\Delta|\underline{\theta})\right)\rho_{n+1}^2\Gamma_{n+1}$$

$$\hat{p}_n(\bar{\theta}) = \frac{1}{r}\left(1-e^{-r\Delta}\right)\nu(\bar{\theta}) + e^{-r\Delta}\left[\pi(\Delta|\bar{\theta})\rho_{n+1}\hat{p}_{n+1}(\bar{\theta}) + (1-\pi(\Delta|\bar{\theta}))\frac{\hat{p}_{n+1}(\underline{\theta})^2}{2\hat{p}_{n+1}(\underline{\theta})-\hat{C}_{n+1}(\underline{\theta})}\right]$$

$$\hat{p}_n(\underline{\theta}) = \frac{1}{r}\left(1-e^{-r\Delta}\right)\nu(\underline{\theta}) + e^{-r\Delta}\left[\pi(\Delta|\underline{\theta})\rho_{n+1}\hat{p}_{n+1}(\bar{\theta}) + (1-\pi(\Delta|\underline{\theta}))\frac{\hat{p}_{n+1}(\underline{\theta})^2}{2\hat{p}_{n+1}(\underline{\theta})-\hat{C}_{n+1}(\underline{\theta})}\right].$$

The only remaining step in the derivation of the equilibrium is to verify that the second-order conditions are satisfied. Substituting the expressions for $\hat{p}_n(\underline{\theta})$ and $\hat{C}_n(\underline{\theta})$ we find that the second order condition is satisfied if

$$\begin{aligned}\frac{2}{r}\left(1-e^{-r\Delta}\right)\nu(\underline{\theta}) + 2e^{-r\Delta}\left[\pi(\Delta|\underline{\theta})p_{n+1}(\bar{\theta}) + (1-\pi(\Delta|\underline{\theta}))p_{n+1}(\underline{\theta})\right] \\ > \frac{1}{r}\left(1-e^{-r\Delta}\right)(\nu(\underline{\theta})-\gamma) + e^{-r\Delta}\left[\pi(\Delta|\underline{\theta})C_{n+1}(\bar{\theta}) + (1-\pi(\Delta|\underline{\theta}))C_{n+1}(\underline{\theta})\right]\end{aligned}$$

Simplifying terms, we get

$$\frac{1}{r}\left(1-e^{-r\Delta}\right)(\nu(\underline{\theta})+\gamma)+e^{-r\Delta}\left[\pi(\Delta|\underline{\theta})\left(2p_{n+1}(\bar{\theta})-C_{n+1}(\bar{\theta})\right) + (1-\pi(\Delta|\underline{\theta}))\left(2p_{n+1}(\underline{\theta})-C_{n+1}(\underline{\theta})\right)\right] > 0.$$

Substituting $p_{n+1}(\underline{\theta}) = C_{n+1}(\underline{\theta}) = \frac{\hat{p}_{n+1}(\underline{\theta})^2}{2\hat{p}_{n+1}(\underline{\theta})-\hat{C}_{n+1}(\underline{\theta})}$, we get

$$\frac{1}{r}\left(1-e^{-r\Delta}\right)(\nu(\underline{\theta})+\gamma)+e^{-r\Delta}\left[\pi(\Delta|\underline{\theta})\left(2p_{n+1}(\bar{\theta})-C_{n+1}(\bar{\theta})\right) + (1-\pi(\Delta|\underline{\theta}))\frac{\hat{p}_{n+1}(\underline{\theta})^2}{2\hat{p}_{n+1}(\underline{\theta})-\hat{C}_{n+1}(\underline{\theta})}\right]$$

If the second-order condition is satisfied at t_{n+1} then $\frac{\hat{p}_{n+1}(\underline{\theta})^2}{2\hat{p}_{n+1}(\underline{\theta})-\hat{C}_{n+1}(\underline{\theta})} > 0$. It suffices then to show that $2p_{n+1}(\bar{\theta}) > C_{n+1}(\bar{\theta})$. This condition is equivalent to

$$2p_{n+1}(\bar{\theta}) - C_{n+1}(\bar{\theta}) = 2\rho_{n+1}\hat{p}_{n+1}(\bar{\theta}) - C_{n+1}(\underline{\theta}) - \rho_{n+1}^2\left(\hat{C}_{n+1}(\bar{\theta}) - \hat{C}_{n+1}(\underline{\theta})\right) > 0$$

From the quadratic equation for ρ_{n+1} we have that

$$C_{n+1}(\underline{\theta}) = 2\hat{p}_{n+1}(\bar{\theta})\rho_{n+1} - \left(2\hat{p}_{n+1}(\bar{\theta}) - \hat{C}_{n+1}(\underline{\theta})\right)\rho_{n+1}^2.$$

Substituting in $2p_{n+1}(\bar{\theta}) - C_{n+1}(\bar{\theta})$ we get

$$2p_{n+1}(\bar{\theta}) - C_{n+1}(\bar{\theta}) = \left(2\hat{p}_{n+1}(\bar{\theta}) - \hat{C}_{n+1}(\bar{\theta})\right)\rho_{n+1}^2.$$

Letting $S_n = 2\hat{p}_n(\bar{\theta}) - \hat{C}_n(\bar{\theta})$, we get the recursion

$$S_n = \frac{1}{r}\left(1 - e^{-r\Delta}\right)\left(\nu(\bar{\theta}) + \gamma\right) + e^{-r\Delta}\left[\pi(\Delta|\underline{\theta})\rho_{n+1}^2 S_{n+1} + (1 - \pi(\Delta|\underline{\theta}))\frac{\hat{p}_{n+1}(\underline{\theta})^2}{2\hat{p}_{n+1}(\underline{\theta}) - \hat{C}_{n+1}(\underline{\theta})}\right].$$

It follows by induction from $S_N > 0$ and $2\hat{p}_{n+1} - \hat{C}_{n+1}(\underline{\theta}) > 0$ that $S_n > 0$ for all $n < N$, which implies that the second order conditions is satisfied. □

Proof of Proposition 3

Proof. Let $\beta(\theta) = \lim_{\Delta\to 0} \frac{\beta_\Delta(\theta)}{\Delta}$ be the limit selling rate of type θ. The limit of Γ_Δ is

$$\Gamma = \frac{\nu(\bar{\theta}) - \nu(\underline{\theta})}{r + \lambda + 2\lim_{\Delta\to 0}\frac{\beta_\Delta(\bar{\theta})}{\Delta}} = \frac{\phi(\bar{\theta}^2 - \underline{\theta}^2)}{r + \lambda + 2\beta(\bar{\theta})}.$$

Letting

$$\Phi_\Delta \equiv \frac{1}{r}\left(1 - e^{-r\Delta}\right)\left(\nu(\underline{\theta}) - \gamma\right) + e^{-r\Delta}\pi(\Delta|\underline{\theta})(1 - \beta_\Delta(\bar{\theta}))^2\Gamma_\Delta,$$

we can write the equations for $\hat{C}_\Delta(\underline{\theta})$

$$\hat{C}_\Delta(\underline{\theta}) = \Phi_\Delta + e^{-r\Delta}\frac{\hat{p}_\Delta(\underline{\theta})^2}{2\hat{p}_\Delta(\underline{\theta}) - \hat{C}_\Delta(\underline{\theta})} \tag{16}$$

and obtain the following equation for $\hat{C}_\Delta(\theta)$ in terms of $\hat{p}_\Delta(\theta)$

$$\hat{C}_\Delta(\underline{\theta}) = \hat{p}_\Delta(\underline{\theta}) + \frac{1}{2}\left(\Phi_\Delta \pm \sqrt{\left(\Phi_\Delta - 2\hat{p}_\Delta(\underline{\theta})\right)^2 - 4e^{-r\Delta}\hat{p}_\Delta(\underline{\theta})^2}\right) \tag{17}$$

From equation (16) we have $\lim_{\Delta\to 0} \hat{C}_\Delta(\underline{\theta}) = \lim_{\Delta\to 0} \hat{p}_\Delta(\underline{\theta})$, so to find the limit of $\hat{p}_\Delta(\underline{\theta})$ we can consider the equation

$$0 = \Phi_\Delta \pm \sqrt{(\Phi_\Delta - 2p_\Delta)^2 - 4e^{-r\Delta}p_\Delta^2}$$

This equation has two solutions $p_\Delta^0 = 0$ and $p_\Delta^1 = \frac{\Phi_\Delta}{1-e^{-r\Delta}}$. It follows that if $\lim_{\Delta\to 0} \frac{\Phi_\Delta}{\Delta} > 0$, then the price coefficient is

$$\hat{p}(\underline{\theta}) = \lim_{\Delta\to 0} \frac{\Phi_\Delta}{1-e^{-r\Delta}} = \frac{1}{r} \lim_{\Delta\to 0} \frac{\Phi_\Delta}{\Delta},$$

while if $\lim_{\Delta\to 0} \frac{\Phi(\Delta)}{\Delta} \leq 0$, the limit solution is $\hat{p}(\underline{\theta}) = 0$. We need to distinguish two cases depending on the sign of $\lim_{\Delta\to 0} \frac{\Phi(\Delta)}{\Delta}$: $\hat{p}(\underline{\theta}) = \lim_{\Delta\to 0} \hat{p}_\Delta = \lim_{\Delta\to 0} \hat{C}_\Delta > 0$ and $\hat{p}(\underline{\theta}) = \lim_{\Delta\to 0} \hat{p}_\Delta = \lim_{\Delta\to 0} \hat{C}_\Delta = 0$. In the first case, we have $\lim_{\Delta\to 0} \beta_\Delta(\underline{\theta}) = 0$, so there is no atom in trade. To find $\lim_{\Delta\to 0} \frac{\beta_\Delta(\theta)}{\Delta}$ we need the limit

$$\lim_{\Delta\to 0} \frac{\hat{p}_\Delta(\underline{\theta}) - \hat{C}_\Delta(\underline{\theta})}{\Delta}$$

Combining the equations for $\hat{C}(\underline{\theta})$ and $\hat{p}(\underline{\theta})$ we get

$$\begin{aligned} \frac{\hat{p}_\Delta(\underline{\theta}) - \hat{C}_\Delta(\underline{\theta})}{\Delta} &= \frac{1-e^{-r\Delta}}{r\Delta}\gamma \\ &+ e^{-r\Delta}\frac{\bar{\pi}\left(1-e^{-\lambda\Delta}\right)}{\Delta}\left[\left(1-\beta_\Delta(\bar{\theta})\right)\hat{p}_\Delta(\bar{\theta}) - \frac{\hat{p}_\Delta(\underline{\theta})^2}{2\hat{p}_\Delta(\underline{\theta}) - \hat{C}_\Delta(\underline{\theta})} - \left(1-\beta_\Delta(\bar{\theta})\right)^2\Gamma_\Delta\right] \end{aligned}$$

Taking limit when $\Delta \to 0$, we get

$$\lim_{\Delta\to 0} \frac{\hat{p}_\Delta(\underline{\theta}) - \hat{C}_\Delta(\underline{\theta})}{\Delta} = \gamma + \lambda\bar{\pi}\left[\hat{p}(\bar{\theta}) - \hat{p}(\underline{\theta}) - \Gamma\right].$$

Hence, we conclude that when $\hat{p}(\underline{\theta}) > 0$ we get

$$\beta(\underline{\theta}) = \frac{\gamma + \lambda\bar{\pi}\left[\hat{p}(\bar{\theta}) - \hat{p}(\underline{\theta}) - \Gamma\right]}{\hat{p}(\underline{\theta})}$$

On the other hand, when $\hat{p}(\underline{\theta}) = 0$, there is an atom in trade so we also need to find the limit

$$\lim_{\Delta \to 0} \frac{2\hat{p}_\Delta(\underline{\theta}) - \hat{C}_\Delta(\underline{\theta})}{\Delta}.$$

Once again, combining the equations for $\hat{C}(\underline{\theta})$ and $\hat{p}(\underline{\theta})$, we get

$$\begin{aligned}\frac{2\hat{p}_\Delta(\underline{\theta}) - \hat{C}_\Delta(\underline{\theta})}{\Delta} &= \frac{1-e^{-r\Delta}}{r\Delta}(\nu(\underline{\theta}) + \gamma) \\ &\quad + e^{-r\Delta}\frac{\pi(\Delta|\underline{\theta})}{\Delta}\left[\left(1-\beta_\Delta(\bar{\theta})\right)2\hat{p}_\Delta(\bar{\theta}) - 2\frac{\frac{\hat{p}_\Delta(\underline{\theta})}{\Delta}}{\frac{2\hat{p}_\Delta(\underline{\theta})-\hat{C}_\Delta(\underline{\theta})}{\Delta}}\hat{p}_\Delta(\underline{\theta}) - \left(1-\beta_\Delta(\bar{\theta})\right)^2\Gamma_\Delta\right] \\ &\quad + e^{-r\Delta}\frac{\frac{\hat{p}_\Delta(\underline{\theta})}{\Delta}}{\frac{2\hat{p}_\Delta(\underline{\theta})-\hat{C}_\Delta(\underline{\theta})}{\Delta}}\frac{\hat{p}_\Delta(\underline{\theta})}{\Delta}\end{aligned}$$

where

$$\begin{aligned}\frac{\hat{p}_\Delta(\underline{\theta})}{\Delta} = \frac{1-e^{-r\Delta}}{r\Delta}\nu(\underline{\theta}) + e^{-r\Delta}\frac{\pi(\Delta|\underline{\theta})}{\Delta}\left[\left(1-\beta_\Delta(\bar{\theta})\right)\hat{p}_\Delta(\bar{\theta}) - \frac{\frac{\hat{p}_\Delta(\underline{\theta})}{\Delta}}{\frac{2\hat{p}_\Delta(\underline{\theta})-\hat{C}_\Delta(\underline{\theta})}{\Delta}}\hat{p}_\Delta(\underline{\theta})\right] \\ + e^{-r\Delta}\frac{\frac{\hat{p}_\Delta(\underline{\theta})}{\Delta}}{\frac{2\hat{p}_\Delta(\underline{\theta})-\hat{C}_\Delta(\underline{\theta})}{\Delta}}\frac{\hat{p}_\Delta(\underline{\theta})}{\Delta}\end{aligned}$$

Letting $y \equiv \lim_{\Delta\to 0}\frac{2\hat{p}_\Delta(\underline{\theta})-\hat{C}_\Delta(\underline{\theta})}{\Delta}$ and $z \equiv \lim_{\Delta\to 0}\frac{\hat{p}_\Delta(\underline{\theta})}{\Delta}$ we get

$$\begin{aligned}y &= \nu(\underline{\theta}) + \gamma + \lambda\bar{\pi}\left[2\hat{p}(\bar{\theta}) - \Gamma\right] + \frac{z^2}{y} \\ z &= \nu(\underline{\theta}) + \lambda\bar{\pi}\hat{p}(\bar{\theta}) + \frac{z^2}{y}\end{aligned}$$

solving for y, z we get

$$\begin{aligned}y &= -\frac{(\gamma + \lambda\bar{\pi}\left[\hat{p}(\bar{\theta}) - \Gamma\right])^2}{\nu(\underline{\theta}) - \gamma + \lambda\bar{\pi}\Gamma} \\ z &= \frac{(\nu(\underline{\theta}) + \gamma + \lambda\bar{\pi}\left[2\hat{p}(\bar{\theta}) - \Gamma\right] - \nu(\underline{\theta}) - \lambda\bar{\pi}\hat{p}(\bar{\theta}))\left(\nu(\underline{\theta}) + \lambda\bar{\pi}\hat{p}(\bar{\theta})\right)}{\nu(\underline{\theta}) + \gamma + \lambda\bar{\pi}\left[2\hat{p}(\bar{\theta}) - \Gamma\right] - 2\left(\nu(\underline{\theta}) + \lambda\bar{\pi}\hat{p}(\bar{\theta})\right)}\end{aligned}$$

so

$$\lim_{\Delta\to 0}\frac{2\hat{p}_\Delta(\underline{\theta})-\hat{C}_\Delta(\underline{\theta})}{\Delta}=-\frac{(\gamma+\lambda\bar{\pi}\left[\hat{p}(\bar{\theta})-\Gamma\right])^2}{\nu(\underline{\theta})-\gamma+\lambda\bar{\pi}\Gamma}$$

It follows that

$$\lim_{\Delta\to 0}\beta_\Delta(\underline{\theta})=-\left(\gamma+\lambda\bar{\pi}\left[\hat{p}(\bar{\theta})-\Gamma\right]\right)(\nu(\underline{\theta})-\gamma+\lambda\bar{\pi}\Gamma).$$

The coefficient of the trading strategy $\beta_\Delta(\bar{\theta})$ satisfies the equation

$$\left(2\hat{p}(\bar{\theta})-\hat{C}(\underline{\theta})\right)\beta_\Delta(\bar{\theta})^2+2\left(\hat{C}(\underline{\theta})-\hat{p}(\bar{\theta})\right)\beta_\Delta(\bar{\theta})+C(\underline{\theta})-\hat{C}(\underline{\theta})=0.$$

We can write the previous equation as

$$\left(2\hat{p}_\Delta(\bar{\theta})-\hat{C}_\Delta(\underline{\theta})\right)\Delta\left(\frac{\beta_\Delta(\bar{\theta})}{\Delta}\right)^2+2\left(\hat{C}_\Delta(\underline{\theta})-\hat{p}_\Delta(\bar{\theta})\right)\frac{\beta_\Delta(\bar{\theta})}{\Delta}+\frac{C_\Delta(\underline{\theta})-\hat{C}_\Delta(\underline{\theta})}{\Delta}=0.$$

Taking the limit when $\Delta\to 0$ we get

$$\beta(\bar{\theta})=\frac{1}{2\left(\hat{C}(\underline{\theta})-\hat{p}(\bar{\theta})\right)}\lim_{\Delta\to}\frac{\hat{C}_\Delta(\underline{\theta})-C_\Delta(\underline{\theta})}{\Delta}$$

Noting that $C_\Delta(\underline{\theta})=\frac{\hat{p}_\Delta(\theta)^2}{2\hat{p}_\Delta(\theta)-\hat{C}_\Delta(\underline{\theta})}$ we obtain from equation (16) that

$$\frac{\hat{C}_\Delta(\underline{\theta})-C_\Delta(\underline{\theta})}{\Delta}=\frac{\Phi_\Delta}{\Delta}-\frac{1-e^{-r\Delta}}{\Delta}C_\Delta(\underline{\theta}).$$

We have that $\lim_{\Delta\to 0}\frac{1-e^{-r\Delta}}{\Delta}C_\Delta(\underline{\theta})=r\hat{p}(\underline{\theta})$. Hence, if $\lim_{\Delta\to 0}\frac{\Phi_\Delta}{\Delta}>0$ then $\lim_{\Delta\to 0}\frac{1-e^{-r\Delta}}{\Delta}C_\Delta(\underline{\theta})=r\hat{p}(\underline{\theta})=\lim_{\Delta\to 0}\frac{\Phi_\Delta}{\Delta}$, which means that $\lim_{\Delta\to 0}\frac{\hat{C}_\Delta(\theta)-C_\Delta(\theta)}{\Delta}=0$. This implies $\beta(\bar{\theta})=0$. On the other hand, if $\lim_{\Delta\to 0}\frac{\Phi_\Delta}{\Delta}\leq 0$ then $\lim_{\Delta\to 0}\frac{1-e^{-r\Delta}}{\Delta}C_\Delta(\underline{\theta})=0$ so $\lim_{\Delta\to 0}\frac{\hat{C}_\Delta(\theta)-C_\Delta(\theta)}{\Delta}=\lim_{\Delta\to 0}\frac{\Phi_\Delta}{\Delta}$. In this case, we have that

$$\beta(\bar{\theta})=-\frac{1}{2\hat{p}(\bar{\theta})}\lim_{\Delta\to 0}\frac{\Phi_\Delta}{\Delta}$$

Having determined $\beta(\theta)$ as function of the sign of $\lim_{\Delta\to 0}\frac{\Phi(\Delta)}{\Delta}$, we turn our attention to the limit $\lim_{\Delta\to 0}\frac{\Phi(\Delta)}{\Delta}$, which is given by

$$\lim_{\Delta\to 0}\frac{\Phi(\Delta)}{\Delta}=\lim_{\Delta\to 0}\frac{\frac{1}{r}\left(1-e^{-r\Delta}\right)(\nu(\underline{\theta})-\gamma)+e^{-r\Delta}\pi(\Delta|\underline{\theta})(1-\beta_\Delta(\bar{\theta}))^2\Gamma_\Delta}{\Delta}\quad=\phi\underline{\theta}^2-\gamma+\lambda\bar{\pi}\Gamma.$$

For a given $\beta(\bar{\theta})$, we have

$$\phi\underline{\theta}^2-\gamma+\lambda\bar{\pi}\Gamma=\phi\underline{\theta}^2-\gamma+\lambda\bar{\pi}\frac{\phi(\bar{\theta}^2-\underline{\theta}^2)}{r+\lambda+2\beta(\bar{\theta})},$$

From here, we get that if

$$\gamma<\phi\underline{\theta}^2+\frac{\lambda\bar{\pi}}{r+\lambda}\phi(\bar{\theta}^2-\underline{\theta}^2)$$

then $\nu(\underline{\theta})-\gamma+\lambda\bar{\pi}\Gamma>0$ for $\beta(\bar{\theta})=0$. In this case the low-type trading rate is

$$\beta(\underline{\theta})=\frac{\gamma+\lambda\bar{\pi}\left[\hat{p}(\bar{\theta})-\hat{p}(\underline{\theta})-\Gamma\right]}{\hat{p}(\underline{\theta})},$$

where

$$\Gamma=\frac{\phi(\bar{\theta}^2-\underline{\theta}^2)}{r+\lambda}.$$

and

$$\hat{p}(\underline{\theta})=\frac{1}{r}\lim_{\Delta\to 0}\frac{\Phi_\Delta}{\Delta}=\frac{1}{r}\left[\phi\underline{\theta}^2-\gamma+\frac{\lambda\bar{\pi}}{r+\lambda}\phi(\bar{\theta}^2-\underline{\theta}^2)\right]=C^{\mathrm{nt}}(\underline{\theta}).$$

To find the coefficient $\hat{p}(\bar{\theta})$ we write the recursive equation for $\hat{p}(\bar{\theta})$ as

$$\hat{p}_\Delta(\bar{\theta})=\frac{\frac{1}{r}\left(1-e^{-r\Delta}\right)\nu(\bar{\theta})+e^{-r\Delta}(1-\bar{\pi})(1-e^{-\lambda\Delta})\frac{\hat{p}_\Delta(\underline{\theta})^2}{2\hat{p}_\Delta(\underline{\theta})-\hat{C}_\Delta(\underline{\theta})}}{1-e^{-r\Delta}\left(\bar{\pi}+(1-\bar{\pi})e^{-\lambda\Delta}\right)\left(1-\beta_\Delta(\bar{\theta})\right)}\tag{18}$$

Taking limit, we get

$$\hat{p}(\bar{\theta})=\frac{\phi\bar{\theta}^2+\lambda(1-\bar{\pi})\hat{p}(\underline{\theta})}{r+\lambda(1-\bar{\pi})}$$

The equilibrium payoffs are

$$C(\underline{\theta})=\hat{p}(\underline{\theta})=C^{\mathrm{nt}}(\underline{\theta})$$
$$C(\bar{\theta})=C^{\mathrm{nt}}(\underline{\theta})+\Gamma=C^{\mathrm{nt}}(\bar{\theta}).$$

The path of x_t converges to

$$x_t = x_0 e^{-\xi(\underline{\theta}) \int_0^t \mathbf{1}_{\{\theta_s = \underline{\theta}\}} ds},$$

On the other hand, if

$$\gamma \geq \phi \underline{\theta}^2 + \frac{\lambda \bar{\pi}}{r + \lambda} \phi (\bar{\theta}^2 - \underline{\theta}^2)$$

then $\lim_{\Delta \to 0} \frac{\Phi(\Delta)}{\Delta} \leq 0$. In this case, if $\gamma + \lambda \bar{\pi} \left[\hat{p}(\bar{\theta}) - \Gamma \right] > 0$, then

$$\lim_{\Delta \to 0} \beta_\Delta(\underline{\theta}) = -\left(\gamma + \lambda \bar{\pi} \left[\hat{p}(\bar{\theta}) - \Gamma \right] \right) \left(\nu(\underline{\theta}) - \gamma + \lambda \bar{\pi} \Gamma \right) > 0,$$

which means that $\beta(\underline{\theta}) = \infty$, and

$$\beta(\bar{\theta}) = -\frac{1}{2\hat{p}(\bar{\theta})} \lim_{\Delta \to 0} \frac{\Phi_\Delta}{\Delta} = -\frac{\phi \underline{\theta}^2 - \gamma + \lambda \bar{\pi} \Gamma}{2\hat{p}(\bar{\theta})},$$

where

$$\lim_{\Delta \to 0} \hat{C}_\Delta(\bar{\theta}) = \Gamma = \frac{\phi(\bar{\theta}^2 - \underline{\theta}^2)}{r + \lambda + 2\beta(\bar{\theta})}.$$

The price coefficient $\hat{p}(\bar{\theta})$ is obtained from equation (18), which yields

$$\hat{p}(\bar{\theta}) = \frac{\phi \bar{\theta}^2}{r + \lambda(1 - \bar{\pi}) + \beta(\bar{\theta})}.$$

Putting the previous expressions together, we find the coefficients $\beta(\bar{\theta}), \hat{p}(\bar{\theta}), \Gamma$ by solving the system

$$\begin{aligned}
\beta(\bar{\theta}) &= \frac{\gamma - \phi \underline{\theta}^2 - \lambda \bar{\pi} \Gamma}{2\hat{p}(\bar{\theta})} \\
\hat{p}(\bar{\theta}) &= \frac{\phi \bar{\theta}^2}{r + \lambda(1 - \bar{\pi}) + \beta(\bar{\theta})} \\
\Gamma &= \frac{\phi(\bar{\theta}^2 - \underline{\theta}^2)}{r + \lambda + 2\beta(\bar{\theta})},
\end{aligned}$$

where the solution must satisfy $\phi \underline{\theta}^2 - \gamma + \lambda \bar{\pi} \Gamma \leq 0$. Using the equations for $\beta(\bar{\theta})$ and $\hat{p}(\bar{\theta})$

we can solve for these coefficients as a function of Γ, to get

$$\beta(\bar{\theta}) = \frac{(r+\lambda(1-\bar{\pi}))\left(\gamma - \phi\underline{\theta}^2 - \lambda\bar{\pi}\Gamma\right)}{\phi\left(2\bar{\theta}^2 + \underline{\theta}^2\right) - \gamma + \lambda\bar{\pi}\Gamma}$$
$$\hat{p}(\bar{\theta}) = \frac{\phi\left(2\bar{\theta}^2 + \underline{\theta}^2\right) - \gamma + \lambda\bar{\pi}\Gamma}{2(r+\lambda(1-\bar{\pi}))}$$

where

$$\Gamma_l \equiv \frac{\gamma - \phi(\underline{\theta}^2 + 2\bar{\theta}^2)}{\lambda\bar{\pi}} \leq \Gamma \leq \frac{\gamma - \phi\underline{\theta}^2}{\lambda\bar{\pi}} \equiv \Gamma_r$$

Letting

$$h(\Gamma) \equiv \frac{(r+\lambda(1-\bar{\pi}))\left(\Gamma_r - \Gamma\right)}{\Gamma - \Gamma_l},$$

we get that Γ is a fixed point

$$\Gamma = \frac{\phi(\bar{\theta}^2 - \underline{\theta}^2)}{r+\lambda+2h(\Gamma)}, \ \Gamma \in [\Gamma_l, \Gamma_r].$$

As long as $h(\Gamma) = \beta(\bar{\theta})$ is finite, we can write the fixed point for Γ as $\Gamma\left(r+\lambda+2h(\Gamma)\right) = \phi(\bar{\theta}^2 - \underline{\theta}^2)$. In this case, collecting terms, we get that Γ solves the quadratic equation

$$Q(\Gamma) \equiv \Gamma^2\left(r+\lambda(1-2\bar{\pi})\right) + \Gamma\left(\phi\left(\bar{\theta}^2 - \underline{\theta}^2\right) + \Gamma_l(r+\lambda) - 2\Gamma_r(r+\lambda(1-\bar{\pi}))\right) - \Gamma_l\phi\left(\bar{\theta}^2 - \underline{\theta}^2\right) = 0.$$

where

$$Q(\Gamma_l) = -2\Gamma_l(r+\lambda(1-\bar{\pi}))\left(\Gamma_r - \Gamma_l\right)$$
$$Q(\Gamma_r) = -\left(\Gamma_r - \Gamma_l\right)\left(\Gamma_r(r+\lambda) - \phi(\bar{\theta}^2 - \underline{\theta}^2)\right) < 0$$

If $\Gamma_l < 0$, then $Q(\Gamma_l) > 0$ so there exist a unique root $Q(\Gamma^*) = 0$ in $[\Gamma_l, \Gamma_r]$. The path of x_t converges to

$$x_t = x_0 e^{-\beta(\bar{\theta})t}\mathbf{1}_{\{t<\underline{\tau}\}},$$

where $\underline{\tau} = \inf\{t > 0 : \theta_t = \underline{\theta}\}$. Finally, as $\Gamma_l \to 0$, we get that $\Gamma^* \to 0$, so $\hat{p}(\bar{\theta}) \to 0$ and $\beta(\bar{\theta}) \to \infty$. If $\Gamma_l > 0$, the high-type limit trading strategy is no longer smooth, so $\beta(\bar{\theta}) = \infty$ and $\hat{p}(\bar{\theta}) = \Gamma = 0$. □

Proof of Proposition 5

Proof. When $N \to \infty$, the coefficients $\hat{C}^{\text{o}}_n(\theta), \hat{p}^{\text{o}}_n(\theta)$ converge to the solution of the system

$$\hat{C}^{\text{o}}_\Delta(\theta) = \frac{1}{r}\left(1-e^{-r\Delta}\right)(\nu(\theta)-\gamma) + e^{-r\Delta}\left[\left(1-\pi(\Delta|\theta)\right)C^{\text{o}}_\Delta(\underline{\theta}) + \pi(\Delta|\theta)C^{\text{o}}_\Delta(\bar{\theta})\right]$$
$$\hat{p}^{\text{o}}_\Delta(\theta) = \frac{1}{r}\left(1-e^{-r\Delta}\right)\nu(\theta) + e^{-r\Delta}\left[\left(1-\pi(\Delta|\theta)\right)C^{\text{o}}_\Delta(\underline{\theta}) + \pi(\Delta|\theta)C^{\text{o}}_\Delta(\bar{\theta})\right].$$

where

$$C^{\text{o}}_\Delta(\theta) = \frac{\hat{p}^{\text{o}}_\Delta(\theta)^2}{2\hat{p}^{\text{o}}_\Delta(\theta) - \hat{C}^{\text{o}}_\Delta(\theta)}.$$

Notice that

$$\hat{C}^{\text{o}}_\Delta(\theta) = \hat{p}^{\text{o}}_\Delta(\theta) - \frac{1}{r}\left(1-e^{-r\Delta}\right)\gamma,$$

so

$$C^{\text{o}}_\Delta(\theta) = \frac{\hat{p}^{\text{o}}_\Delta(\theta)^2}{\hat{p}^{\text{o}}_\Delta(\theta) + \frac{1}{r}\left(1-e^{-r\Delta}\right)\gamma}.$$

so we only need to consider the system for $\hat{p}_\Delta(\theta)$

$$\hat{p}^{\text{o}}_\Delta(\theta) = \frac{1}{r}\left(1-e^{-r\Delta}\right)\nu(\theta) + e^{-r\Delta}\left[\left(1-\pi(\Delta|\theta)\right)\frac{\hat{p}^{\text{o}}_\Delta(\underline{\theta})^2}{\hat{p}^{\text{o}}_\Delta(\underline{\theta}) + \frac{1}{r}\left(1-e^{-r\Delta}\right)\gamma} + \pi(\Delta|\theta)\frac{\hat{p}^{\text{o}}_\Delta(\bar{\theta})^2}{\hat{p}^{\text{o}}_\Delta(\bar{\theta}) + \frac{1}{r}\left(1-e^{-r\Delta}\right)\gamma}\right]$$

We can rewrite the previous equation

$$\frac{\hat{p}^{\text{o}}_\Delta(\underline{\theta})\left(\hat{p}^{\text{o}}_\Delta(\underline{\theta}) + \frac{1}{r}\gamma\right)}{\hat{p}^{\text{o}}_\Delta(\underline{\theta}) + \frac{1}{r}\left(1-e^{-r\Delta}\right)\gamma} = \frac{1}{r}\nu(\underline{\theta}) + \frac{e^{-r\Delta}\pi(\Delta|\underline{\theta})}{1-e^{-r\Delta}}\left[-\frac{\hat{p}^{\text{o}}_\Delta(\underline{\theta})^2}{\hat{p}^{\text{o}}_\Delta(\underline{\theta}) + \frac{1}{r}\left(1-e^{-r\Delta}\right)\gamma} + \frac{\hat{p}^{\text{o}}_\Delta(\bar{\theta})^2}{\hat{p}^{\text{o}}_\Delta(\bar{\theta}) + \frac{1}{r}\left(1-e^{-r\Delta}\right)\gamma}\right]$$
$$\frac{\hat{p}^{\text{o}}_\Delta(\bar{\theta})\left(\hat{p}^{\text{o}}_\Delta(\bar{\theta}) + \frac{1}{r}\gamma\right)}{\hat{p}^{\text{o}}_\Delta(\bar{\theta}) + \frac{1}{r}\left(1-e^{-r\Delta}\right)\gamma} = \frac{1}{r}\nu(\bar{\theta}) + \frac{e^{-r\Delta}\left(1-\pi(\Delta|\bar{\theta})\right)}{1-e^{-r\Delta}}\left[\frac{\hat{p}^{\text{o}}_\Delta(\underline{\theta})^2}{\hat{p}^{\text{o}}_\Delta(\underline{\theta}) + \frac{1}{r}\left(1-e^{-r\Delta}\right)\gamma} - \frac{\hat{p}^{\text{o}}_\Delta(\bar{\theta})^2}{\hat{p}^{\text{o}}_\Delta(\bar{\theta}) + \frac{1}{r}\left(1-e^{-r\Delta}\right)\gamma}\right]$$

Taking limit when $\Delta \to 0$, we get that if $\lim_{\Delta\to 0} \hat{p}_\Delta(\theta) = \hat{p}(\theta) > 0$, then it must solve the equation

$$\begin{aligned} r\hat{p}^{\text{o}}(\underline{\theta}) &= \phi\underline{\theta}^2 - \gamma + \lambda\bar{\pi}\left[\hat{p}^{\text{o}}(\bar{\theta}) - \hat{p}^{\text{o}}(\underline{\theta})\right] \\ r\hat{p}^{\text{o}}(\bar{\theta}) &= \phi\bar{\theta} - \gamma + \lambda(1-\bar{\pi})\left[\hat{p}^{\text{o}}(\underline{\theta}) - \hat{p}^{\text{o}}(\bar{\theta})\right], \end{aligned}$$

so it follows that if $C^{\text{nt}}(\theta) > 0$, then $\hat{p}^{\text{o}}(\theta) = C^{\text{o}}(\theta) = C^{\text{nt}}(\theta)$. If $C^{\text{o}}(\theta) < 0$, then we can multiply on both sides by $\hat{p}^{\text{o}}_\Delta(\theta) + \frac{1}{r}\left(1 - e^{-r\Delta}\right)\gamma$ before taking $\Delta \to 0$ to obtain that the limit is $\hat{p}(\theta) = 0$. Hence, if $C^{\text{o}}(\underline{\theta}) < 0 < C^{\text{o}}(\bar{\theta})$, we get that

$$\hat{p}^{\text{o}}(\bar{\theta}) = \frac{\phi\bar{\theta} - \gamma}{r + \lambda(1-\bar{\pi})},$$

and if $C^{\text{o}}(\bar{\theta}) < 0$ then $\hat{p}(\bar{\theta}) = 0$. Next, we look at the limit of the trading policy.

$$\beta^{\text{o}}_\Delta(\theta) = \frac{\hat{p}^{\text{o}}_\Delta(\theta) - \hat{C}^{\text{o}}_\Delta(\theta)}{2\hat{p}^{\text{o}}_\Delta(\theta) - \hat{C}^{\text{o}}_\Delta(\theta)} = \frac{\frac{1}{r}\left(1 - e^{-r\Delta}\right)\gamma}{\hat{p}^{\text{o}}_\Delta(\theta) + \frac{1}{r}\left(1 - e^{-r\Delta}\right)\gamma}.$$

If $\hat{p}^{\text{o}}(\theta) > 0$, we can take the limit

$$\beta^{\text{o}}(\theta) \equiv \lim_{\Delta\to 0} \frac{\beta^{\text{o}}_\Delta(\theta)}{\Delta} = \frac{\gamma}{\hat{p}^{\text{o}}(\theta)},$$

whereas if $\lim_{\Delta\to 0} \hat{p}^{\text{o}}_\Delta(\theta) = 0$ then we get that $\beta^{\text{o}}(\theta) = \infty$. □

Proof of Proposition 6

Proof. The fact that payoffs are the same in the smooth trading case follows directly from Propositions 3 and 5. In the case where condition (**??**) is satisfied, the equilibrium payoff of the high type is $V(x, \bar{\theta}) = (\mu/r)x + \frac{1}{2}C(\bar{\theta})x^2$. For $\gamma \in [\kappa_{**}, \kappa_\dagger)$ we have $C^0(\bar{\theta}) = 0$ so the ranking is immediate. For $\gamma \in [\kappa_*, \kappa_{**})$ we have $C(\bar{\theta}) \geq C^o(\bar{\theta})$ where

$$C^{\text{o}}(\bar{\theta}) = \frac{\phi\bar{\theta}^2 - \gamma}{r + \lambda(1-\bar{\pi})}$$

and $C(\bar{\theta}) = \Gamma$ where

$$\beta(\bar{\theta}) = \frac{\gamma - \phi\underline{\theta}^2 - \lambda\bar{\pi}\Gamma}{2\hat{p}(\bar{\theta})}$$
$$\hat{p}(\bar{\theta}) = \frac{\phi\bar{\theta}^2}{r + \lambda(1-\bar{\pi}) + \beta(\bar{\theta})}$$
$$\Gamma = \frac{\phi(\bar{\theta}^2 - \underline{\theta}^2)}{r + \lambda + 2\beta(\bar{\theta})},$$

Rewritting the equation for Γ, we get that $C(\bar{\theta})$ satisfies

$$\begin{aligned}(r + \lambda(1-\bar{\pi}))C(\bar{\theta}) &= \phi(\bar{\theta}^2 - \underline{\theta}^2) - (\lambda\bar{\pi} + 2\beta(\bar{\theta})\Gamma \\ &= \phi\bar{\theta}^2 - \gamma - 2\beta(\bar{\theta})\Gamma + \left(\gamma - \underline{\theta}^2 - \lambda\bar{\pi}\Gamma\right) \\ &= \phi\bar{\theta}^2 - \gamma + 2\beta(\bar{\theta})\left(\hat{p}(\bar{\theta}) - \Gamma\right) \\ &> \phi\bar{\theta}^2 - \gamma,\end{aligned}$$

where the third lines follows from the equation for $\beta(\bar{\theta})$ and the last inequality follows from $\hat{p}(\bar{\theta}) > \Gamma$. Hence, $C(\bar{\theta}) > C^o(\bar{\theta})$ □

Proof of Proposition 8

Proof. Letting $\Psi_\Delta(\alpha) \equiv C_\Delta(\bar{\theta}, \alpha) - C_\Delta(\underline{\theta}, \alpha)$ and $\Gamma_\Delta(\alpha) \equiv \hat{C}_\Delta(\bar{\theta}, \alpha) - \hat{C}_\Delta(\underline{\theta}, \alpha)$, we get

$$\Psi_\Delta(\alpha) = \frac{\left(\hat{C}_\Delta(\bar{\theta}, \alpha) - \hat{C}_\Delta(\underline{\theta}, \alpha)\right) C_\Delta(\bar{\theta}, \alpha)}{2\hat{p}_\Delta(\alpha) - \hat{C}_\Delta(\bar{\theta}, \alpha)} = \frac{C_\Delta(\bar{\theta}, \alpha)}{2\hat{p}_\Delta(\alpha) - \hat{C}_\Delta(\bar{\theta}, \alpha)}\Gamma_\Delta(\alpha),$$

and

$$\Gamma_\Delta(\alpha) = \frac{1}{r}\left(1 - e^{-r\Delta}\right)\left(\nu(\bar{\theta}) - \nu(\underline{\theta})\right) + e^{-(r+\lambda)\Delta}\Psi_\Delta(e^{-\lambda\Delta}\alpha).$$

The coefficients $\hat{C}_\Delta(\alpha), \hat{p}_\Delta(\alpha)$ satisfies the equation

$$\begin{aligned}\hat{C}_\Delta(\bar{\theta}, \alpha) &= \frac{1}{r}\left(1 - e^{-r\Delta}\right)\left(\nu(\bar{\theta}) - \gamma\right) + e^{-r\Delta}C_\Delta(\bar{\theta}, e^{-\lambda\Delta}\alpha) - e^{-r\Delta}(1 - e^{-\lambda\Delta})\Psi_\Delta(e^{-\lambda\Delta}\alpha) \\ \hat{p}_\Delta(\alpha) &= \frac{1}{r}\left(1 - e^{-r\Delta}\right)\nu(\alpha) + e^{-r\Delta}C_\Delta(\bar{\theta}, e^{-\lambda\Delta}\alpha).\end{aligned}$$

Thus, we have that

$$\hat{p}_\Delta(\alpha) - \hat{C}_\Delta(\bar{\theta}, \alpha) = \frac{1}{r}\left(1 - e^{-r\Delta}\right)\left(\gamma + \nu(\alpha) - \nu(\bar{\theta})\right) + e^{-r\Delta}(1 - e^{-\lambda\Delta})\Psi_\Delta(e^{-\lambda\Delta}\alpha)$$
$$\xrightarrow[\Delta\to 0]{} 0,$$

which means that $\lim_{\Delta\to 0} C_\Delta(\bar{\theta}, \alpha) = \lim_{\Delta\to 0} \hat{C}_\Delta(\bar{\theta}, \alpha) = \lim_{\Delta\to 0} \hat{p}_\Delta(\alpha)$. It follows from here that $\lim_{\Delta\to 0} \Psi_\Delta(\alpha) = \hat{C}(\bar{\theta}, \alpha) - \hat{C}(\underline{\theta}, \alpha)$, so

$$\lim_{\Delta\to 0} \frac{\hat{p}_\Delta(\alpha) - \hat{C}_\Delta(\bar{\theta}, \alpha)}{\Delta} = \gamma - (1-\alpha)\phi(\bar{\theta}^2 - \underline{\theta}^2) + \lambda[\hat{C}(\bar{\theta}, \alpha) - \hat{C}(\underline{\theta}, \alpha)].$$

The previous limit implies that the limit trading rate is given by

$$\lim_{\Delta\to 0} \frac{\beta_\Delta(\bar{\theta}, \alpha)}{\Delta} = \bar{\beta}(\alpha) \equiv \frac{\gamma - (1-\alpha)\phi(\bar{\theta}^2 - \underline{\theta}^2) + \lambda\Gamma(\alpha)}{\hat{p}(\alpha)}.$$

From the equation for $\Gamma_\Delta(\alpha)$ we get

$$\Gamma_\Delta(\alpha) - e^{-(r+\lambda)\Delta}\Gamma_\Delta(e^{-\lambda\Delta}\alpha) = \frac{1}{r}\left(1 - e^{-r\Delta}\right)\left(\nu(\bar{\theta}) - \nu(\underline{\theta})\right) - 2e^{-(r+\lambda)\Delta}\beta_\Delta(\bar{\theta}, \alpha)\Gamma_\Delta(e^{-\lambda\Delta}\alpha).$$

Dividing by Δ, and taking limit when $\Delta \to 0$, we get

$$(r + \lambda + 2\bar{\beta}(\alpha))\Gamma(\alpha) = \phi\left(\bar{\theta}^2 - \underline{\theta}^2\right) - \lambda\alpha\Gamma'(\alpha).$$

Similarly,

$$r\hat{C}(\bar{\theta}, \alpha) = \phi\bar{\theta}^2 - \gamma - \lambda\alpha C'(\bar{\theta}, \alpha) - \lambda\Gamma(\alpha),$$

moreover, as $\hat{p}(\alpha) = \hat{C}(\bar{\theta}, \alpha)$, we get

$$r\hat{p}(\alpha) = \phi\bar{\theta}^2 - \gamma - \lambda\alpha\hat{p}'(\alpha) - \lambda\Gamma(\alpha).$$

From here, we get the system of ODEs for the equilibrium

$$
\begin{aligned}
r\hat{p}(\alpha) &= \phi\bar{\theta}^2 - \gamma - \lambda\alpha\hat{p}'(\alpha) - \lambda\Gamma(\alpha) \\
(r + \lambda + 2\bar{\beta}(\alpha))\Gamma(\alpha) &= \phi\left(\bar{\theta}^2 - \underline{\theta}^2\right) - \lambda\alpha\Gamma'(\alpha) \\
\bar{\beta}(\alpha) &= \frac{\gamma - (1-\alpha)\phi(\bar{\theta}^2 - \underline{\theta}^2) + \lambda\Gamma(\alpha)}{\hat{p}(\alpha)}
\end{aligned}
$$

The low type always has the option to reveal himself and trade the full information optimal

$$
\tilde{x}_\Delta(\underline{\theta}, 0) = \frac{\hat{p}_\Delta(0)}{2\hat{p}_\Delta(0) - \hat{C}_\Delta(\underline{\theta}, 0)} x,
$$

which yields a deviation payoff

$$
\tilde{V}_\Delta(\underline{\theta}) = \frac{\mu}{r}x + \frac{1}{2}\tilde{C}_\Delta(\underline{\theta})x^2,
$$

where

$$
\tilde{C}_\Delta(\underline{\theta}) = \frac{\hat{p}_\Delta(0)^2}{2\hat{p}_\Delta(0) - \hat{C}_\Delta(\underline{\theta}, 0)}.
$$

In a pooling equilibrium, it must be the case that $C_\Delta(\underline{\theta}, \alpha) \geq \tilde{C}_\Delta(\underline{\theta})$. To simplify the analysis, we consider the case with permanent negative shocks $\bar{\pi} = 0$. Moreover, we assume that once the market belief is $\alpha = 0$, it remains there forever. Under this assumption, the continuation equilibrium given $\alpha = 0$ corresponds to the one for the low type with observable shocks. When we take $\Delta \to 0$, we get that $\tilde{C}_\Delta(\underline{\theta}) \to \max\{C^{\text{nt}}(\underline{\theta}), 0\}$, so the condition for deviations become

$$
C(\underline{\theta}, \alpha) = \hat{p}(\alpha) - \Gamma(\alpha) \geq \max\{C^{\text{nt}}(\underline{\theta}), 0\}.
$$

Under the assumption that the shock is persistent, the system of ODEs reduces to

$$
\begin{aligned}
r\hat{p}(\alpha) &= \phi\bar{\theta}^2 - \gamma - \lambda\Gamma(\alpha) - \lambda\alpha\hat{p}'(\alpha) \\
(r + \lambda + 2\bar{\beta}(\alpha))\Gamma(\alpha) &= \phi\left(\bar{\theta}^2 - \underline{\theta}^2\right) - \lambda\alpha\Gamma'(\alpha) \\
\bar{\beta}(\alpha) &= \frac{\gamma - (1-\alpha)\phi(\bar{\theta}^2 - \underline{\theta}^2) + \lambda\Gamma(\alpha)}{\hat{p}(\alpha)}
\end{aligned}
$$

□