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CEO Horizon, Optimal Pay Duration, and the Escalation of Short-Termism

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ABSTRACT

This paper studies optimal contracts when managers manipulate their performance measure at the expense of firm value. Optimal contracts defer compensation. The manager's incentives vest over time at an increasing rate, and compensation becomes very sensitive to short-term performance. This generates an endogenous horizon problem whereby managers intensify performance manipulation in their final years in office. Contracts are designed to encourage effort while minimizing the adverse effects of manipulation. We characterize the optimal mix of short- and long-term compensation along the manager's tenure, the optimal vesting period of incentive pay, and the dynamics of short-termism over the CEO's tenure.

SHORT-TERMISM IS PREVALENT AMONG MANAGERS. Graham, Harvey, and Rajgopal (2005) find that 78% of U.S. CEOs are willing to sacrifice long-term value to beat market expectations. For example, Dechow and Sloan (1991) argue that, by the end of their tenure, CEOs tend to cut R&D investment, which, though profitable, has negative implications for the firm's reported earnings. Managerial short-termism has been the suspect of concern for many years, but it has assumed a particularly prominent role in recent years following the Enron scandal and the financial crisis in 2008.

To understand this phenomenon, the theoretical literature has adopted two approaches. One approach studies CEO behavior, taking managerial incentives as given, and thus is silent about optimal incentives (see, e.g., Stein (1989)). However, the complexity of CEO contracts in the real world (which include accounting-based bonuses, stock options, restricted stock, deferred compensation, clawbacks, etc.) suggests that shareholders are aware of potential manipulation by CEOs and design compensation to mitigate the consequences

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of such manipulation. An alternative approach studies optimal compensation contracts that are designed to fully remove CEO manipulations. In this class of models, manipulation is not observed on the equilibrium path (Edmans et al. (2012)). This approach is particularly helpful in settings in which CEO manipulation is too costly to the firm or easy to rule out, but it cannot explain why manipulation seems so frequent in practice or why real-world contracts tolerate or even induce manipulation (see Bergstresser and Philippon (2006)).

In this paper, we study optimal compensation contracts when CEOs exert hidden effort but can also manipulate the firm's performance to increase their compensation, sometimes at the expense of firm value. Building on Holmstrom and Milgrom (1987), we consider a setting with a risk-averse CEO who can save privately and consume continuously, and who exerts two costly actions: effort and manipulation. Both actions increase the CEO's performance in the short run, but manipulation also has negative consequences for firm value. As in Stein (1989), we assume that these consequences are not perfectly/immediately captured by the performance measure but rather take time to be verified, potentially creating an externality when the CEO tenure is shorter than the firm's life span.

Our paper makes two contributions to the literature. First, on the normative side, we study the contract that maximizes firm value in the presence of manipulation. We characterize the optimal mix of long- and short-term incentives, the duration of CEO pay over CEO tenure, and the ideal design of clawbacks and postretirement compensation. Second, on the positive side, we make predictions about the evolution of CEO manipulations along CEO tenure, and we establish the existence of an endogenous CEO horizon problem.

We study the timing of manipulation: how it evolves over CEO tenure and whether optimal contracts generate a horizon effect whereby the CEO distorts performance at the end of his tenure. Previous literature shows that in dynamic settings one can implement positive effort and zero manipulation at the same time (unlike in static settings) by appropriately balancing the mix of short- and long-run incentives. However, in our setting, inducing zero manipulation is not optimal; rather, tolerating some manipulation is desirable because doing so allows the firm to elicit higher levels of effort than a manipulation-free contract. Furthermore, to fully discourage manipulation, the firm would have to provide the CEO with a large postretirement compensation package that ties his wealth to the firm's postretirement performance. Such postretirement compensation is costly to the firm, as it imposes risk on the CEO during a period when effort does not need to be incentivized and the CEO must be compensated for bearing this extra risk (see, e.g., Dehaan, Hodge, and Shevlin (2013)).

In our model, performance pay at some date t has the benefits of providing incentives at t and of deterring manipulation prior to t. On the other hand, it has the cost of encouraging manipulation at time t. This trade-off shapes the contract design and the evolution of performance pay along the CEO's tenure.

In the absence of manipulation, short-term incentives—measured as the contract's pay-performance sensitivity (PPS)—are constant over time, as in

Holmstrom and Milgrom (1987). Unfortunately, the simplicity of this contract vanishes under the possibility of manipulation. A constant PPS contract is no longer optimal because it induces excessive manipulation, particularly in the final years in office. Indeed, offering the CEO a stationary contract would lead him to aggressively shift performance across periods, boosting current performance at the expense of firm value. To mitigate this behavior, an optimal contract implements lower levels of short-term compensation and higher levels of long-term compensation, measured roughly as the present value of the contract's future *slopes*.

Also, in the absence of manipulation, CEO incentives vest deterministically, whereas under the possibility of manipulation, vesting depends on firm performance. This is empirically relevant. Bettis et al. (2010) assert that, even though restricted stock awards with time-vesting provisions account for the majority of performance-based pay in U.S. companies, shareholder advocacy groups and proxy research services have expressed concern that these provisions do not provide sufficiently strong incentives and have suggested that compensation contracts include performance-based vesting conditions. In fact, since the mid-1990s, U.S. firms have increasingly issued option and stock awards with sophisticated performance-based vesting conditions. Our paper provides a rationale for this phenomenon. Under the possibility of manipulation, the optimal contract defers compensation and includes performance-based vesting provisions. In the absence of manipulation, the vesting date of incentives is known at the start of the CEO's tenure and is independent of the firm's performance. When the CEO can manipulate performance, the optimal contract includes performance-based vesting. Thus, the duration of incentives is random: vesting accelerates with positive shocks and is delayed with negative shocks. Random vesting is helpful in the presence of manipulation because it allows the principal to change the level of long-term incentives without having to simultaneously distort short-term incentives to avoid creating an imbalance, which would trigger extra manipulation. Hence, performance-based vesting provides the principal with an additional degree of freedom to reduce the CEO's long-term incentives without having to distort effort to contain manipulation.

The optimal contract also includes a postretirement package that ties the manager's wealth to the performance of the firm, observed for some time after his retirement. This contracting tool is helpful but has limited power when the CEO is risk-averse: even when the firm has the ability to tie the manager's wealth forever—and to any degree—to the firm's postretirement performance, the contract generally induces some manipulation. Although it would be possible to defer compensation long enough to deter manipulation altogether, firms might not do so given the cost. A key insight of this paper is that firms find it more beneficial to defer compensation, while the CEO is still on the job rather than after he retires. This result implies that long-term incentives are larger at the beginning of the CEO's tenure and decay toward the end.

Under the possibility of manipulation, optimal CEO contracts are nonlinear, unlike in Holmstrom and Milgrom (1987). Following Edmans et al. (2012), we first characterize the optimal contract within the subclass of contracts that implement deterministic sequences of effort and manipulation. Under such deterministic contracts, long-term incentives and effort are intertwined. Long-term incentives can be reduced only by increasing the current slope of short-term compensation, which necessarily distorts the level of effort. This is why, in general, the optimal contract implements incentives that are history dependent. The benefit of providing incentives that are history dependent and lead to stochastic effort and manipulation resides precisely in allowing the principal to control the evolution of long-term incentives independent of the CEO's effort. We find that at the beginning of CEO's tenure, the performance sensitivity of long-term incentives is close to zero. However, toward the end of their tenure, such sensitivity becomes negative; positive shocks accelerate vesting and thereby reduce long-term incentives. Generally, we find that long-term incentives are mean-reverting and follow a target level. If, due to their stochastic nature, long-term incentives increase relative to their target, the sensitivity of long-term incentives with respect to shocks becomes negative, in order to drive the long-term incentives back down.

A. Related Literature

Beginning with Narayanan (1985), Dye (1988), and Stein (1989), a large number of studies in accounting, economics, and finance have examined the causes and consequences of performance manipulation in corporate settings. Most of the literature studying managerial short-termism has taken incentives as exogenously given (Stein (1989), Fischer and Verrecchia (2000), Guttman, Kadan, and Kandel (2006), Kartik, Ottaviani, and Squintani (2007)) or has restricted attention to static or two-period settings with linear contracts, which are unsuited for studying the dynamics of short-termism and its relation to optimal long-term incentives (Baker (1992), Goldman and Slezak (2006), Dutta and Fan (2014), Peng and Röell (2014)). A related strand of the literature examines optimal compensation contracts in the presence of moral hazard and adverse selection (Maggi and Rodriguez-Clare (1995), Crocker and Slemrod (2007), Beyer, Guttman, and Marinovic (2014)).

A more recent stream of literature studies dynamic contracts under the possibility of manipulation (Edmans et al. (2012), Varas (2017), Zhu (2018)). These studies restrict attention to contracts that prevent manipulation altogether. Because we are interested in making predictions about the evolution of short-termism, we consider more general contracts that implement optimal levels of manipulation. Sabac (2008) studies CEO horizon effects in a multiperiod model with renegotiation where effort has long-term consequences. He finds that effort can decrease, while incentive rates increase as managers approach retirement. DeMarzo, Livdan, and Tchistyi (2013) study a dynamic agency model in which the CEO can take on "tail risk," thereby gambling with the firm's money. The optimal contract must strike a balance between providing incentives to exert effort and controlling the manager's risk-taking behavior. The authors show that when the manager's continuation value reaches low levels due to poor performance, the manager engages in excessive risk-taking.

On the technical side, we borrow heavily from Holmstrom and Milgrom (1987), Williams (2011), He, Wei, and Yu (2014), and Sannikov (2014). The problem of managerial short-termism is closely related to the long-run moral hazard problem in Sannikov (2014), and we model the intertemporal effect of the CEO's action in a similar way as Sannikov (2014). In Sannikov (2014), the future CEO's productivity is determined by today's effort, so long-term incentives are required to incentivize effort today. He, Wei, and Yu (2014) study the design of long-term contracts when the manager's ability is unknown and learned over time and show that a combination of private saving and CARA utility provides great tractability to analyze dynamic contracting problems with persistent private information.

The paper is organized as follows. Section I presents the model. Section II studies incentive compatibility. Section III presents the principal's optimization problem. Section IV considers the case of infinite CEO tenure. Section V analyzes the general case finite CEO tenure. Section VI characterizes the optimal contract. Section VII discusses the special case of deterministic incentives. Section VIII studies, numerically, some comparative statics. Section IX concludes. The proofs of the main results are presented in the Appendix.

I. Model

We study a dynamic agency problem in which the agent (hereafter, CEO) can manipulate the firm's performance to boost his own compensation. The CEO exerts two costly actions, effort a_t and manipulation m_t. The principal observes neither action but a noisy measure of firm performance.

Let $\{B_t\}_{t\geq 0}$ be a standard Brownian motion in a probability space with measure $\mathbf{P}$, and let $\{\mathcal{F}_t\}_{t\geq 0}$ be the filtration generated by B. For any $\mathcal{F}_t$-adapted effort, a_t, and manipulation, m_t, processes, the firm's cash flow process is given by

$$dX_t = (a_t + m_t - \theta M_t)dt + \sigma dB_t$$
$$M_t = \int_0^t e^{-\kappa(t-s)} m_s ds,$$

where M_t is the stock of manipulation accumulated through time t. The stock of manipulation M_t reduces firm cash flows at each point in time, the marginal effect of manipulation on firm cash flows is $-\theta$. At the same time, the stock M_t depreciates at rate κ. That is, the consequences of manipulation are more persistent when κ is smaller. Also, as θ vanishes, manipulation ceases to have future cash-flow consequences as it is qualitatively equivalent to effort.

This representation of short-termism goes back to Stein (1989) and captures the idea that manipulation may increase current cash flows but eventually destroys firm value. The accounting literature refers to this behavior as real earnings management. Some examples of this behavior arise when managers cut advertising expenditures or R&D to boost reported earnings, offer excessive discounts to meet earnings expectations, or risk the firm's reputation by lowering product quality/safety. More generally, we can think of manipulation as any potentially unproductive actions aimed at boosting the firm's short-run profits. Notice that unlike effort, manipulation is inherently dynamic; it increases today's performance but decreases the firm's cash flows in future periods. In a nutshell, manipulation is a bad investment, or a mechanism the manager might use to *borrow* performance from the future to boost current performance.[1]

Following Holmstrom and Milgrom (1987), we assume that the CEO has exponential preferences given by

$$u(c, a, m) = -e^{-\gamma(c-h(a)-g(m))}/\gamma,$$

where $h(a) \equiv a^2/2$ and $g(m) \equiv gm^2/2$. By assuming that manipulation is costly to the manager, we follow the literature on costly state falsification (Dye (1988), Fischer and Verrecchia (2000), Lacker and Weinberg (1989), Crocker and Morgan (1998), Kartik (2009)). The cost of manipulation $g(m)$ captures the various personal costs the CEO bears from manipulating the firm's performance. These costs include the effort required to find ways of distorting cash-flows, litigation risk, fines imposed by the U.S. Securities and Exchange Commission (SEC), or the natural distaste associated with behaving unethically.[2] We allow the cost parameter g to be zero, which captures the case in which manipulation is costless.

The CEO is infinitely lived but works for the firm for a finite period of time, $t \in [0, T]$. We refer to T as the manager's retirement date, and to $T - t$ as his horizon at time t. We assume that the contract can stipulate compensation beyond retirement, until time $T + \tau$ for $\tau \geq 0$. In other words, the manager's compensation can be made contingent on outcomes observed after his retirement. This possibility captures the principal's ability to implement a clawback: a higher τ represents an environment where clawbacks can be enforced for a longer period of time.

In practice, T is random. For tractability reasons, we assume that the retirement date T is known. One can think of T as an approximation to some predictable separation date. For example, Cziraki and Xu (2014) find that CEO terminations are mostly concentrated around the end date of their contracts. The CEO's expected utility given a consumption flow $\{c_t\}_{t\geq 0}$ is

$$U(c, a, m) = E^{(a,m)}\left[\int_0^T e^{-rt} u(c_t, a_t, m_t)dt + \int_T^{T+\tau} e^{-rt} u^R(c_t)dt + e^{-r(T+\tau)} \frac{u^R(c_{T+\tau})}{r}\right],$$

[1] Prior literature (see Dutta and Fan (2014)) has modeled the reversal of manipulation as taking place in the second period of a two-period setting. Although not fundamental, one of the benefits of our specification is its flexibility to accommodate different reversal speeds. In our model, the effect of manipulation vanishes gradually based on κ. As we will see, this parameter is a key determinant of the manager's manipulation patterns.

[2] This last interpretation is consistent with introspection and extant experimental evidence (Gneezy (2005)).

where $u^R(c) \equiv u(c, 0, 0)$ is the flow utility accrued to the CEO after retirement. Notice again that the CEO tenure is finite but the principal effectively controls the CEO's compensation over his entire life, even after $T + \tau$. As will become clear, this assumption makes it possible to study the effect of changes in the contracting environment while holding the CEO's life expectancy constant. This allows us to remove dynamic effects on the structure of the CEO's compensation driven merely by the shortening of the period length available to pay the CEO as he grows old. Edmans et al. (2012) show that with a finite life, PPS rises over time as the number of periods to pay the agent his promised compensation is reduced. The same would be true in our setting if we assumed that the CEO's life ends at $T + \tau$.

Following He, Wei, and Yu (2014), we assume that the CEO can borrow and save privately—that is, saving is a hidden action—at the common interest rate r. This allows the CEO to smooth consumption intertemporally and hence restricts the ability of the principal to implement compensation schemes that lead to steep expected consumption patterns. Being able to privately save and borrow, the CEO can smooth consumption over time.

The principal designs the contract to maximize firm value. A contract is a consumption process $\{c_t\}_{t\geq 0}$ and an effort-manipulation pair $\{(a_t, m_t)\}_{t\in[0,T]}$ adapted to the filtration generated by the performance measure X_t. Formally, the principal chooses the contract to maximize the present value of the firm's discounted cash flow net of the CEO's compensation, as given by

$$V(c, a, m) = E^{(a,m)}\left[\int_0^{\infty} e^{-rt}(dX_t - c_t \mathbf{1}_{\{t\leq T+\tau\}}dt) - e^{-r(T+\tau)}\frac{c_{T+\tau}}{r}\right].$$

Firm value is thus equal to the discounted net stream of cash flows, which includes a terminal bonus granting the manager a consumption flow $c_{T+\tau}$ from time $T + \tau$ onward (the CEO's pension benefits).

We assume that the negative long-run effect of manipulation dominates the instant benefit, so manipulation destroys value. In other words, manipulation is a negative net present value project. Using integration by parts, we find that

$$E\left[\int_0^{\infty} e^{-rt}dX_t\right] = E\left[\int_0^{T} e^{-rt}(a_t - \lambda m_t)dt\right],$$

where $\lambda \equiv \frac{\theta}{r+\kappa} - 1$ captures the value-destroying effect of manipulation. When θ is large relative to $r + \kappa$, manipulation is detrimental to firm value—yet potentially attractive to the manager. In other words, manipulation destroys value when either the stock of manipulation M_t is highly persistent (i.e., $r + \kappa$ is small) or the marginal effect of M_t on the firm current cash flows is large (i.e., θ is large). Throughout the paper, we make the following parametric assumption.

CONDITION 1: $\theta \geq r + \kappa$.

If $\theta = r + \kappa$, that is, if $\lambda = 0$, then manipulation has no cash flow effect—it only shifts income across periods, being akin to accrual earnings management. In contrast, if $\theta > r + \kappa$, ($\lambda > 0$), then manipulation not only shifts cash flows across periods but also destroys firm value, being akin to real earnings management.

The principal's expected payoff given (c_t, a_t, m_t) is

$$V(c,a,m) = E\left[\int_0^T e^{-rt}(a_t - \lambda m_t)dt - \int_0^{T+\tau} e^{-rt} c_t dt - e^{-r(T+\tau)}\frac{c_{T+\tau}}{r}\right].$$

II. The CEO's Problem

To solve for the optimal contract, in this section, we characterize the CEO's behavior given an arbitrary contract. Specifically, we state the CEO's problem given an arbitrary contract, we state the necessary incentive compatibility constraints, and we provide an informal discussion of these conditions. After describing the necessary conditions for incentive compatibility in Proposition 1, we provide a formal analysis of the CEO's optimization problem and we derive these conditions using the stochastic maximum principle.

The CEO can save and borrow privately. We denote by S_t the balance in the CEO's savings account, so given an arbitrary contract prescribing actions (c_t, a_t, m_t), the CEO solves the following problem:

$$\begin{aligned} &\sup_{\hat{c},\hat{a},\hat{m}} U(\hat{c},\hat{a},\hat{m}) \\ &\quad \text{subject to} \\ &dX_t = (\hat{a}_t + \hat{m}_t - \theta \hat{M}_t)dt + \sigma dB_t \\ &dS_t = (rS_t - \hat{c}_t + c_t)dt, \;\; S_0 = 0. \end{aligned} \tag{1}$$

We rely on the first-order approach to characterize the necessary incentive compatibility constraints, that is, we characterize incentive compatibility by the first-order conditions of the CEO's problem. We verify the sufficiency of the first-order conditions in Section VI.

As is common in dynamic contracting problems, we use the CEO's continuation utility, W_t, as a state variable in the recursive formulation of the contract. The continuation utility can be written as:

$$W_t = E_t\left[\int_t^{T+\tau} e^{-r(s-t)} u(c_s, a_s, m_s) ds + e^{-r(T+\tau-t)}\frac{u^R(c_{T+\tau})}{r}\right]. \tag{2}$$

The contract stipulates the sensitivity of W_t to cash flow shocks, dW_t/dX_t, as captured by the slope of incentives, which we denote by $-W_t\beta_t$. We refer to β_t as the contract's PPS.

Often in models of dynamic contracting, the continuation value is a sufficient statistic. In our setting, however, manipulation has a persistent effect on firm performance. Thus, the CEO's incentive to manipulate is tied to the manager's expectation of the contract's future PPS, β_t. Because of the persistent effect of manipulation, we need to include an additional state variable that proxies for the effect of manipulation on future outcomes. This intertemporal effect is captured by the state variable p_t as given by

$$p_t = E_t\left[\int_t^{T+\tau} e^{-(r+\kappa)(s-t)}\frac{dW_s}{dX_s}ds\right]. \tag{3}$$

When choosing manipulation, the CEO faces a trade-off between boosting current compensation and reducing future compensation. The latter effect is discounted by the interest rate r and the "depreciation rate" of the manipulation stock, κ.

The presence of long-term incentives is crucial to attenuate manipulation. When the CEO inflates dX_t by an extra dollar, he expects compensation to go down in the future. As the manipulation reverses, the larger the CEO's long-term incentives p_t become. The speed κ and intensity θ of the manipulation reversal determine how much effort the CEO will exert before resorting to manipulation.

The contract also specifies how long-term incentives p_t react to shocks, that is, the performance sensitivity of long-term incentives, which we denote by σ_{pt}.

We can now state the necessary incentive compatibility constraints. In Section VI, we turn to the issue of sufficiency.

PROPOSITION 1: *A necessary condition for the contract (c_t, a_t, m_t) to be incentive compatible is that for any $t \in [0, T]$,*

$$r\gamma h'(a_t) = \beta_t \tag{4a}$$

$$g'(m_t) = \phi\frac{p_t}{W_t} + \frac{\beta_t}{r\gamma} \quad \textit{if } m_t > 0 \tag{4b}$$

$$g'(m_t) \geq \phi\frac{p_t}{W_t} + \frac{\beta_t}{r\gamma} \quad \textit{if } m_t = 0, \tag{4c}$$

where $\phi \equiv \frac{\theta}{r\gamma}$ and $(W_t, p_t, \beta_t, \sigma_{pt})_{t\geq 0}$ solve the backward SDE

$$dW_t = -\sigma\beta_t W_t dB_t,\ \ W_{T+\tau} = \frac{u^R(c_{T+\tau})}{r} \tag{5a}$$

$$dp_t = \big[(r+\kappa)p_t + \beta_t W_t\big]dt + \sigma\sigma_{pt}W_t dB_t,\ \ p_{T+\tau} = 0. \tag{5b}$$

Finally, the private savings condition requires that for any $t \in [0, T+\tau]$, consumption satisfies

$$rW_t = u(c_t, a_t, m_t). \tag{6}$$

The previous result states necessary incentive compatibility constraints using the first-order approach. It remains to prove that the first-order approach is valid, so the above conditions are sufficient. We address this issue in Section VI.

Equation (4a) states that the CEO's marginal cost of effort must equal the sensitivity of his continuation utility to performance β_t, which captures the CEO's marginal benefit of effort. This condition is analogous to the incentive compatibility constraint in a static setting with linear contracts, which states that the marginal cost of effort is equal to the slope of incentives. Equations (4b) and (4c) are analogous incentive constraints for manipulation: the marginal cost of manipulation equals the marginal benefit of manipulation. The latter has two components: (1) a positive component capturing the extra compensation the manager gets by manipulating the performance measure today β_t, and (2) a negative component capturing the decrease in future compensation arising from the reversal of future cash flows triggered by manipulations, which is proportional to p_t in equation (3). This represents the negative effect in present value that today's manipulation has on future payoffs. Equations (5a) and (5b) provide the evolution of the state variables (W_t, p_t), which is derived in Appendix A. Equation (6) is a Euler equation arising from the private savings assumption and the absence of wealth effects under CARA preferences. The Euler equation for consumption implies that the marginal utility of consumption is a martingale, while CARA utility implies that the marginal utility is proportional to current utility. Because current utility u_t equals rW_t, it follows that continuation value W_t is a martingale (its drift is given by $rW_t - u_t = 0$). As He, Wei, and Yu (2014) point out, for general utility functions, allowing for private savings introduces an additional state variable, but this is not the case in a CARA setting, where the marginal value of savings is proportional to the level of current utility. Introducing private savings greatly simplifies our problem and the characterization of the optimal contract. When the agent cannot smooth consumption, the principal chooses the agent's consumption to control the ratio between the agent's current utility and his continuation value to provide incentives.

III. Principal's Problem

Having characterized the CEO incentive compatibility conditions, we next study the principal's optimization problem. We need to solve a two-dimensional stochastic control problem because manipulation generates a persistent state variable, namely, the stock of manipulation M_t. This means that we must keep track of the contract's long-term incentives p in addition to the CEO's continuation utility W. Fortunately, the absence of wealth effects in the manager's CARA preferences, along with the possibility of private savings, allows us to work with a single state variable, as we demonstrate below.

The principal's original optimization problem can be written as follows:

$$V(W_0, p_0) = \sup_{c,a,m,\beta,\sigma_p} V(c, a, m)$$

subject to

$$\begin{aligned}
dW_t &= -\sigma\beta_t W_t dB_t,\ \ W_{T+\tau} = \frac{u^R(c_{T+\tau})}{r} \\
dp_t &= \big[(r+\kappa)p_t + \beta_t W_t\big]dt + \sigma\sigma_{pt} W_t dB_t,\ \ p_{T+\tau} = 0 \\
rW_t &= u(c_t, a_t, m_t)
\end{aligned}$$

Incentive compatibility constraints (4a) to (4c).

In principle, we need to keep track of two state variables, W and p. However, using the properties of CARA preferences under private savings, we can rewrite the principal's optimization problem as a function of the single state variable $z_t \equiv -p_t/W_t$. This variable represents the contract's long-term incentives p scaled by the agent's continuation utility W. We can think of z as measuring the importance of deferred compensation (e.g., the present value of future equity grants scaled by the total continuation value of the CEO). Hereafter, we refer to z_t as long-term incentives.

Before reformulating the principal's problem, notice that the private savings condition immediately pins down the manager's consumption process as given by

$$c_t = h(a_t) + g(m_t) - \frac{\log(-r\gamma W_t)}{\gamma}. \tag{7}$$

Given private savings, consumption depends on the CEO's continuation value, effort, and manipulation, so we do not need to consider it as a separate control variable. This further simplifies the formulation of the problem.

LEMMA 1: *The principal value function can be written as*

$$V(W, p) = constant + \frac{\log(-r\gamma W)}{r\gamma} + F\Big(-\frac{p}{W}\Big),$$

where, defining $z \equiv -p/W$, $F(z)$ solves the maximization problem

$$F(z) = \sup_{a_t, m_t, \beta_t, \sigma_{zt}} E\left[\int_0^T e^{-rt}\big(a_t - \lambda m_t - h(a_t) - g(m_t)\big)dt - \int_0^{T+\tau} e^{-rt}\frac{\sigma^2\beta_t^2}{2r\gamma}dt\right] \tag{8}$$

subject to the incentive compatibility constraints (4a) and (4b), and the law of motion for z_t,

$$dz_t = [(r+\kappa)z_t + \beta_t(\sigma\sigma_{zt} - 1)]dt + \sigma_{zt}dB_t,\ \ z_{T+\tau} = 0, \tag{9}$$

where $\sigma_{zt} \equiv \sigma(\beta_t z_t - \sigma_{pt})$.

The optimal contract boils down to the following problem. The principal chooses both short-term incentives β_t and the sensitivity of long-term incentives σ_{zt} to maximize firm value. We interpret changes in long-term incentives dz_t as capturing vesting of the CEO's incentives. PPS can be zero today, but if the

CEO has equity grants that vest in the future, then z_t will be positive. When z_t goes down, the duration of the CEO's incentives decreases, as if the incentives vested earlier. In contrast, an increase in z_t amounts to delaying vesting and thus is equivalent to increasing the duration of the CEO incentives.

The principal also chooses $\sigma_{zt,}$ which captures the sensitivity of incentives z_t to performance shocks, dB_t. The principal can implement history-dependent incentives by setting $\sigma_{zt} \neq 0$, effectively implementing performance vesting. As is apparent from equation (9), σ_{zt} increases the drift of incentives z_t. So, by choosing σ_{zt}, the principal indirectly controls the rate of vesting. By contrast, when $\sigma_{zt} = 0$, incentives evolve in a deterministic fashion over time.

We make the following technical assumptions. First, we restrict attention to contracts with bounded incentive slopes. That is, for some arbitrarily large constant $\bar{\beta}$, we consider contracts with sensitivity β_t bounded by $\bar{\beta}$ for all time t. This assumption is similar to the restriction in He, Wei, and Yu (2014),[3] and it implies that z_t remains bounded for all feasible contracts. Notice that we can always choose a value of $\bar{\beta}$ large enough that the constraint is binding with very small probability. The upper bound $\bar{\beta}$ also implies an upper bound $\bar{a} \equiv (r\gamma)^{-1}\bar{\beta}$ on the implemented effort. We further assume that the agent can freely dispose of the output. The assumption of free disposal is standard in the static contracting literature (see, e.g., see Innes (1990)), and more recently has been used in a dynamic setting by Zhu (2018). Free disposal imposes a non-negativity constraint on β_t. This means that we restrict attention to contracts that have a sensitivity $\beta_t \in [0, \bar{\beta}]$ before retirement.

IV. Infinite Tenure and Irrelevance of Manipulation

What is the role of the retirement date T on the structure of CEO pay? As a benchmark, here we study in which $T = \infty$. We start with a variation of the moral hazard without manipulation problem studied by Holmstrom and Milgrom (1987) but allow for both intermediate consumption and private savings. This problem serves as a benchmark for evaluating the effect of short-termism on the optimal contract.

In the absence of manipulation (i.e., when the manager cannot manipulate the performance measure), deferring compensation beyond T plays no role, in which case hence, $z_T = 0$ and the principal's problem boils down to

$$\max_{a_t} \; E\left[\int_0^T e^{-rt}\left(a_t - h(a_t) - \frac{\sigma^2 r\gamma h'(a_t)^2}{2}\right)dt\right].$$

The principal can optimize the agent's effort point-wise, which yields

$$a^{HM} \equiv \frac{1}{1 + r\gamma\sigma^2},$$

[3] As in He, Wei, and Yu (2014), this assumption serves the role of a transversality condition.

where a^{HM} is the optimal effort arising in the absence of manipulation concerns, as shown by Holmstrom and Milgrom (1987) (this is also the level of effort arising in the case without learning studied by He, Wei, and Yu (2014)). We have thus confirmed that under CARA preferences and Brownian shocks, the optimal contract is linear and implements constant effort over time.

In practice, the benefits of nonlinear contracts are controversial. The executive compensation literature often argues that firms should get rid of nonlinearities in compensation schemes to prevent performance manipulation. For example, Jensen (2001, 2003) argues that nonlinearities induce managers to manipulate compensation over time, distracting them from the optimal long-term policies. Consistent with this view, we show that even when manipulation is possible, the linear contract above remains optimal as long as the CEO works forever. More precisely, we show that when $T = \infty$, the optimal contract is linear and induces no manipulation. In fact, a perpetual linear contract aligns the incentives of the principal and the CEO, eliminating his incentive to manipulate performance over time. Below, we show that a no-manipulation contract is no longer optimal when $T < \infty$, even though it might still be feasible.

The no-manipulation constraint can be written as follows:

$$\beta_t - \theta E_t\left[\int_t^\infty e^{-(r+\kappa)(s-t)} \frac{\beta_s W_s}{W_t} ds\right] \leq 0.$$

If PPS, β_t, is constant, then the no-manipulation constraint can be reduced to

$$\beta\left(1 - \theta \int_t^\infty e^{-(r+\kappa)(s-t)} ds\right) \leq 0$$

because W_t is a martingale. This is always satisfied under the condition that manipulation has negative net present value, as stated in Assumption 1 (i.e., $\theta \geq r + \kappa$). Hence, we have verified that, under infinite tenure, the optimal contract in the relaxed problem that ignores the possibility of manipulation continues to be feasible when the manager is allowed to manipulate. When the manager's tenure is infinite, the possibility of manipulation is irrelevant and the optimal contract implements no manipulation, as it is identical to that in Holmstrom and Milgrom (1987). We summarize the previous discussion in the following proposition.

PROPOSITION 2: *When $\theta \geq r + \kappa$ and $T = \infty$, the optimal contract entails no manipulation. The optimal contract is linear and stationary implementing $a_t = a^{HM}$.*

V. Finite Tenure

In this section, we solve the principal's problem for finite T using backward induction. First, we characterize the optimal contract after retirement, in $t \in (T, T + \tau]$. We then solve for the contract before retirement, in $t \in [0, T]$, taking as given the optimal postretirement compensation.

A. Postretirement Compensation

An important aspect of a CEO's contract is postretirement compensation and the extent to which it is tied to the firm's performance. By linking the CEO's wealth to the firm's postretirement performance, the firm can mitigate manipulation by the CEO in his final years in office. In this section, we fix the contract's promised postretirement incentives, z_T, and study how the contract optimally allocates incentives after retirement over time.

We focus on contracts in which z_t is deterministic after retirement, that is, on $(T, T+\tau]$. This means that we set $\sigma_{zt} = 0$ over the interval $(T, T+\tau]$, so vesting is indeed deterministic once the CEO is out of office. We make this assumption for tractability, as accommodating the terminal condition $z_{T+\tau} = 0$ is extremely difficult in the case of stochastic contracts.[4] Moreover, the restriction to contracts with deterministic vesting makes it possible to solve for the value function at time T in closed form, which simplifies the analysis of the Hamilton-Jacobi-Bellman (HJB) equation (11a). However, the restriction to deterministic contracts is not fundamental for the qualitative aspects of the contract during the employment period $[0, T]$, which is the main focus of our paper. The main qualitative results follow from the fact that the cost of providing long-term incentives z_T is convex in z_T, which should hold for stochastic contracts. In fact, the structure of the stochastic problem for postretirement compensation in the case $\tau = \infty$ is similar to that in He, Wei, and Yu (2014), who show in their setting that the cost of providing long-term incentives is convex.

For a fixed promise z_T, the principal must spread payments to the manager over the period $(T, T+\tau]$ to minimize the overall cost of providing incentives. As previously mentioned, we focus on contracts that implement deterministic vesting after retirement. We can therefore formulate the problem as the following cost minimization problem:

$$\min_{\beta} \int_T^{T+\tau} e^{-r(t-T)} \frac{\sigma^2 \beta_t^2}{2r\gamma} dt$$

subject to

$$z_T = \int_T^{T+\tau} e^{-(r+\kappa)(t-T)} \beta_t dt.$$

This formulation shows that providing postretirement incentives is particularly costly when cash flows are noisy or the manager is more risk-averse. Risk aversion explains why the flow cost to the firm is convex (quadratic) in β_t. The principal prefers to smooth the stream of incentives rather than cluster them in a short time period.

[4] We are not aware of a treatment of this terminal value problem in the stochastic control literature.

The Lagrangian of this minimization problem is

$$\mathcal{L} = \int_T^{T+\tau} e^{-r(t-T)} \frac{\sigma^2 \beta_t^2}{2r\gamma} dt + \ell \left(z_T - \int_T^{T+\tau} e^{-(r+\kappa)(t-T)} \beta_t dt \right),$$

where ℓ is the Langrange multiplier. We can minimize the objective with respect to β_t pointwise, which yields

$$\beta_t = \ell \frac{2r\gamma}{\sigma^2} e^{-\kappa(t-T)}.$$

We find the value of the multiplier ℓ by replacing β_t in the constraint. The multiplier is proportional to the level of deferred compensation at time T, represented by z_T. Thus, the level of the postretirement PPS is proportional to z_T and decreases exponentially over time at speed κ, which is the depreciation rate of the stock of manipulation.

The cost of postretirement incentives is given by $\frac{1}{2}\mathcal{C} z_T^2$, where

$$\mathcal{C} \equiv \frac{\sigma^2(r + 2\kappa)}{r\gamma\left(1 - e^{-(r+2\kappa)\tau}\right)}. \tag{10}$$

Hence, the cost increases, in a convex fashion, in the magnitude of incentives z_T. This convexity implies that the principal dislikes volatility in z_T. On the surface, this suggests that implementing (random) history-dependent incentives is suboptimal because it may lead to a large postretirement compensation package z_T, but—as we will discuss later—having the possibility of implementing history-dependent incentives also has benefits.

Postretirement incentives (to deter manipulation) are more costly (on net) than preretirement incentives because they do not also incentivize effort. In general, postretirement incentives are more costly when the performance measure is more volatile (σ), the manager is more risk-averse (γ), the stock of manipulation depreciates faster (κ), and τ is smaller. Notice that the principal is always better off as τ increases. However, the cost of postretirement incentives does not go to zero as $\tau \to \infty$.

VI. Optimal Contract

Next, we analyze the compensation contract over the CEO's active life. We investigate how the mix of long- versus short-term incentives and the duration of incentives respond to shocks throughout the CEO's tenure.

Given the optimal postretirement scheme derived above, we can specialize the principal's problem in Lemma 1. Hereafter, it is convenient to eliminate manipulation as one of the controls by noting that manipulation is given by[5] $m_t = (a_t - \phi z_t)^+/g$ and defining the payoff function as

$$\pi(a, z) \equiv a - \frac{\lambda}{g}(a - \phi z)^+ - \frac{1}{2g}\left[(a - \phi z)^+\right]^2 - \frac{(1 + r\gamma\sigma^2)a^2}{2}.$$

[5] We use the usual notation $x^+ \equiv \max\{x, 0\}$.

We can rewrite the principal's problem in Lemma 1 as

$$F(z,0)=\sup_{z_0,(a_t,\sigma_{zt})_{t\in[0,T]}} E\left[\int_0^T e^{-rt}\pi(a_t,z_t)dt-e^{-rT}\frac{1}{2}\mathcal{C}z_T^2\right]$$

subject to

$$dz_t=[(r+\kappa)z_t+r\gamma a_t(\sigma\sigma_{zt}-1)]dt+\sigma_{zt}dB_t.$$

The HJB equation associated with the above problem is as follows:[6]

$$rF=\max_{a,\sigma_z}\ \pi(a,z)+F_t+[(r+\kappa)z+r\gamma a(\sigma\sigma_z-1)]F_z+\frac{1}{2}\sigma_z^2F_{zz}, \tag{11a}$$

$$F(z,T)=-\frac{1}{2}\mathcal{C}z^2. \tag{11b}$$

The terminal condition captures the cost of providing postretirement incentives z_t, as derived in Section A. Once we solve for the value function for arbitrary z_0, we initialize the contract at $z_0=\arg\max_z F(z,0)$.

Optimizing with respect to σ_z requires that the value function be concave. We can always ensure that the value function is weakly concave by introducing public randomization. However, we have not been able to prove that the solution is strictly concave. Hereafter, we assume that the value function is concave so that the following characterization applies to regions of the state space in which such randomization is not required. We have not found instances in our examples where the use of randomization is required. Moreover, $F(z,t)$ is strictly concave in z at time T (because it satisfies the terminal condition), so $F(z,t)$ is also strictly concave for t close to T.[7,8]

Let $\sigma_z(z_t,t)$ be the value of σ_{zt} evaluated at the path of z_t under the optimal contract. The contract entails history-dependent incentives, that is, $\sigma_z(z_t,t)\neq 0$. This is optimal because it helps the principal to control z_t before retirement. Exposing the CEO to risk after retirement ($t>T$) by setting $z_T>0$ may deter some manipulation, but it is costly as it is inefficient from a risk-sharing point of view, and does not stimulate effort. Hence, the principal would like to reduce long-term incentives just before the CEO's retirement, at time T. Now, if we

[6] We omit the specification of the value function at the upper bound on z_t as this is not needed for the following analysis. We provide the specification of the boundary condition at the upper bound in the Internet Appendix when we provide the numerical algorithm used for the solution of the HJB equation. The Internet Appendix is available in the online version of the article on the *Journal of Finance* website.

[7] The main challenge in establishing concavity on $[0,T]$, for an arbitrary time T, comes from the interaction term $a_t\sigma_{zt}$ in the drift of z_t.

[8] The HJB equation presents the difficulty that the diffusion coefficient σ_{zt} is not bounded away from zero, which means that the HJB equation is a degenerate parabolic PDE. Because a classical solution may fail to exist, we resort to the theory of viscosity solutions for the analysis. The fact that the value function is a viscosity solution of the HJB equations follows from the principle of dynamic programming. The uniqueness of the solution follows from the comparison principle in Fleming and Soner (2006).

look at the SDE for z_t in (9), we see that the drift of z_t depends both on the level of effort a_t and the sensitivity of incentives, $\sigma_z(z_t, t)$. The principal can reduce z_t by either increasing effort or by implementing a negative sensitivity, $\sigma_z(z_t, t)$. The latter amounts to implementing a negative correlation between incentives p_t and the CEO continuation value W_t.

By setting $\sigma_z(z_t, t) \neq 0$, the principal effectively implements performance vesting. The optimality of (random) performance vesting arises in this model because, by changing $\sigma_z(z_t, t)$, the principal can control the evolution (i.e., drift) of incentives z_t. This is useful because it allows the principal to accelerate vesting toward retirement and reduce the level of incentives z_t without having to increase the short-term incentives β, which would trigger more manipulation. In other words, by controlling the sensitivity of incentives $\sigma_z(z_t, t)$, the principal can partially decouple incentives for effort provision (which are driven by β_t) from manipulation incentives.

As previously mentioned, implementing history-dependent incentives is costly. A high sensitivity $\sigma_z(z_t, t)$ leads to volatile incentives z_t, and as discussed in Section A, the cost of giving the CEO postretirement incentives increases, in a convex manner, in the size of those incentives, z_T, given the agent's risk aversion. This explains why the absolute magnitude of $\sigma_z(z_t, t)$ is relatively small, as seen in Figure 1.

Given the value function F, we find the optimal effort, manipulation, and sensitivity by solving the optimization problem in the HJB equation. The optimal policy is then given by

$$a(z, t) = \min\{\tilde{a}(z, t), \bar{a}\}^+ \tag{12a}$$

$$\tilde{a}(z, t) = \begin{cases} \frac{g-\lambda+\phi z - r\gamma g F_z}{1+gH(F_z, F_{zz})} & \text{if } 1 - r\gamma F_z \geq \phi z H(F_z, F_{zz}) + \frac{\lambda}{g} \\ \frac{1-r\gamma F_z}{H(F_z, F_{zz})} & \text{if } 1 - r\gamma F_z < \phi z H(F_z, F_{zz}) \\ \phi z & \text{otherwise} \end{cases}$$

$$m(z, t) = \frac{1}{g}(a(z, t) - \phi z)^+ \tag{12b}$$

$$\sigma_z(z, t) = -r\gamma\sigma a(z, t)\frac{F_z}{F_{zz}}, \tag{12c}$$

where $H(F_z, F_{zz}) \equiv 1 + r\gamma\sigma^2 + r^2\gamma^2\sigma^2(F_{zz})^{-1}F_z^2$. The second-order condition requires that $1 + gH(F_z, F_{zz}) \geq 0$. The volatility $\sigma_z(z, t)$ remains determined by (12c) if this condition is not satisfied. However, the optimal effort now is either ϕz or $\bar{a}$. In the particular case in which $g > \lambda$, effort is given by $\min(\phi z, \bar{a})$.

Due to its nonlinearity, it is difficult to obtain analytical results by analyzing the HJB equation (11a) directly. However, we can derive some insights about the optimal contract indirectly by analyzing the sample paths of the dual variable $\psi_t = F_z(z_t, t)$, which captures the principal's marginal value of providing long-term incentives to the CEO. The approach of analyzing the

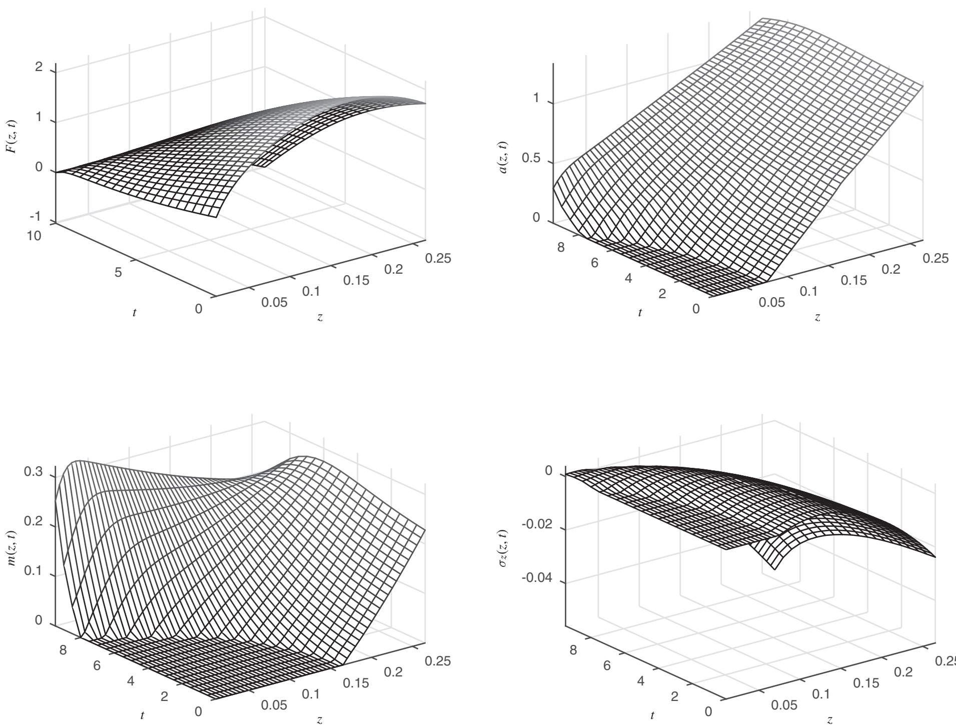


Figure 1. Solution to principal's optimization problem. Parameters: $r = 0.1$, $\gamma = 1$, $g = 1$, $\theta = 0.4$, $\kappa = 0.3$, $\sigma = 2$, $T = 10$, and $\tau = 5$. This plot shows the solution of the optimal contract in the t, z space. The bottom left panel reveals that manipulation is zero when both t and z are low. As t approaches retirement date T, manipulation escalates. In general, incentives are stochastic, as reflected by $\sigma_{zt} \neq 0$. However, the sign of σ_{zt} depends on both t and z_t. Early in the CEO's career, the contract is virtually deterministic, but as the manager approaches retirement, the contract implements performance contingent vesting, which leads to vesting being positively correlated with performance.

optimal contract by studying the sample paths of ψ_t follows the approach used by Farhi and Werning (2013) and Sannikov (2014), and it is similar to the analysis of deterministic contracts using optimal control developed in Section VII (and the analogous stochastic maximum principle). In fact, ψ_t is the stochastic analogue to the traditional costate variable in optimal control problems.

Because the payoff function $\pi(a, z)$ fails to be differentiable at $a = \phi z$ when $\lambda > 0$, we assume $\lambda = 0$ throughout the rest of this section. This assumption is made purely for technical reasons and is not required when we solve the model numerically. Using Ito's lemma and the Envelope theorem, we derive the following representation for the dual variable ψ_t, which is key in the characterization of the optimal contract. We find that the pair (ψ_t, z_t) solves the forward–backward SDE

$$\begin{aligned} d\psi_t &= -(\kappa\psi_t + \phi m_t)dt - r\gamma\sigma a_t\psi_t dB_t, \;\; \psi_0 = 0, \\ dz_t &= [(r+\kappa)z_t + r\gamma a_t(\sigma\sigma_z(z_t,t) - 1)]dt + \sigma_z(z_t,t)dB_t, \;\; z_T = -\mathcal{C}^{-1}\psi_T. \end{aligned}$$

This means that, for any $t \in [0, T]$, the value of ψ_t is given by

$$\begin{aligned} \psi_t &= -\phi \int_0^t e^{-\kappa(t-s)}\mathcal{E}_{s,t} m_s ds \\ \mathcal{E}_{s,t} &\equiv \exp\left\{-\int_s^t r\gamma\sigma a_u dB_u - \frac{1}{2}\int_s^t r^2\gamma^2\sigma^2 a_u^2 du\right\}, \end{aligned} \tag{13}$$

and hence $\psi_t \leq 0$ for all $t \in [0, T]$. Notice that $\psi_t = 0$ if and only if $m_s = 0$ for all $s < t$. This implies that $\sigma_z(z_t, t)$ is zero (so incentives are deterministic) if there has been no manipulation before time t.[9] Equation (13) makes it possible to derive the qualitative properties of the optimal contract and implies that the marginal value of incentives has an upper boundary at zero, and this implies in turn that if the value function is concave, then there is a lower bound $\bar{z}(t)$ for the long-term incentives, that is, $z_t \geq \bar{z}(t)$ where $F_z(\bar{z}(t), t) = 0$. We provide a qualitative characterization of the lower boundary $\bar{z}(t)$ in Proposition 3 and show that this boundary decreases over time. This is consistent with the notion that the principal wishes to reduce long-term incentives over time to avoid leaving the manager with a large postretirement package.

Furthermore, combining the solution to the maximization problem in (12c) with the representation for $\psi_t = F_z(z_t, t)$ in (13), we find that $\sigma_z(z_t, t)$ is nonpositive: positive shocks reduce the long-term incentive, reducing the duration of incentives. This establishes the optimality of performance vesting discussed at the beginning of this section. The following proposition summarizes these results.

PROPOSITION 3: *If $\theta = r + \kappa$, then the optimal contract has the following properties:*

1. *Lower bound on long-term incentives: Let $\bar{z}(t)$ be the solution to $F_z(\bar{z}(t), t) = 0$. $\bar{z}(t) \geq 0$ is a decreasing function such that $\bar{z}(T) = 0$, $\bar{z}(t) > 0$ for all $t < T$, and $z_t \geq \bar{z}(t)$. Moreover, as $z_0 = \bar{z}(0)$ and $\sigma_z(z_t, t) \neq 0$ when $z_t > \bar{z}(t)$, the lower bound $\bar{z}(t)$ is tight.*[10]
2. *Long-term incentives and performance are negatively correlated: $\sigma_z(z_t, t) \leq 0$ for all $t \in [0, T]$.*
3. *When $m_t > 0$, the drift of long-term incentives is negative, that is, $E_t(dz_t) < 0$.*

[9] We can show that this never happens if $\lambda = 0$, but the analysis of deterministic contracts suggests that it could be the case if $\lambda > 0$.

[10] $\sigma_z(z_t, t) = 0$ only if $z_t = \bar{z}(t)$ or if z_t hits the upper bound implied by the upper bound on incentives $\bar{\beta}$.

The first part of the proposition indicates that there is a positive lower bound for long-term incentives and that this lower bound decreases over time, consistent with the notion that incentives need to vest over time as the CEO approaches retirement to mitigate the risk that the manager bears after retirement.

Performance vesting is optimal (i.e., $\sigma_z(z_t, t) < 0$) because it provides an extra degree of freedom to control the level of incentives z_t without triggering excessive manipulation m_t. In contrast, under deterministic vesting, the only way to reduce the long-term incentives is by increasing short-term incentives β_t, which exacerbates manipulation and lowers the level of effort the principal can implement in the future. This is precisely where performance vesting helps: long-term incentives can be reduced over time without necessarily distorting the level of effort. By adjusting the sensitivity of incentives $\sigma_z(z_t, t)$, the principal can control the drift of z_t while holding the trajectory of effort constant.

It is precisely the possibility of manipulation that justifies performance vesting in our setting. On the surface, one might think that performance vesting exacerbates the CEO's incentives to manipulate since, by inflating performance, the CEO can accelerate vesting. This logic is flawed. The manager's manipulation incentive at a given point depends on the sensitivity of his continuation value to performance β_t and duration z_t, not on the sensitivity of duration $\sigma_z(z_t, t)$. Making vesting more or less sensitive to performance at time t by modifying $\sigma_z(z_t, t)$ does not affect the CEO manipulation incentives at time t. For example, consider the case in which $\beta_t = 0$. The manager has no incentive to manipulate performance, and this is true independent of the sensitivity of incentives $\sigma_z(z_t, t)$. In other words, as long as β_t does not change, the choice of $\sigma_z(z_t, t)$ will not affect manipulation incentives at time t. Of course, $\sigma_z(z_t, t)$ has an indirect effect on incentives to manipulate in future periods due to its effect on the duration of incentives (z_t).

By setting a negative sensitivity $\sigma_z(z_t, t)$, the principal effectively implements a negative correlation between p_t and W_t. Hence, positive shocks that boost the agent's continuation value W_t reduce the duration of incentives p_t. In brief, good performance accelerates vesting.

The evolution of z_t resembles a mean-reverting process that follows a time-varying target $\bar{z}(t)$ converging to zero as T becomes closer. Figure 2 shows the evolution of the lower boundary $\bar{z}(t)$ together with the drift of z_t. The lower bound decreases over time toward zero, and the drift of z is negative above the lower bound on incentives, $\bar{z}(t)$. Moreover, we find that whenever the optimal contract implements positive manipulation, the drift of long-term incentives is negative. In particular, we find that the drift is negative when z_t is close to the lower boundary.[11] Long-term incentives revert toward the target $\bar{z}(t)$ over time, and the magnitude of the negative drift of z_t increases when we are close

[11] We have not been able to sign the drift for values of z_t such that $m_t = 0$. However, we show that in the case of contracts with deterministic vesting, that is, $\sigma_z(z_t, t) = 0$, the drift of z_t is always negative on the optimal path.

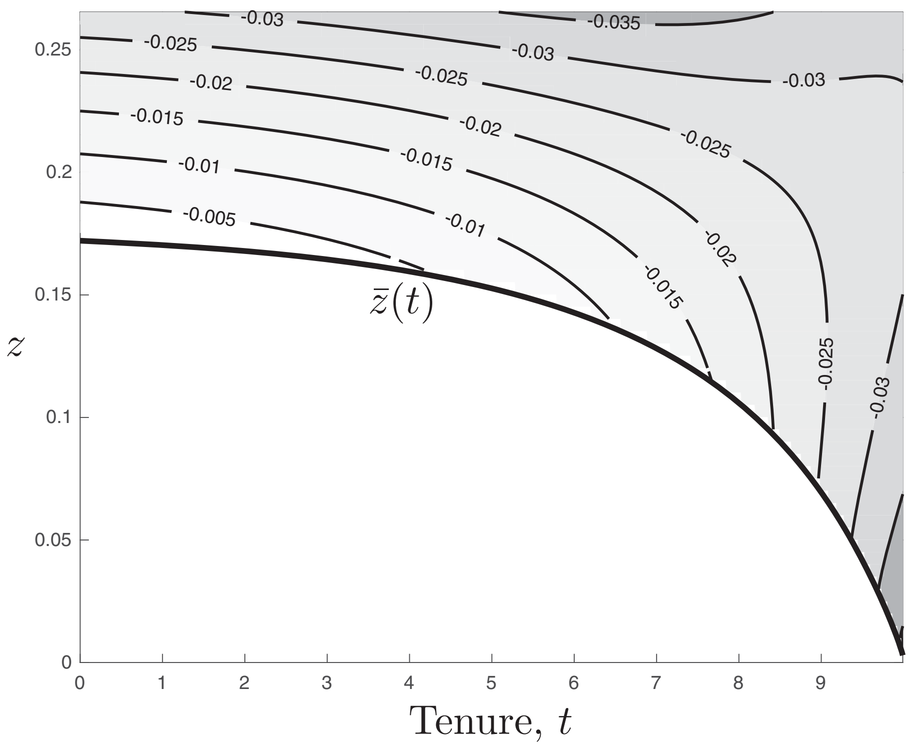


Figure 2. Lower bound ($\bar{z}(t)$) and drift ($E_t(dz_t)$) of incentives. Parameters: $r = 0.1$, $\gamma = 1$, $g = 1$, $\theta = 0.4$, $\kappa = 0.3$, $\sigma = 2$, $T = 10$, and $\tau = 5$. This plot shows the evolution of $\bar{z}(t)$ together with the drift of the continuation value. The lower bound $\bar{z}(t)$ decreases over time toward zero, and the drift of z_t is negative, which means that long-term incentives revert toward the target $\bar{z}(t)$. The black curve represents $\bar{z}(t)$, while the contour lines represent the value of drift, $E_t(dz_t)$, for different pairs (t, z). The relevant state space on path corresponds to the pairs (t, z) above the curve $\bar{z}(t)$. A darker background represents a lower (more negative) drift.

to the retirement date T. The relative importance of short-term incentives increases as the CEO gets closer to retirement, explaining the CEO horizon effect. Figure 3 shows the evolution of expected long-term incentives, effort, manipulation, and sensitivity. Long-term incentives and effort decrease, and manipulation increases over time. The volatility of incentives is low at the beginning of a CEO's tenure—so the contract's evolution is close to deterministic, that is, it decreases over time (its absolute value increases). This means that the contract becomes more sensitive to performance over time.

Consider the effect of enforcement on the optimal contract. Our model includes an upper bound τ for the length of the clawback period $[T, T + \tau]$. This parameter captures the fact that the principal cannot impose risk on the manager's wealth forever, as this would be impossible to enforce. It is not surprising then that a lower τ reduces the level of long-term incentives. When the clawback period is shorter, providing long-term incentives toward the end of the

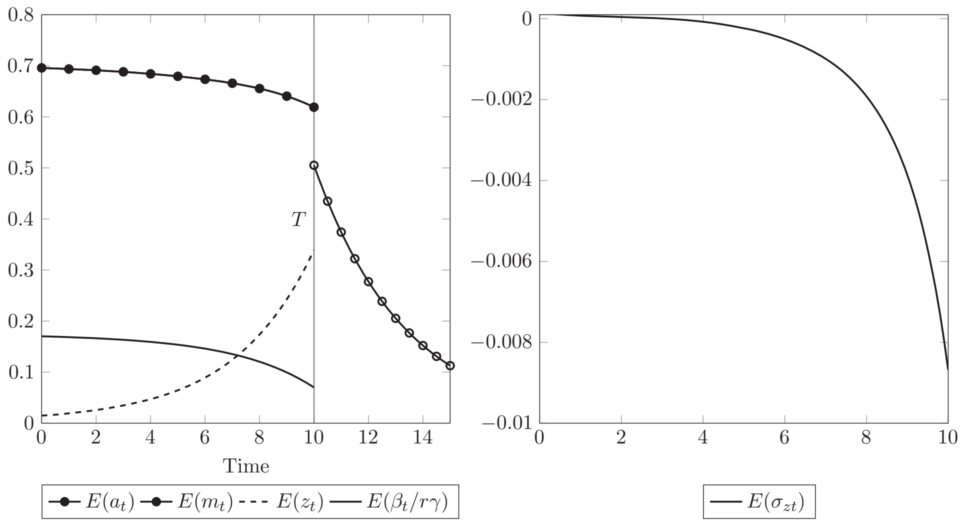


Figure 3. Expected path optimal contract. The expected path is computed simulating 1,000 paths and using antithetic variates for variance reduction. Parameters: $r = 0.1$, $\gamma = 1$, $g = 1$, $\theta = 0.4$, $\kappa = 0.3$, $\sigma = 2$, $T = 10$, and $\tau = 5$. The left panel shows the escalation of manipulation over time. To attenuate manipulation, the contract reduces short-term incentives over time, leading to a profile of decreasing effort. The right panel shows that the contract's volatility is very low at the beginning, but as times goes by the volatility of incentives increases and long-term incentives become negatively correlated to performance.

CEO's tenure is more costly because it makes the CEO's compensation more risky. Moreover, a lower τ reduces the overall duration of CEO incentives. In fact, a lower τ reduces the importance of long-term incentives at the beginning of the CEO's tenure and the lower bound on long-term incentives $\bar{z}(t)$, as the following proposition proves.

PROPOSITION 4: *If $\theta = r + \kappa$, then*

1. *The lower boundary on long-term incentives, $\bar{z}(t)$, is increasing in τ.*
2. *Initial long-term incentives z_0 are increasing in τ and T.*
3. *The above implies that initial effort is increasing in τ and T, while initial manipulation is decreasing in both τ and T.*

We conclude this section by revisiting the CEO's problem and establishing sufficient conditions for the validity of the first-order approach. This approach makes it possible to find a recursive formulation for the principal's problem and analyze the relaxed problem in which we only consider the first-order conditions. Solving for the optimal contract is not possible if the first-order approach is not valid because one lacks a recursive formulation that can be analyzed using the tools of stochastic control theory. The next proposition provides sufficient conditions for the validity of the first-order approach.

PROPOSITION 5: *Assume that $r\gamma\sigma^2 > 1$ and that the cost of manipulation satisfies*

$$g \geq \frac{1}{r\gamma\sigma^2 - 1}. \tag{14}$$

Then, given the optimal contract characterized in Proposition 3, the necessary incentive compatibility constraint is also sufficient. If either $r\gamma\sigma^2 \leq 1$ or (14) is not satisfied, then there is a bound $L_v > 0$ such that the necessary incentive compatibility constraint is also sufficient if $-L_v \leq \sigma_z(z_t, t)$.

As in the previous literature (He, Wei, and Yu (2014), Sannikov (2014)), the sufficiency of the first-order approach requires that the sensitivity of long term incentives be bounded. However, because in our setting this sensitivity is never positive, we only need to bound the sensitivity from below. Moreover, when $r\gamma\sigma^2$ and g are high enough, the first-order approach is valid for any nonpositive sensitivity, and there is no need to impose any additional bound on the sensitivity of incentives. Finally, notice that the first-order approach is always valid if $\sigma_z(z_t, t) = 0$, so the first-order approach is always valid for the deterministic contracts considered next in Section VII.

VII. Deterministic Incentives

In general, effort and manipulation are history dependent. However, one can gain further insights into the dynamics of compensation and CEO behavior by following Edmans et al. (2012) and He, Wei, and Yu (2014) and looking at the subclass of contracts that implement deterministic sequences of effort and manipulation. In this section, we characterize the best contract among the class of contracts that implement deterministic incentives. Hence, in this section, by optimal contract we mean the "best deterministic contract." With deterministic incentives, $\sigma_z(z_t, t) = 0$, the evolution of long-term incentives specializes to

$$\dot{z}_t = (r + \kappa)z_t - r\gamma a_t. \tag{15}$$

Equation (15) reveals a fundamental limitation of a deterministic contract: effort and long-term incentives are intertwined—to reduce long-term incentives, z_t, the contract must increase current effort a_t, and vice versa. The following proposition characterizes the path of incentives and CEO actions induced by the optimal deterministic contract.

PROPOSITION 6: *Based on the incentives to manipulate, the manager's tenure T can be divided into three regions characterized by thresholds $t^* \leq t^{**}$:*

- *In the first region, $[0, t^*)$, there is no manipulation, and the effort level is the same as that arising when manipulation is impossible, that is, $a_t = a^{HM}$.*
- *In the second region, $(t^*, t^{**}]$, there is zero manipulation, but the level of effort is bounded by the magnitude of long-term incentives ($a_t = \phi z_t$).*

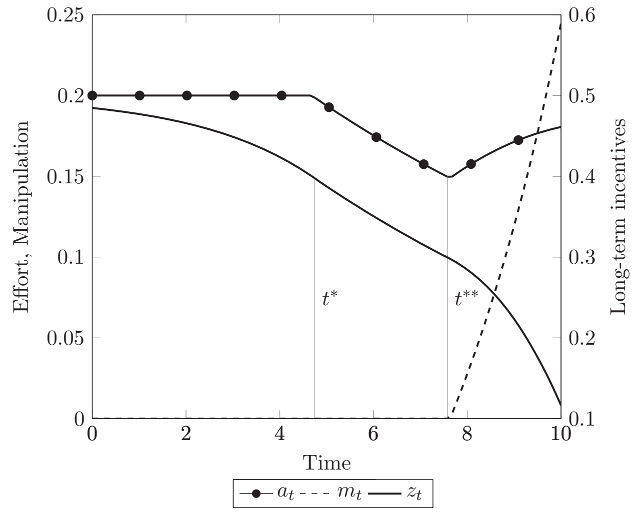


Figure 4. Deterministic contract. Parameters: $r = 0.1$, $\gamma = 1$, $g = 1$, $\theta = 0.5$, $\kappa = 0.3$, $\sigma = 2$, $T = 10$, and $\tau = 1$. The deterministic contract exhibits three regions. First, the CEO horizon is long enough such that manipulation is not an issue and effort is relatively high. Second, the contract implements zero manipulation but the manipulation constraint is binding, which leads to a decreasing effort profile. Third, implementing zero manipulation is too costly, and the manipulation and effort profiles are increasing.

- *In the third region,* $(t^{**}, T]$*, manipulation is positive and increasing over time.*
- *Depending on parameters, the regions* $(t^*, t^{**}]$ *and* $(t^{**}, T]$ *can be empty.*

Over time, long-term incentives z_t *are weakly decreasing, while manipulation* m_t *is weakly increasing.*

There are three distinct regions. In the first region, manipulation is not a concern. In the second region, there is no manipulation, but preventing manipulation forces the principal to lower the level of effort implemented. In the third region, preventing manipulation is too costly: both effort and manipulation go up over time. The region $(t^{**}, T]$ is empty when the no-manipulation contract identified in the previous section is optimal. Figure 4 provides a numerical example in which the three regions identified above are present. Surprisingly, effort is nonmonotone: it decreases at the beginning and increases toward the end (this is not true in general, and depending on the parameters, effort can be either increasing or decreasing in the final region $(t^{**}, T]$).

Why is effort increasing in the final region? The reason is simple: vesting of long-term incentives accelerates toward the end of the CEO's tenure, to avoid leaving the manager with a large postretirement package, and thus boosts short-term incentives. Edmans et al. (2012) find a result that is similar, but driven by a different mechanism. In their model, the CEO is finitely lived, so vesting accelerates by the end of his tenure because fewer periods are available to compensate the CEO. Hence, payments have to be spread over a shorter time period to keep the manager from shirking. In our setting, the CEO is infinitely lived so there is no need to accelerate vesting to satisfy the promise-keeping constraint. In our setting, vesting accelerates because deferring compensation after retirement is more costly than deferring compensation while the CEO is active. Having excessive deferred compensation after the CEO retires is costly, and hence vesting accelerates toward the end of his tenure to lower the level of postretirement incentives. In turn, this means that PPS increases by the end of tenure, thereby increasing both effort and manipulation.

The optimal contract induces manipulation sometimes, but not necessarily in every instant of the manager's tenure. As mentioned above, the CEO's tenure consists of three phases, ranked by the intensity of manipulation. During the first phase, manipulation incentives are weak because the CEO horizon is long, which means the principal has enough time to "detect" and penalize the managerial manipulation. As a consequence, short-run incentives are strong and the manager exerts high effort and zero manipulation. During the second phase, the manager's manipulation incentives are binding but the contract still implements zero manipulation. However, to prevent manipulation, the principal is forced to distort the contract PPS downward, which leads to a pattern of decreasing effort. During the third phase, vesting speeds up, manipulation incentives become stronger, and manipulation escalates.

Long-term incentives have to mature over time, as the manager approaches retirement, and this process tilts incentives toward the short run. In turn, this triggers manipulation, but may also boost effort in the final years. We can think of these two effects as mirror images. Providing high postretirement compensation is costly. To reduce it, some of the contract's long-term incentives must mature, which increases short-term incentives. The relative length of the three phases in the manager's tenure depends on the severity of the manipulation problem. Thus, for instance, when the reversal of manipulation is slow (low θ), enforcement is weak (low τ), or manipulation is easy (low g), the relative importance of the third phase increases at the expense of the other two phases, especially the first one.

We find that the optimal contract follows similar patterns to those in the contract with deterministic vesting. In the next section, we discuss the predictions of the model and provide some numerical examples and comparative statics. In most of our examples, the long-term incentive sensitivity implemented by the stochastic contract—which is itself random—is very small on average. Hence, the deterministic contract seems like a good approximation of the state-contingent contract, and it captures the evolution of CEO behavior and incentive pay very accurately, particularly at the beginning of CEO's tenure.

VIII. The Model at Work: Numerical Examples and Empirical Implications

In this section, we discuss the empirical implications of the model and relate them to existing evidence.

A. Vesting and Short-Termism

With regard to short-termism and vesting, Edmans, Fang, and Lewellen (2013) show that CEO manipulation increases during years with significant amounts of shares and option vesting. The authors find that, in years in which CEOs experience significant equity vesting, they cut investments in R&D, advertising, and capital expenditures. Seemingly, vesting induces CEOs to act myopically in order to meet short-term targets.

B. Horizon, Short-Termism, and Pay Duration

The executive compensation literature hypothesizes the existence of a "CEO horizon problem" whereby CEO short-termism is particularly severe in a CEO's final years in office, in so far as the manager is unable to internalize the consequences of his actions. Gibbons and Murphy (1992), for instance, hypothesize that existing compensation policies induce executives to reduce investments during their last years of office, but do not find conclusive evidence of greater manipulation. Gonzalez-Uribe and Groen-Xu (2015) find that "CEOs with more years remaining in their contract pursue more influential, broad and varied innovations." Dechow and Sloan (1991) document that managers tend to reduce R&D expenditures as they approach retirement, and these reductions in R&D are mitigated by CEO stock ownership.

Although intuitive, the CEO horizon hypothesis seems to ignore the fact that managers' incentives are endogenous. If shareholders anticipate the CEO horizon problem, they will arguably adjust compensation contracts accordingly. This could explain why empirical evidence regarding the relation between manipulation and tenure is mixed (Gibbons and Murphy (1992)). Cheng (2004), for instance, finds that compensation contracts become particularly insensitive to accounting performance measures that are easily manipulable by the end of the manager's tenure, suggesting that compensation committees are able to anticipate the manager's incentives. In this paper, we show that a CEO horizon problem exists even in the presence of endogenous incentives. The finite nature of a CEO's tenure and the fact that deferring compensation after retirement is costly explain why optimal contracts implement manipulation in our setting. In Section IV, we show that when the manager horizon grows large ($T \to \infty$), the possibility of manipulation is irrelevant. Linear contracts such as that analyzed by Holmstrom and Milgrom (1987) suffice to eliminate manipulation. This result is consistent with Jensen (2001, 2003), who recommends linear contracts to prevent managers from gaming compensation systems. In sum, our analysis suggests that when the CEO has a limited horizon, linear contracts are unable to prevent short-termism and may even induce too much.[12]

From a contracting perspective, two tools are effective at addressing the possibility of manipulation: (1) deferred compensation and (2) clawbacks. Both tools are used in practice. Some empirical evidence suggests that after SOX

[12] Kothari and Sloan (1992) provide evidence that accounting earnings commonly take up to three years to reflect changes in firm value.

the average duration of CEO compensation increased and firms started to rely more on restricted stock to compensate managers. Gopalan et al. (2014) provide evidence that the duration of stock-based compensation is about three to five years. They document a negative association between the duration of incentives and measures of manipulation such as discretionary accruals. In particular, they find that this duration is shorter for older executives and those with longer tenures. The second instrument is clawbacks. A clawback is a contractual clause included in employment contracts whereby the manager is obliged to return previously awarded compensation due to special circumstances, as described in the contract, for example, a fraud or restatement. The growing popularity of clawback provisions is due, at least in part, to the Sarbanes–Oxley Act of 2002, which requires the SEC to pursue the repayment of incentive compensation from senior executives who are involved in a fraud or a restatement.[13] Although we do not incorporate clawbacks—as a discrete event triggered by a restatement—in our model, the fact that the manager's income depends on postretirement performance captures the essence of clawbacks as an incentive mechanism.

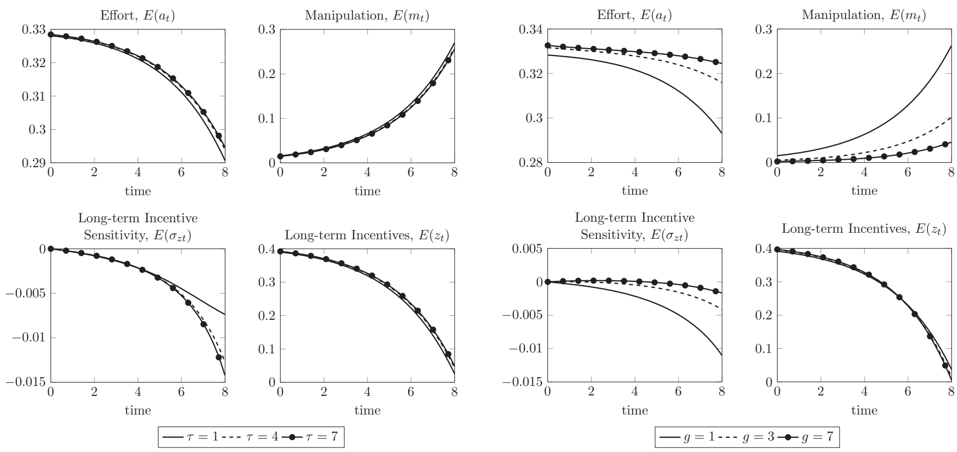
C. Pay-for-Performance

The executive compensation literature has documented at least two puzzles regarding PPS. First, pay-for-performance evolves with CEO tenure (Brickley, Linck, and Coles (1999)). Unlike in Holmstrom and Milgrom (1987), a constant PPS is not optimal in our setting. Indeed, a constant PPS would lead to excessive manipulation, especially around the retirement date. Our model predicts a profile of increasing manipulation along with a relatively low but potentially increasing PPS. Some evidence suggests that the PPS of CEO compensation increases over time, as manager's stock ownership grows (Gibbons and Murphy (1992)). At first blush, this fact seems to contradict the predictions of our model. In our setting, PPS may increase over time; however, it is never higher than at the start of the CEO's tenure, and it is nonmonotonic in time, it increases only at the end of the CEO's tenure. A time profile of increasing PPS is consistent with an extended version of the model in which the performance measure is a distorted version of the firm's cash flows (e.g., the firm earnings). A second empirical puzzle that was identified in the 1990s is the low PPS in CEO contracts (see, e.g., Jensen (2001)). Our model predicts that such low PPS could be the result of the possibility of manipulation, as already suggested by Goldman and Slezak (2006).

D. Corporate Governance and Short-Termism

The CEO horizon problem is ultimately a corporate governance weakness reflecting the inability of the firm to monitor the CEO's actions. If we take

[13] The prevalence of clawback provisions among Fortune 100 companies increased from less than 3% prior to 2005 to 82% in 2010.



(A) Comparative static clawback period τ. **(B)** Comparative static manipulation cost.

Figure 5. Comparative statics. Baseline parameters: $r = 0.1$, $\gamma = 5$, $g = 1$, $\theta = 0.4$, $\kappa = 0.3$, $\sigma = 2$, $T = 8$, and $\tau = 2$.

corporate governance to be a set of mechanisms (some of which are exogenous to the firm) that make it more costly for the manager to manipulate performance (e.g., by increasing the cost of manipulation g), then our model predicts that a stronger corporate governance should result in higher short-term compensation (lower duration) and greater firm value. Maybe paradoxically, it does not predict that the level of manipulation will be lower. If stronger corporate governance makes short-run incentives relatively more effective at stimulating effort, vis-a-vis manipulation, then the firm may find it optimal to offer stronger short-term incentives, even at the expense of tolerating greater manipulation. This effect is present in previous static models of costly state falsification. For example, Lacker and Weinberg (1989) show that no manipulation is optimal when the cost of manipulation is not overly convex. In our setting, with a quadratic falsification cost, this condition translates into a low value of g. From the incentive compatibility constraint, we find that the sensitivity of manipulation to changes in effort (for a fixed z) is $1/r\gamma g$. This means that when the marginal cost of manipulation g is low, the trade-off between higher effort and higher manipulation is too high. A small increment in effort generates so much manipulation that it makes the no-manipulation contract optimal. In fact, when $g = 0$, the optimal contract implements no manipulation. Hence, in this case, the optimal contract implements no manipulation. Of course, one needs to be careful when interpreting this observation as evidence that short-run incentives cause manipulation (Bergstresser and Philippon (2006)). As the cost of manipulation g grows large, the manager's manipulation incentives are vanishingly low. The contract then becomes stationary—with constant PPS because short-term incentives suffice to induce effort.

Another parameter that relates to corporate governance is τ. Recall that τ captures the length of the clawback period, that is, how long after retirement principal the principal can tie the CEO's wealth to the firm's performance. Figure 5 shows that a longer clawback period allows the principal to induce higher effort and less manipulation. The contract tends to rely more on performance-contingent vesting, and long-term incentives tend to be higher. As mentioned previously, extending τ is not a panacea. The consequences of manipulation are present even as τ grows large because providing a large postretirement package over a long clawback period can eliminate manipulation, although, as a downside, this would impose excessive risk on the manager.

IX. Conclusion

This paper studies optimal CEO contracts when managers can increase short-term performance at the expense of firm value. Our model is flexible, nesting both the case in which the CEO can manipulate performance by distorting the timing of cash flows and the case in which the manager can manipulate accruals. We consider a setting in which manager horizon is finite. We find that long-term incentives decrease over time, managerial short-termism increases, and effort may be nonmonotonic in time, increasing at the end of the CEO's career. The optimal compensation scheme includes deferred compensation. Vesting of the manager's incentives accelerates at the end of tenure, thus shifting the balance of incentives toward short-term compensation. This process gives rise to a CEO horizon problem—as an inherent feature of optimal contracts—whereby managers intensify performance manipulation in their final years in office. We characterize the optimal mix of short- and long-term incentives and the optimal duration and vesting of incentives along the manager's tenure.

We explore the optimality of deferred compensation as a contracting tool for alleviating the effects of CEO manipulation. Though potentially effective, postretirement compensation may impose significant risk on the CEO during a time when incentives are not needed to stimulate effort. This makes it costly from the firm's perspective, limiting the effectiveness of such compensation.

Unlike in Holmstrom and Milgrom (1987), the optimal contract is nonlinear in performance. Moreover, it implements path-dependent effort and manipulation, effectively making the firm's performance more noisy. Under a deterministic contract, the firm can modify the CEO's long-term incentives only by distorting the CEO's effort (for the contract to preserve incentive compatibility). History-dependent incentives help because they allow the firm to control the evolution of long-term incentives without exacerbating manipulation. The optimal contract is one in which the sensitivity of long-term incentives to firm performance at the beginning of the CEO's tenure and at the end are qualitatively different. At the beginning, positive performance shocks increase the use of long-term incentives, in other words, the duration of incentives increases when the firm is performing well. In contrast, at the end of the CEO's tenure, long-term incentives are negatively correlated to the firm's performance. Positive performance shocks lead to an acceleration of incentive vesting.

We conclude by noting that the design of monetary incentives alone is not enough to eliminate managerial short-termism. In practice, other corporate governance tools may complement the disciplining role of compensation. For example, we can presume that CEOs' discretion to make short-term investments or cut long-term investments evolves over time, being a function of the manager's horizon. There are different ways in which this could be addressed. For example, the level of discretion a CEO receives affects the freedom he has to manipulate performance, but it also makes him less productive. In other words, the CEO might not be able to manipulate performance as freely as before, and the associated lack of flexibility could also reduce his productivity. Specifically, assume that under low discretion the manager's effort produces only a fraction of what it produces otherwise (i.e., the marginal productivity of effort is αa_t for some $\alpha < 1$). Our analysis suggests that CEOs should be given more discretion at the beginning of their tenure, with an increment in board oversight taking place as he gets close to retirement. Of course, this policy recommendation must be taken with a grain of salt. There are additional factors that we have ignored in the model. One of these factors is learning: a young and inexperienced CEO's talent may be uncertain, and the board may want to monitor his actions more closely.

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Appendix A: Necessary Conditions for Incentive Compatibility

First, we begin analyzing the CEO's optimization problem in (A2) and deriving the conditions stated in Proposition 1. Given a contract that prescribes actions (c_t, a_t, m_t) and any CEO strategy $(\hat{c}, \hat{a}, \hat{m})$, we denote a deviation from the contract's recommended actions by $\Delta c \equiv \hat{c} - c$, $\Delta a \equiv \hat{a} - a$, and $\Delta m \equiv \hat{m} - m$. We simplify the notation by denoting the CEO's utility flow if he follows the contract's recommendation by $u_t \equiv u(c_t, a_t, m_t)$ and if he deviates by $u_t^{\Delta} \equiv u(c_t + \Delta c_t, a_t + \Delta a_t, m_t + \Delta m_t)$.

A contract is a function of the entire performance path X_t, which makes analysis of the CEO's problem involved. To overcome this challenge, we follow the approach proposed by Williams (2011) and introduce the following change of measure. Let $\mathbf{P}$ be the probability measure under recommendation $\{(a, m)\}_{t\in[0,T]}$ and let $\mathbf{P}^{\Delta}$ be the probability measure induced by the deviation $\{(\hat{a}, \hat{m})\}_{t\in[0,T]}$. For any such deviation, we define the exponential martingale:

$$\xi_t \equiv \exp\left(-\frac{1}{2}\int_0^t \eta_s^2 ds + \int_0^t \eta_s dB_s\right)$$

$$\eta_t \equiv \frac{1}{\sigma}(\Delta a_t + \Delta m_t - \theta \Delta M_t).$$

By Girsanov's theorem, the Radon-Nikodym derivative between $\mathbf{P}^{\Delta}$ and $\mathbf{P}$ is given by $d\mathbf{P}^{\Delta}/d\mathbf{P} = \xi_{T+\tau}$. Using the fact that $E(\xi_{T+\tau}|\mathcal{F}_t) = \xi_t$ and the law of iterated expectations, we can write the CEO's expected payoff given a deviation as

$$U(\hat{c}, \hat{a}, \hat{m}) = E^{(a,m)}\left[\int_0^{T+\tau} e^{-rt}\xi_t u_t^{\Delta} dt + e^{-r(T+\tau)}\xi_{T+\tau}\frac{u_{T+\tau}^{\Delta}}{r}\right]. \tag{A1}$$

The change of variables in equation (A1) is useful because it allows to fix the expectation operator by introducing the new state variable ξ_t in the CEO's optimization problem. Under this change of variables, the agent's problem is a stochastic control problem with random coefficients (Williams (2011)).

Notice that, without loss of generality, we can take the recommendation (c_t, a_t, m_t) as a reference point and consider the optimization with respect to $(\Delta c_t, \Delta a_t, \Delta m_t)$. A contract is incentive compatible if and only if $\Delta c = \Delta a = \Delta m = 0$ is the CEO's optimal choice. Using equation (A1), we can write the CEO's problem as

$$\begin{aligned}
&\sup_{\Delta c, \Delta a, \Delta m} U(c + \Delta c, a + \Delta a, m + \Delta m) \\
&\quad \text{subject to} \\
&d\xi_t = \frac{\xi_t}{\sigma}(\Delta a_t + \Delta m_t - \theta \Delta M_t) dB_t \\
&d\Delta M_t = (\Delta m_t - \kappa \Delta M_t) dt \\
&dS_t = (rS_t - \Delta c_t) dt.
\end{aligned} \tag{A2}$$

As mentioned above, this is a stochastic control problem that can be analyzed using the stochastic maximum principle—a generalization of Pontryagin's maximum principle to stochastic control problems (Yong and Zhou (1999)).

We begin by defining the (current-value) Hamiltonian function $\mathcal{H}$ as follows:

$$\mathcal{H} = \xi u^{\Delta} + q^M(\Delta m - \kappa \Delta M) + q^S(rS - \Delta c) + \nu^{\xi}\frac{\xi}{\sigma}(\Delta a + \Delta m - \theta \Delta M). \tag{A3}$$

There are three control variables $(\Delta a, \Delta m, \Delta c)$, three state variables $(\Delta M, S, \xi)$, and their associated adjoint variables (q^M, q^S, q^{ξ}). The first two state variables $(\Delta M, S)$ have drift but are not (directly) sensitive to cash flow shocks. The third state variable ξ is a martingale; its sensitivity to shocks is ν^{ξ}.

We maximize the Hamiltonian with respect to the control variables. Because the Hamiltonian is jointly concave in $(\Delta c, \Delta a, \Delta m)$, it suffices to consider the first-order conditions evaluated at $\Delta a = \Delta m = \Delta c = 0$. This yields the following first-order conditions:

$$u_{a_t} = -\frac{\nu_t^{\xi}}{\sigma}, \qquad u_{m_t} = -\frac{\nu_t^{\xi}}{\sigma} - q_t^M, \qquad u_{c_t} = q_t^S,$$

along with the complementary slackness conditions for the non-negativity constraint of m_t. The three adjoint variables (q_t^S, q_t^M, q_t^ξ) follow stochastic differential equations (SDEs), which are the stochastic analogue of the differential equations in optimal control theory:

$$\begin{aligned} dq_t^S &= rq_t^S dt - \frac{\partial \mathcal{H}_t}{\partial S_t} dt + \nu_t^S dB_t = \nu_t^S dB_t \\ dq_t^M &= rq_t^M dt - \frac{\partial \mathcal{H}_t}{\partial \Delta M_t} dt + \nu_t^M dB_t = \left((r+\kappa) q_t^M + \theta \frac{\nu_t^\xi}{\sigma} \right) dt + \nu_t^M dB_t \\ dq_t^\xi &= rq_t^\xi dt - \frac{\partial \mathcal{H}_t}{\partial \xi_t} dt + \nu_t^\xi dB_t = (rq_t^\xi - u_t) dt + \nu_t^\xi dB_t. \end{aligned}$$

The adjoint equations must satisfy the transversality conditions $q_{T+\tau}^\xi = \frac{u_{T+\tau}}{r}$, $q_{T+\tau}^M = 0$, and $q_{T+\tau}^S = \frac{u_{c,T+\tau}}{r}$. Theses equations are standard in stochastic control theory, but their economic meaning will become clear later.

First, we solve for q_t^ξ by integrating its SDE and using the corresponding transversality condition, which yields

$$q_t^\xi = E_t \left[\int_t^{T+\tau} e^{-r(s-t)} u_s ds + e^{-r(T+\tau)} \frac{u_{T+\tau}}{r} \right].$$

It is now apparent that the adjoint variable q_t^ξ captures the evolution of the CEO's continuation value, so we follow standard notation in the contracting literature and denote $W_t = q_t^\xi$. It is also convenient to write the sensitivity of the continuation value to cash flow shocks as $\nu_t^\xi \equiv -\beta_t W_t \sigma$. The SDE of the continuation value can thus be rewritten as:

$$dW_t = (rW_t - u_t)dt - \beta_t W_t \sigma dB_t, \tag{A4}$$

where β_t captures the sensitivity of W_t to shocks dB_t (recall that W_t is negative, given the negative exponential utility). The coefficient β_t is often referred to as the pay-for-performance sensitivity (PPS).[14] Hereafter, we refer to β_t as the CEO's short-run incentives or PPS.

We arrive at equation (5a) by plugging the private savings condition (6) in the above equation. Equation (5b), in contrast, is derived from the SDE for the adjoint variable of manipulation, q_t^M. Specifically, $p_t \equiv q_t^M/\theta$ captures the contract's long-term incentives and measures the incentive power of deferred compensation to deter manipulation.

Finally, we study the CEO's saving strategy. Using the first-order condition for consumption $u_c = q^S$ and the SDE for the adjoint variable q_t^S, we conclude that the marginal utility of consumption follows the following SDE:

$$du_{c_t} = \nu_t^S \sigma dB_t. \tag{A5}$$

[14] In the case of contracts implementing a deterministic sequence of effort and manipulation, β_t is proportional to the sensitivity of consumption to cash-flows that corresponds to the traditional definition of PPS in the empirical literature adapted to our setting.

Equation (A5) is the continuous-time version of the classic Euler equation for consumption, which states that the marginal utility of consumption must be a martingale when the CEO's discount rate is equal to the market's interest rate, as otherwise the CEO would save or borrow money to smooth out his consumption path.

We solve equation (A5) using a guess and verify approach. As in He, Wei, and Yu (2014), we conjecture that $W_t = ru_t$, and given CARA utility, we find that $u_c = -\gamma u = -\gamma rW$. Substituting this relation into equation (A5) and setting $\nu_t^S = r\gamma\beta_t W_t\sigma$, we obtain

$$du_{ct} = -r\gamma dW_t = r\gamma\beta_t W_t\sigma dB_t. \tag{A6}$$

Dividing by $-r\gamma$, we find that equation (A6) coincides with equation (A4) and verify that $u_{ct} = -r\gamma W_t$ solves the adjoint equation for q_t^S. Hence, the marginal utility of consumption u_c and the continuation utility W_t are martingales. This result, due to He, Wei, and Yu (2014), combines two observations. First, the CEO can smooth consumption intertemporally, so his marginal utility of consumption is a martingale. Second, under CARA preferences, the CEO's continuation value W_t is linear in flow utility u_t.

The first-order conditions along with the private savings condition, $u = rW$, yield the necessary incentive conditions for effort and manipulation stated in Proposition 1:

$$r\gamma h'(a_t) = \beta_t \tag{A7a}$$

$$r\gamma g'(m_t) = \beta_t + \theta\frac{p_t}{W_t}. \tag{A7b}$$

The complementary slackness condition in equation (5a) is obtained by replacing $u_t = rW_t$ in equation (A4) and defining $\nu_t^M \equiv \sigma_{pt}W_t\sigma$.

Appendix B: Sufficiency of Agent Incentive Compatibility

To prove sufficiency, we follow Sannikov (2014) and He, Wei, and Yu (2014) in the construction of an upper bound for the payoff after a deviation. In particular, we construct an upper bound of the form

$$\hat{W}_t = W_t e^{-r\gamma(S_t + L_1\Delta M_t^2) + \theta z_t\Delta M_t}.$$

Let $\mathbf{P}^\Delta$ be the measure induced by a deviation $(\Delta a_t, \Delta m_t)_{t\in[0,T]}$ and let

$$B_t^\Delta \equiv B_t - \int_0^t \frac{(\Delta a_s + \Delta m_s - \theta\Delta M_s)}{\sigma}ds.$$

By Girsanov's theorem, B_t^Δ is a Brownian motion under $\mathbf{P}^\Delta$, which means that under $\mathbf{P}^\Delta$,

$$dW_t = -\beta_t W_t(\Delta a_t + \Delta m_t - \theta\Delta M_t)dt - \beta_t W_t\sigma dB_t^\Delta$$

$$dz_t = \Big[(r+\kappa)z_t + \beta_t(\sigma\sigma_{zt}-1) + \frac{\sigma_{zt}}{\sigma}(\Delta a_t + \Delta m_t - \theta\Delta M_t)\Big]dt + \sigma_{zt}dB_t^{\Delta}$$

$$W_{T+\tau} = \frac{u^R(c_{T+\tau})}{r} z_{T+\tau} = 0.$$

Using Ito's lemma, we find that

$$d\hat{W}_t = \hat{W}_t(\mu_t^{\hat{W}}dt + \sigma_t^{\hat{W}}dB_t^{\Delta}),$$

where

$$\begin{aligned}\mu_t^{\hat{W}} = & -r\gamma(rS_t - \Delta c_t) + 2r\gamma\kappa L_1\Delta M_t^2 - r\gamma(-\phi z_t + 2L_1\Delta M_t)\Delta m_t + \frac{1}{2}r^2\gamma^2\phi^2\Delta M_t^2\sigma_{zt}^2 \\ & + r\gamma\phi\Delta M_t[rz_t + \beta_t(\sigma\sigma_{zt}-1)] + r\gamma\phi\Delta M_t\frac{\sigma_{zt}}{\sigma}(\Delta a_t + \Delta m_t - \theta\Delta M_t) \\ & - r\gamma a_t(\Delta a_t + \Delta m_t - \theta\Delta M_t) - r\gamma\phi\sigma\beta_t\Delta M_t\sigma_{zt}.\end{aligned}$$

Let W_t^{Δ} be the expected payoff given the deviation $(\Delta a_t, \Delta m_t, \Delta c_t)_{t\in[0,T+\tau]}$. For any fixed t_0 and $t \in [t_0, T+\tau]$, define the process

$$G_{t_0,t} \equiv \int_{t_0}^{t} e^{-r(s-t_0)}u_s^{\Delta}ds + e^{-r(t-t_0)}\hat{W}_t,$$

where

$$u_t^{\Delta} \equiv u\big(c_t + \Delta c_t, (a_t + \Delta a_t)\mathbf{1}_{t\le T}, (m_t + \Delta m_t)\mathbf{1}_{t\le T}\big),$$

and notice that by definition $E[G_{t_0,T+\tau}] = W_t^{\Delta}$. Differentiating $G_{t_0,t}$, we get

$$e^{rt}dG_{t_0,t} = u_t^{\Delta} - r\hat{W}_t dt - r\gamma\mu_t^{\hat{W}}\hat{W}_t dt + \sigma_t^{\hat{W}}\hat{W}_t dB_t.$$

Using the first-order condition for the agent's consumption, $rW_t = u_t \equiv u(c_t, a_t\mathbf{1}_{t<T}, m_t\mathbf{1}_{t<T})$, we can write the previous expression as:

$$e^{rt}dG_{t_0,t} = r\hat{W}_t\left[\left(\frac{u_t^{\Delta}}{u_t}e^{r\gamma(S_t+L_1\Delta M_t^2)-\theta\Delta z_t M_t} - 1\right)dt - \gamma\mu_t^{\hat{W}}\right] + \sigma_t^{\hat{W}}\hat{W}_t dB_t^{\Delta}.$$

Given that $\hat{W}_t < 0$, we need to show that

$$Q_t \equiv \frac{u_t^{\Delta}}{u_t}e^{r\gamma(S_t+L_1\Delta M_t^2)-\theta\Delta z_t M_t} - 1 + \frac{\mu_t^{\hat{W}}}{r} \ge 0.$$

Using the inequality $e^x \ge 1 + x$, we find that

$$\begin{aligned}& e^{r\gamma(S_t+L_1\Delta M_t^2)-\theta\Delta z_t M_t-\gamma\left(\Delta c_t - a_t\Delta a_t - \frac{\Delta a_t^2}{2} - gm_t\Delta m_t - g\frac{\Delta m_t^2}{2}\right)} - 1 \ge \\ & \gamma\left[r(S_t + L_1\Delta M_t^2) - \theta z_t\Delta M_t - \left(\Delta c_t - a_t\Delta a_t - \frac{\Delta a_t^2}{2} - gm_t\Delta m_t - g\frac{\Delta m_t^2}{2}\right)\right].\end{aligned}$$

Substituting $\mu_t^{\hat{W}}$, we get

$$\begin{aligned}
\frac{Q_t}{\gamma} &= \left((r+2\kappa)L_1 - \phi\theta\frac{\sigma_{zt}}{\sigma}\right)\Delta M_t^2 + \left(\frac{\Delta a_t^2}{2} + g m_t \Delta m_t + g\frac{\Delta m_t^2}{2}\right) \\
&\quad + (\phi z_t - 2L_1\Delta M_t)\Delta m_t + \frac{1}{2} r\gamma\phi^2\Delta M_t^2\sigma_{zt}^2 + \theta a_t \Delta M_t(\sigma\sigma_{zt} - 1) \\
&\quad + \phi\Delta M_t\frac{\sigma_{zt}}{\sigma}(\Delta a_t + \Delta m_t) \\
&\quad - a_t(\Delta m_t - \theta\Delta M_t) - \theta\sigma a_t \Delta M_t \sigma_{zt} \\
&= \left((r+2\kappa)L_1 - \phi\theta\frac{\sigma_{zt}}{\sigma} + \frac{1}{2}r\gamma\phi^2\sigma_{zt}^2\right)\Delta M_t^2 + \frac{\Delta a_t^2}{2} + g\frac{\Delta m_t^2}{2} \\
&\quad + \big(g m_t - (a_t - \phi z_t)\big)\Delta m_t \\
&\quad - 2L_1\Delta M_t\Delta m_t + \theta a_t\sigma\Delta M_t\sigma_{zt} + \phi\Delta M_t\frac{\sigma_{zt}}{\sigma}(\Delta a_t + \Delta m_t) - \theta\sigma a_t\Delta M_t\sigma_{zt} \\
&\geq \left((r+2\kappa)L_1 - \phi\theta\frac{\sigma_{zt}}{\sigma} + \frac{1}{2}r\gamma\phi^2\sigma_{zt}^2\right)\Delta M_t^2 + \frac{\Delta a_t^2}{2} + g\frac{\Delta m_t^2}{2} \\
&\quad + \left(\phi\frac{\sigma_{zt}}{\sigma} - 2L_1\right)\Delta m_t\Delta M_t \\
&\quad + \phi\frac{\sigma_{zt}}{\sigma}\Delta a_t\Delta M_t,
\end{aligned}$$

where in the last line we have used the fact that $(g m_t - (a - \phi z_t))\Delta m_t \geq 0$. Completing squares,

$$\begin{aligned}
\frac{Q_t}{\gamma} &\geq \left[(r+2\kappa)L_1 - \phi\theta\frac{\sigma_{zt}}{\sigma} + \frac{1}{2}r\gamma\phi^2\sigma_{zt}^2 - \frac{\phi^2\sigma_{zt}^2}{2\sigma^2} - \frac{1}{2g}\left(\phi\frac{\sigma_{zt}}{\sigma} - 2L_1\right)^2\right]\Delta M_t^2 \\
&+ \frac{1}{2}\left(\Delta a_t + \phi\frac{\sigma_{zt}}{\sigma}\Delta M_t\right)^2 + \frac{g}{2}\left(\Delta m_t + \frac{1}{g}\left(\phi\theta\frac{\sigma_{zt}}{\sigma} - 2L_1\right)\Delta M_t\right)^2.
\end{aligned}$$

Hence, if

$$(r+2\kappa)L_1 - \phi\theta\frac{\sigma_{zt}}{\sigma} + \frac{1}{2}r\gamma\phi^2\sigma_{zt}^2 - \frac{\phi^2\sigma_{zt}^2}{2\sigma^2} - \frac{1}{2g}\left(\phi\frac{\sigma_{zt}}{\sigma} - 2L_1\right)^2 \geq 0, \qquad \text{(B1)}$$

then $G_{t_0,t}$ has a negative drift, so

$$W_{t_0}^{\Delta} = E_{t_0}^{\Delta}[G_{t_0,T+\tau}] \leq G_{t_0,t_0} = \hat{W}_{t_0}.$$

Accordingly, if inequality (B1) is satisfied, the optimality of $\Delta a_t = 0,\ \Delta m_t = 0$, and$\Delta c_t = 0$ follows directly from the fact that $W_0 = \hat{W}_0 \geq W_0^{\Delta}$, and for any t such that $\Delta a_s = 0,\ \Delta m_s = 0$, and $\Delta c_s = 0$ for all $s \leq t$, we have $W_t = \hat{W}_t \geq W_t^{\Delta}$.

Next, we find conditions such that (B1) holds. We can write the left-hand side of equation (B1) as a function of L_1:

$$H(L_1) \equiv (r+2\kappa)L_1 - \phi\left(\theta - \frac{L_1}{g}\right)\frac{\sigma_{zt}}{\sigma} + \left(\frac{r\gamma}{2} - \frac{1}{2\sigma^2} - \frac{1}{2\sigma^2 g}\right)\phi^2\sigma_{zt}^2 - \frac{2L_1^2}{g}.$$

First, if $r\gamma\sigma^2 - 1 > 0$ and

$$g \geq \frac{1}{r\gamma\sigma^2 - 1}, \tag{B2}$$

then, because $\sigma_{zt} \leq 0$, we can simply consider $L_1 = 0$ and get

$$H(0) = -\phi\theta\frac{\sigma_{zt}}{\sigma} + \left(r\gamma - \frac{1}{\sigma^2} - \frac{1}{\sigma^2 g}\right)\frac{\phi^2\sigma_{zt}^2}{2} \geq 0.$$

Second, if $r\gamma\sigma^2 - 1 > 0$ or (B2) does not hold, then we can take L_1 to maximize

$$(r+2\kappa)L_1 - \frac{2L_1^2}{g},$$

which yields

$$L_1 = \frac{(r+2\kappa)g}{4}.$$

After substituting in $H(L_1)$, we get

$$H(L_1) = \frac{(r+2\kappa)^2 g}{8} - \frac{\phi}{4}(4\theta - r - 2\kappa)\frac{\sigma_{zt}}{\sigma} + \left(\frac{r\gamma}{2} - \frac{1}{2\sigma^2} - \frac{1}{2\sigma^2 g}\right)\phi^2\sigma_{zt}^2.$$

Because $\sigma_{zt} \leq 0$, we only need to consider the negative root, so there exists $L_v > 0$ such that if $\sigma_{zt} \geq -L_v$, then $H(L_1) \geq 0$, where L_v is the absolute value of the negative root as given by

$$L_v = \frac{r\gamma\sigma}{4\theta}\frac{(4\theta - r - 2\kappa) + \sqrt{(4\theta - r - 2\kappa)^2 - 4(r+2\kappa)^2\left(r\gamma g\sigma^2 - g - 1\right)}}{1 + g^{-1} - r\gamma\sigma^2}.$$

Appendix C: Principal Problem

PROOF LEMMA 1: Plugging equation (7) in the objective function yields the principal's expected payoff as a function of a_t, m_t, and W_t:

$$E\left[\int_0^T e^{-rt}\left(a_t - \lambda m_t - h(a_t) - g(m_t)\right)dt + \int_0^{T+\tau} e^{-rt}\left(\frac{\log(-W_t)}{\gamma} + \frac{\log(r\gamma)}{\gamma}\right)dt \right.$$
$$\left. + e^{-r(T+\tau)}\frac{\log(-r\gamma W_{T+\tau})}{r\gamma}\right]. \tag{C1}$$

Using Ito's lemma, we compute the expected value of $\log(-W_t)$, which is given by

$$E[\log(-W_t)] = \log(-W_0) - \frac{1}{2}E\left[\int_0^t \sigma^2 \beta_s^2 ds\right].$$

If we change the order of integration, we obtain

$$E\left[\int_0^{T+\tau} e^{-rt} \int_0^t \sigma^2 \beta_s^2 ds dt\right] = E\left[\int_0^{T+\tau} \left(e^{-rt} - e^{-r(T+\tau)}\right) \frac{\sigma^2 \beta_t^2}{r} dt\right].$$

Substituting this expression in equation (C1) and ignoring constant terms, we can write the principal's expected payoff as

$$E\left[\int_0^T e^{-rt}\big(a_t - \lambda m_t - h(a_t) - g(m_t)\big)dt - \int_0^{T+\tau} e^{-rt} \frac{\sigma^2 \beta_t^2}{2r\gamma} dt\right]. \tag{C2}$$

The first term inside brackets captures the cash flow realized throughout the CEO's tenure. The second term captures the monetary impact of the compensation risk borne by the manager until the end of the clawback period, at time $T + \tau$.

After removing the dependence of the principal's payoff on the manager's continuation utility W_t, we remove such dependence from the incentive constraints as well. As mentioned above, we use $z_t \equiv -p_t/W_t$ as state variable. Hereafter, we refer to z as the contract's long-term incentives.

Using the law of motion of W_t and p_t in (5a) and (5b), along with Ito's lemma, we find that the law of motion of z follows the following SDE:

$$dz_t = [(r + \kappa)z_t + \beta_t(\sigma\sigma_{zt} - 1)]dt + \sigma_{zt} dB_t, \tag{C3}$$

where $\sigma_{zt} \equiv \sigma(\beta_t z_t - \sigma_{pt})$. Also, the incentive compatibility constraint is defined by $g'(m_t) = h'(a_t) - \phi z_t = (a_t - \phi z_t)/g$. We have thus reduced the optimal contract to a finite-horizon one-dimensional stochastic control problem that can be written as:

$$F(z) = \sup_{a,m,\beta,v} E\left[\int_0^T e^{-rt}\big(a_t - \lambda m_t - h(a_t) - g(m_t)\big)dt - \int_0^{T+\tau} e^{-rt} \frac{\sigma^2 \beta_t^2}{2r\gamma} dt\right]$$

subject to the law of motion of z_t in (9) and the manager's participation and incentive constraints (equations (4a) and (4b)).

A. Optimal Contract: Maximization of the HJB Equation

We have the HJB equation

$$rF = \max_{a,m,v} \pi(a, m) + F_t + [(r + \kappa)z + ar\gamma(\sigma v - 1)]F_z + \frac{1}{2}\sigma_z F_{zz},$$

subject to

$$m \geq \frac{a - \phi z}{g}$$
$$m \geq 0,$$

with boundary conditions

$$F(z, T) = -\frac{1}{2}\mathcal{C}z^2$$
$$F(0, t) = 0.$$

The Lagrangean for the optimization problem is

$$L = a - \lambda m - \frac{gm^2}{2} - \frac{(1 + r\gamma\sigma^2)a^2}{2} + [(r+\kappa)z + ar\gamma(\sigma\sigma_z - 1)]F_z + \frac{1}{2}\sigma_z^2 F_{zz} + \eta\left(m - \frac{a - \phi z}{g}\right) + \nu m.$$

The first-order condition with respect to a and m is

$$1 - (1 + r\gamma\sigma^2)a - \frac{\eta}{g} + r\gamma(\sigma\sigma_z - 1)F_z = 0$$
$$-\lambda - gm + \eta + \nu = 0$$
$$ar\gamma\sigma F_z + \sigma_z F_{zz} = 0,$$

where $\nu = 0$ if $m > 0$ and $\eta = 0$ if $m > (a - \phi z)/g$ (which means that $m = 0$). The volatility of the continuation value is

$$v = -\frac{ar\gamma\sigma F_z}{F_{zz}}.$$

Suppose that $m > 0$. If this is the case, we have that $\nu = 0$ and

$$\eta = \lambda + gm = \lambda + a - \phi z.$$

Substituting in the first-order condition for effort, we get

$$g - \left(1 + g\left(1 + r\gamma\sigma^2 + \frac{r^2\gamma^2\sigma^2 F_z^2}{F_{zz}}\right)\right)a - \lambda + \phi z - r\gamma g F_z = 0,$$

and hence we get

$$a = \frac{g - \lambda + \phi z - r\gamma g F_z}{1 + g\left(1 + r\gamma\sigma^2 + \frac{r^2\gamma^2\sigma^2 F_z^2}{F_{zz}}\right)}$$
$$m = \frac{a - \phi z}{g}$$

$$\sigma_z = -\frac{ar\gamma\sigma F_z}{F_{zz}}.$$

This solution satisfies the constraints if and only if

$$\frac{g-\lambda-r\gamma g F_z}{g} \geq \phi z\left(1+r\gamma\sigma^2+\frac{r^2\gamma^2\sigma^2 F_z^2}{F_{zz}}\right).$$

If this condition is violated, it must be the case that $m_t = 0$. Suppose that $\eta = 0$, which means that the constraint $m \geq (a-\phi z)/g$ is slack. In this case, we find that

$$a = \frac{1-r\gamma F_z}{1+r\gamma\sigma^2+\frac{r^2\gamma^2\sigma^2 F_z^2}{F_{zz}}}.$$

Substituting in the constraints, we find that $m \geq (a-\phi z)/g$ is slack if and only if

$$1-r\gamma g F_z < \phi z\left(1+r\gamma\sigma^2+\frac{r^2\gamma^2\sigma^2 F_z^2}{F_{zz}}\right).$$

Finally, if the two conditions above are violated, that is, if

$$1-\frac{\lambda+r\gamma g F_z}{g} < \phi z\left(1+r\gamma\sigma^2+\frac{r^2\gamma^2\sigma^2 F_z^2}{F_{zz}}\right) < 1-r\gamma g F_z,$$

then it must be the case that $m_t = 0$ and $a_t = \phi z_t$. We find then that the solution to the maximization problem in the HJB equation is

$$a(z,t) = \begin{cases} \frac{g-\lambda+\phi z-r\gamma g F_z}{1+gH(z,t)} & \text{if } 1-\frac{\lambda+r\gamma g F_z}{g} \geq \phi z H(z,t) \\ \frac{1-r\gamma F_z}{H(z,t)} & \text{if } 1-r\gamma F_z < \phi z H(z,t) \\ \phi z & \text{otherwise} \end{cases}$$

$$m(z,t) = \frac{1}{g}(a(z,t)-\phi z)^+$$

$$\sigma_z(z,t) = -a(z,t) r\gamma\sigma \frac{F_z}{F_{zz}},$$

where

$$H(z,t) = 1+r\gamma\sigma^2+\frac{r^2\gamma^2\sigma^2 F_z^2}{F_{zz}}.$$

Finally, we need to show that the second-order conditions are satisfied. Consider the function

$$G(a,m,\sigma_z) \equiv a-\lambda m-\frac{gm^2}{2}-\frac{(1+r\gamma\sigma^2)a^2}{2}+ar\gamma(\sigma\sigma_z-1)F_z+\frac{1}{2}\sigma_z F_{zz}.$$

If we maximize with respect to σ_z, we get that

$$\sigma_z = -r\gamma\sigma a \frac{F_z}{F_{zz}}.$$

The second-order condition for this maximization is that $F_{zz} < 0$. Substituting in G, we get the following optimization problem for a and m

$$\max_{a,m}\ a - \lambda m - \frac{gm^2}{2} - \frac{(1+r\gamma\sigma^2)a^2}{2} - r\gamma a F_z - \frac{1}{2}r^2\gamma^2\sigma^2 a^2 \frac{F_z^2}{F_{zz}}$$

subject to

$$gm - a + \phi z \geq 0.$$

Because $z_t \geq \bar{z}(t)$ (so $F_z(z_t,t) \leq 0$), we can restrict attention to z such that $F_z(z_t,t) \leq 0$ when we check the second-order condition. We consider three cases:

1. $H(z,t) > 0$
2. $H(z,t) < 0$ and $1 + gH(z,t) > 0$
3. $1 + gH(z,t) < 0$.

Consider the case in which the incentive compatibility constraint for m holds with equality. In this case, we can consider the bordered Hessian which is given by

$$\begin{bmatrix} 0 & g & -1 \\ g & -g & 0 \\ -1 & 0 & -(1+r\gamma\sigma^2) - r^2\gamma^2\sigma^2\frac{F_z^2}{F_{zz}} \end{bmatrix}.$$

Thus, the second-order condition for the maximization problem for a, m is

$$\begin{vmatrix} 0 & g & -1 \\ g & -g & 0 \\ -1 & 0 & -(1+r\gamma\sigma^2) - r^2\gamma^2\sigma^2\frac{F_z^2}{F_{zz}} \end{vmatrix} > 0,$$

which corresponds to the condition

$$1 + g\left(1 + r\gamma\sigma^2 + r^2\gamma^2\sigma^2\frac{F_z^2}{F_{zz}}\right) = 1 + gH(z,t) > 0.$$

If this condition is not satisfied, then the solution to the optimization problem is an extreme point. We can notice this considering the optimization problem

$$\max_{a}\ a - \frac{\lambda}{g}(a-\phi z)^+ - \frac{[(a-\phi z)^+]^2}{2g} - \frac{(1+r\gamma\sigma^2)a^2}{2} - r\gamma a F_z - \frac{1}{2}r^2\gamma^2\sigma^2 a^2\frac{F_z^2}{F_{zz}}. \tag{C4}$$

The optimal effort is an extreme point when $1 + gH(z,t) < 0$, which means that there are two candidates for the solutions ϕz and $\bar{a}$. Suppose that $\phi z < \bar{a}$. In this case, we have that

$$\bar{a} - \frac{\lambda}{g}(\bar{a} - \phi z) - \frac{(\bar{a} - \phi z)^2}{2g} - \frac{(1 + r\gamma\sigma^2)\bar{a}^2}{2} - r\gamma\bar{a}F_z - \frac{1}{2}r^2\gamma^2\sigma^2\bar{a}^2\frac{F_z^2}{F_{zz}} \geq \phi z - \frac{(1 + r\gamma\sigma^2)(\phi z)^2}{2} - r\gamma\phi z F_z - \frac{1}{2}r^2\gamma^2\sigma^2(\phi z)^2\frac{F_z^2}{F_{zz}},$$

or

$$g - \lambda - r\gamma g F_z - \frac{1}{2}\left(1 + g(1 + r\gamma\sigma^2) + gr^2\gamma^2\sigma^2\frac{F_z^2}{F_{zz}}\right)(\bar{a} + \phi z) > 0.$$

This condition is satisfied if $g > \lambda$ and $1 + gH(z,t) < 0$.

Finally, we consider the case in which the constraint for m_t is slack. In this case, the second-order condition for a in (C4) is $1 + r\gamma\sigma^2 + r^2\gamma^2\sigma^2\frac{F_z^2}{F_{zz}} = H(z,t) > 0$ (which immediately implies that $1 + gH(z,t) > 0$). If this condition is not satisfied, then it must be the case that $a \in [\phi z, \bar{a}]$, because the solution cannot be an interior point of $[0, \phi z]$, and hence

$$1 - r\gamma F_z - \frac{\phi z}{2}H(z,t) > 0,$$

so the value of $a = \phi z$ is higher than the value of $a = 0$.

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Supporting Information

Additional Supporting Information may be found in the online version of this article at the publisher's website:

Appendix S1: Internet Appendix.