Multilateral Contracting in Stage Financing

Paolo Fulghieri Yunzhi Hu Felipe Varas *

November 14, 2025

Abstract

Venture capital financing typically features complex securities and staging. We develop a dynamic contracting model where an entrepreneur seeks financing from active investors (who provide costly monitoring and screening) and passive investors (who offer cheaper capital). Under multilateral moral hazard, we show that the optimal contract can be implemented through a sequential offering of securities, including common and preferred equity, options, warrants, as well as a combination of senior debt and credit lines (venture debt). Our model predicts when entrepreneurs optimally separate monitoring and screening across multiple active investors ("rounds financing") versus consolidating these functions with a single active investor ("milestone financing"). Rounds financing dominates when informed capital is scarce.

Keywords: Security design, multilateral contracting, staged financing.

*Fulghieri: Kenan-Flagler Business School, UNC Chapel Hill, CEPR and ECGI; Paolo_Fulghieri@kenan-flagler.unc.edu. Hu: Kenan-Flagler Business School, UNC Chapel Hill; Yunzhi_Hu@kenan-flagler.unc.edu. Varas: Naveen Jindal School of Management, University of Texas at Dallas; felipe.varas@utdallas.edu. Nancy Gahlot provided excellent research assistance. We thank Per Strömberg for very helpful comments. All errors are only our own.

1 Introduction

Complex financial contracting is an essential feature of venture capital financing in entrepreneurial firms (see, for example, Kaplan and Strömberg (2003)). Venture capital deals involve several classes of investors, each playing a specific role and at a specific point in time. Some venture capital investors are actively engaged in the management of their portfolio companies; others provide financing while maintaining a more passive posture. A common feature is stage financing, whereby new capital for follow-up investment is made available only upon satisfactory progression of the underlying investment project.

Both entrepreneurs and venture capital investors add value to portfolio companies. The role of the entrepreneur is particularly important at the early stages of investment, when human capital is a key input in project development. Venture capital investors add value by either being actively involved in the management of the firm, such as by monitoring and advising the entrepreneur, or by assessing project profitability before providing additional funding.[1] In addition, entrepreneurs, venture capital investors, and portfolio companies are all linked by a web of financial arrangements and securities. These include equity, preferred stock, debt tranches with different seniority, options, and warrants. Such financial arrangements must provide sufficient incentives to all parties involved in the financing and execution of the investment project. In this paper, we study a model of staged venture capital financing that captures these features in a parsimonious way.

An important question is the dynamic structure of financing stages.[2] Financing may be provided sequentially by different VCs in separate financing stages ("rounds financing"). Each financing round (for example, Series A, then followed by Series B) takes place only after satisfactory progression of the investment project, as assessed by the new incoming VC investor. In alternative, financing may be provided by a single VC, or syndicate (for example, a "supersized" Series A financing) whereby each financing stage is contingent on the successful completion of predetermined milestones ("milestone financing").

[1]Lerner (1995) and Gompers (1995) documents the importance of VC monitoring of their portfolio companies. Hellmann and Puri (2002) show that VC plays a positive role in the successful development of their portfolio companies.

[2]Kaplan and Strömberg (2003) distinguish between *ex-ante*, long-term financing, where financing stages are determined by achieving predetermined milestones, and *ex-post*, short-term financing, where subsequent financing is provided in new VC rounds. They find that ex-ante financing occurs in about 15% of the deals in their sample.

In our model, an entrepreneur has access to an investment project. At the earlier stages, the project requires both entrepreneurial human capital, in the form of effort, and capital investment. The project then requires an additional follow-up investment at an interim stage before a final risky payoff is realized at the final stage. The entrepreneur is penniless and must raise funds from outside investors. There are two types of external investors: active and passive investors. Active investors represent informed capital: these are specialized agents, such as venture capitalists, contributing to the project value creation in addition to providing financing. Passive investors are uninformed agents (for example, institutional investors such as pension funds) and only provide financing. Active investors can contribute to project development in two ways. First, they can monitor the entrepreneur at the earlier stages of project development. Monitoring is costly but valuable in that it promotes entrepreneurial effort.[3] Second, before financing the subsequent follow-up investment, active investors can obtain a costly signal to assess satisfactory project development. Absent the second round of financing, the project must be liquidated.

While uninformed capital is abundant in the economy, informed capital is scarce. We capture this notion by assuming that active investors discount cash flows at a lower discount factor (i.e, a greater discount rate) than passive investors, whose discount factor is normalized to one. We interpret the active investors' discount factors as a measure of the tightness of the market for informed capital. Entrepreneurs are impatient and have the lowest discount factor. Raising capital for investment is impaired by multilateral moral hazard. First, both entrepreneurial effort choice and monitoring by the active investor are made privately by each agent at the early stage of project development. Second, production of the costly signal on project quality (at the interim stage) and observation of its realization are private to the active investor performing the evaluation. Final output of the project, for simplicity, is observable and contractible.

A key question in our paper is to determine whether monitoring and screening functions should be performed by two separate active investors or, rather, should be combined and performed by a single active investor. The choice is made by the entrepreneur at the beginning of the period. An important difference between the two financing arrangements is the respective set of incentive constraints. Specifically, separating monitoring and screening requires that each active investor does

[3] Bernstein et al. (2016) show that VC monitoring is valuable as it increases the likelihood of innovations.

not have the incentive to unilaterally deviate from screening *or* monitoring (given that the other agent performs the required task). In contrast, combining monitoring and screening also requires that the active agent does not have the incentive to deviate and refrain from both monitoring *and* screening. We refer to this additional constraint as the double-deviation constraint. Imposing this additional constraint will affect both the size and timing of cash payments in optimal incentive contracts.

We first characterize the solution to the optimal contracting problem that minimizes the financing costs in each of the two arrangements, and we show that its solution can be implemented by the entrepreneur through sequential issuance of securities. We show that the financing frictions we examine generate, endogenously, a rich set of securities that resemble the complex financial arrangements of venture capital financing. In our model, optimal securities include common and preferred equity, options and warrants, in addition to senior debt and credit lines. The optimal financing arrangement also includes the allocation of control rights in the form of assigning the decision to continue the project or to terminate it through liquidation.

We find that the financial arrangement with the separation of project monitoring and screening may be implemented as follows. At the outset, the entrepreneur finances the initial investment by issuing a package of debt, equity, and options. The entrepreneur issues equity to the active investor (engaged with monitoring) and retains an amount of equity that is sufficient to ensure both monitoring and effort ("skin-in-the-game"). The entrepreneur also issues to the passive investor a package of securities which includes debt and a credit line with the same seniority as the debt tranche ("venture debt"), and options.[4] The options component gives the passive investor the right to buy out (at the interim date) both the entrepreneur and the active investor (i.e., the monitor).

At the interim date, the entrepreneur seeks to finance the follow-up investment through both debt and equity, as follows. First, the entrepreneur announces the offer to issue preferred stock (mezzanine financing). To decide whether or not to subscribe to the preferred equity offer, a second active investor (specializing in second-round financing) produces information (i.e., screens) the project. If a good signal is observed, the investor subscribes to the offer, and preferred equity is issued. The successful equity offer triggers the option held by the passive investor to buy out the

[4] The use of venture debt is discussed in Davis et al. (2020).

entrepreneur and the monitor. The entrepreneur uses the proceeds from the preferred equity offer to finance the follow-up investment in conjunction with drawing from the credit line. If the equity offer fails, which is the case if a bad signal is observed by the second-round investor, the investor exercises the control rights and chooses to liquidate the project, collecting the full liquidation value.

If the project is continued, its payoff is realized at the last period. If a low payoff is obtained, the passive investor collects the full project value as a senior claimant. If a high payoff is obtained, investors are paid according to priority rules: the passive investor's senior debt is paid in full, the active investor holding preferred stock is paid next, and then the residual is paid to common equity (held by the passive investor).

The financial arrangement with a combination of the monitoring and screening functions to a single active investor proceeds as follows. At the beginning, the entrepreneur grants to the active investor a warrant on preferred stock, giving the investor the option to buy at the interim date newly issued preferred equity. The entrepreneur retains sufficient equity value to guarantee sufficient incentives to exert effort. In addition, the firm issues to the passive investors a package of securities that includes, again, senior debt, a credit line, and the option to buy out the entrepreneur at the interim date. Proceeds from the sale of securities are used to finance the initial capital expenditure.

At the interim date, the active investor must decide whether or not to exercise the warrant and thus force the firm to issue preferred stock, a decision made after producing information on the project. Similar to the previous case, if the signal is good, the warrant on preferred stock is exercised, and preferred stock is issued. The entrepreneur finances the follow-up investment with the proceeds from the sale of preferred stock and by drawing on the credit line. The passive investor exercises the option to buy out the entrepreneur. After that, the project continues, and security holders are paid out on the last date as in the previous case. If the active investor observes a bad signal, the warrant on preferred stock is not exercised, and the project is again liquidated.

In the final step of our paper, we compare the two financing arrangements. We show that the choice depends on its impact on both the timing and size of payoff streams to the passive investor, who is effectively the residual claimant (due to the greater discount factor). We show that combining monitoring and screening, by affecting incentive constraints, has two effects on net cash flows to the passive investor. The first effect is that the passive investor can avoid the incentive payment that must otherwise be made to the monitoring investor at the intermediate period and combine it with the incentive payment made in the last period. Combining payments to the last period has the advantage of providing incentives for both screening and monitoring. The delay is, however, costly because of the differences in discount factors of active and passive investors. The second effect is that to satisfy the monitoring and screening incentive constraint, the single active investor must receive in the last period an incremental rent with respect to the rent necessary to induce screening alone in the case of two active investors. Importantly, the size of the incremental rent depends on whether or not the double deviation constraint binds.

We find that combining screening and monitoring within a single active investor is optimal for investment projects that are characterized by a greater upside potential ("unicorns") and a larger downside risk ("lemons"). This happens because these projects allow firms to offer more effective incentive contracts, leading to lower delayed compensation and, thus, to reduce the cost of implementing optimal contracts.

The choice of financing arrangement also depends on the importance of entrepreneurial effort, that is, on the "human-capital intensity" of the investment project. We find that human-capital intensity has a non-monotonic effect on the optimal financing arrangement. Separating monitoring and screening is optimal for investment projects characterized by a human-capital intensity that is either sufficiently low or sufficiently large. In the first case, the double-deviation constraint does not bind. Low human-capital sensitivity raises the incentive pay necessary to induce monitoring and, correspondingly, increases the incremental surplus necessary to induce monitoring and screening in the single-investor case with no corresponding benefit at the interim date. The overall effect is to increase the cost of implementing incentive contracts with a single active investor relative to two active investors. In the second case, the double deviation constraint binds. High human-capital intensity lowers the incentive compensation necessary to induce monitoring with two active investors, reducing the cash flow benefit at the interim date of combining monitoring and screening. The effect is again to make the two-investor arrangement more desirable.

Finally, we show that delegating monitoring and screening to two separate active investors is more desirable when the market for informed capital is tighter (i.e. when the active investors' discount factor is lower). This property depends on the fact that combining monitoring and screening involves delaying payments to an active investor to later stages of project development, lengthening the duration of the investment. If the capital is committed for a longer period of time, that is particularly costly when informed capital is scarce.

Our paper is linked to the rapidly growing literature on optimal venture capital contracting.[5] In a seminal paper, Admati and Pfleiderer (1994) highlight the advantages of having an insider investor in a multistage financial contracting with information asymmetry. The optimal contract is equity, ensuring that the VC receives a constant percentage of the project's payoff while contributing the same percentage to future investments, thus aligning incentives and promoting optimal investment decisions. Bergemann and Hege (1998) studies stage financing in a dynamic agency model with learning and experimentation. They show that the optimal contract is a time-varying share contract that provides inter-temporal risk-sharing between the venture capitalist and entrepreneur. The entrepreneur's share reflects the value of a real option based on control of the funds and information flow. Neher (1999) proposes staged financing as a device to protect outsiders from being held up by the entrepreneur, who provides inalienable human capital (as in Hart and Moore (1994)).[6] Dewatripont and Maskin (1995) study the role of decentralized financing to reduce a time-inconsistency problem generated by soft budget constraints. The paper shows that decentralized financing hardens the ex-post budget constraint, reducing continuations of unprofitable projects, but at the cost of reducing the ex-ante incentives to monitor projects.

Several papers examine the problem of optimal security design in venture capital contracting. Cornelli and Yosha (2003) show that financing with convertible preferred stock reduces an entrepreneur's incentive to engage in inefficient "window dressing." Repullo and Suarez (2004) study a security design problem between an entrepreneur and a single investor under double-sided moral hazard. It shows that optimal contracts for the investor are convertible preferred stock, giving seniority on the bad states, and an option to convert into equity in the good states. Berglöf and Von Thadden (1994) show that raising funds through a mixture of short-term and long-term financing from different investors dominates raising funds from a single investor. The reason is that the separation of (senior) short-term and long-term claims reduces the entrepreneur's incentives to

[5] Da Rin et al. (2013), Lerner and Nanda (2023), and Janeway et al. (2021) offer excellent surveys of the literature.

[6] The effects of hold-ups and renegotiations in venture capital contracting are examined Fulghieri and Sevilir (2009a,b).

engage in harmful contract renegotiations. In contrast, contract renegotiation plays no role in our model. Hellmann and Thiele (2015) consider an equilibrium search-and-bargaining model where early-stage investment ("angel financing") interacts with later-stage VC financing by affecting incentives (through their effects on outside options and surplus extraction). Our model is a partial equilibrium, optimal contracting model where entrepreneurs capture all the surplus.

Our paper is closely related to Schindele (2006), in which the VC performs two types of tasks. Advising enhances the probability of success, whereas monitoring reduces potential losses but imposes costs on the entrepreneur. The paper shows that contracting with a multitasking financier allows the entrepreneur to borrow more than contracting with an advisor and a monitor separately. The reason is that advising and monitoring are strategic substitutes. In contrast, in our model, monitoring and screening ("advising") are strategic complements. The separation of these tasks exacerbates this conflict, reducing the entrepreneur's ability to raise funds.

The paper is organized as follows. In Section 2 we introduce the basic model. In Section **??**, we study the optimal financial arrangement with separation of monitoring and screening. Section 4 examines the optimal financing when monitoring and screening are combined in a single agent. In Section 5, we determine the optimal choice of the number of outside investors. Section 6 concludes the paper. All proofs are in the Appendix.

2 The Model

2.1 Agents and Technology

We consider a two-period model with three dates: $t \in \{0, 1, 2\}$. There are three groups of agents in our economy: one entrepreneur, active, and passive outside investors. All agents are risk-neutral and benefit from limited liability.

The entrepreneur (she) has access to an investment opportunity (the "project") that necessitates both entrepreneurial effort and capital. The project requires at $t = 0$ an initial capital expenditure $K_0 > 0$. Continuation of the project requires at $t = 1$ an additional follow-up investment $K_1 \geq 0$; if no additional investment is made, the project is liquidated at a fixed value $L < K_0$. Upon continuation, the project matures at $t = 2$, and it has either a high payoff, $X = X_H$ ("success"), or a low payoff, $X = X_L$ ("failure"), with $0 \leq X_L < L < X_H$ and $X_L < K_1$. The project success probability, p, depends on project quality. Projects can either be of "good" quality, with success probability p_G, or of "bad" quality, with success probability $p_B < p_G$.

Project quality is uncertain and depends on entrepreneurial human capital in the form of effort, e. We assume that the entrepreneur's effort is only needed at the initial stage of project execution and that no additional effort is needed for project continuation.[7] Specifically, by exerting effort at the initial stage, $t = 0$, the entrepreneur affects the probability, λ_e, that a project is of good quality. For simplicity, we assume that the entrepreneur can exert either high effort, $e = \bar{e}$, or low effort, $e = \underline{e}$, and thus that $\lambda_e \in \{\lambda_{\underline{e}}, \lambda_{\bar{e}}\}$. We interpret the difference $\lambda_{\bar{e}} - \lambda_{\underline{e}}$ as capturing the sensitivity of a project to entrepreneurial effort, and we will refer to it as the "human-capital intensity" of the investment project. The choice of effort is made privately by the entrepreneur, creating moral hazard. In particular, we assume that if the entrepreneur exerts low effort, she will earn an unpledgeable private benefit $b + \Delta b > 0$, with $b > 0$ and $\Delta b > 0$.

We assume that the entrepreneur has no initial wealth and must seek financing from outside investors who, in contrast, have deep pockets. Entrepreneurs and investors are characterized by different discount rates. The entrepreneur is impatient and discounts one-period-ahead cash flows using a discount factor δ_E. Passive investors are patient and do not discount future cash flows, setting the discount factor to 1. Active investors, in contrast, discount future cash flows at a smaller factor $\delta_E < \delta \leq 1$. The assumption that $\delta \leq 1$ captures the notion that active investors represent "informed capital," which may be scarce in the economy.[8] As such, active investors have a (weakly) greater opportunity cost of capital than passive investors. We interpret the difference $|\delta - 1|$ as a measure of the "tightness" of the market for informed capital in the economy.

[7]This assumption captures the phases of the R&D cycle that typically characterizes new ventures. Entrepreneurial human capital (or expertise) is critical at the early stages of the cycle for the identification of new business opportunities and the successful development of innovative technologies (the "research" stage). Once such hurdles are overcome, entrepreneurial human capital becomes less important. Rather, later stages of the R&D cycle require product development and commercialization, and depend on a more accurate assessment of its economic viability (the "development").

[8]The importance of informed capital in the economy is discussed in Holmstrom and Tirole (1997).

2.2 Active Investors and Project Valuation

Active investors are endowed with costly technologies to monitor and produce information on project quality. As in Hellmann (1998), we assume that at $t = 0$ an active investor can pay a private monetary cost c_M to reduce the entrepreneur's private benefit from shirking from $b + \Delta b$ to b, thus alleviating the moral hazard problem.[9] If an active investor chooses to monitor the entrepreneur, which we refer to as *monitoring,* it is not observed by other financiers.

Before the follow-up investment K_1 is made, an active investor can also obtain at the interim date, $t = 1$, a costly signal on project quality that can be used to assess its continuation value. We assume that by paying a monetary cost c_I, an active investor can obtain a binary signal Y on project quality. Both the action of acquiring the signal, which we refer to as *screening*, and its realization are not observed by other investors. For simplicity, we assume that the signal is perfectly informative on project's quality, $Y \in \{B, G\}$, and on the residual success probability $p \in \{p_G, p_B\}$. This implies that the continuation value of the project depends only on the realization of the signal, making it a sufficient statistic for entrepreneurial effort, e.[10] The difference $p_G - p_B$ affects the dispersion of project valuations after their initial stage and before the continuation decision is made. We interpret this difference as characterizing the degree of "innovativeness " of the investment project.[11]

In our paper, we focus on the more interesting case where it is optimal to monitor the entrepreneur (who exerts high effort), to screen the project, and to continue it only if a good signal is obtained. Accordingly, we will make the following parametric assumptions. Denote by $\mathbb{E}_Y[X] \equiv p_Y X_H + (1 - p_Y) X_L$ the expected value of the project payoff conditional on signal Y.

[9] Note that we formalize the entrepreneur's moral hazard problem as one with an effort choice. Alternatively, the moral hazard problem could equivalently be interpreted as one with a project choice, whereby the entrepreneur can choose among three different projects: Good, Bad, and BAD. The Good project has a probability of success $p_{\bar{e}}$ but no private benefit. The Bad project has a probability of success $p_{\underline{e}}$ and private benefit b. Finally, the BAD project has a probability of success $\pi_{\underline{e}}$ and private benefit $b + \Delta b$. Monitoring can eliminate the BAD project but not the Bad one. All results in our paper continue to hold in this project-choice setup.

[10] As a result, if a signal is obtained, the entrepreneur's compensation will only depend on the realization of the signal Y instead of the project's final outcome, X.

[11] For example, new and unproven technologies may be characterized by investment projects that are either potentially very valuable ("unicorns") or with little or no value ("lemons"). In contrast, mature industries are characterized by more homogeneous investment projects, with smaller valuation differences between "good" and "bad" projects.

Similarly, let $\mathbb{E}_e[X] \equiv \pi_e X_H + (1 - \pi_e) X_L$, where

$$\pi_e \equiv \lambda_e p_G + (1 - \lambda_e) p_B, \quad e \in \{\underline{e}, \bar{e}\},$$

denote the ex-ante expected value of the project payoff, conditional on the entrepreneur's choice of effort, e. Throughout the paper, we will make the following assumptions:

(A1) if the project is screened, it is optimal to liquidate it after a bad signal is observed and continue it after a good signal:

$$\mathbb{E}_B[X] - K_1 \leq L \leq \mathbb{E}_G[X] - K_1;$$

(A2) it is optimal to screen the project rather than to implement it without screening:

$$\lambda_e(\mathbb{E}_G[X] - K_1) + (1 - \lambda_e)L - c_I \geq \mathbb{E}_e[X] - K_1, \quad e \in \{\underline{e}, \bar{e}\};$$

(A3) it is optimal to exert effort:

$$(\lambda_{\bar{e}} - \lambda_{\underline{e}}) \left[\mathbb{E}_G[X] - L\right] \geq b + \Delta b.$$

Condition (A3)) ensures that the incremental project value created by exerting effort, $(\lambda_{\bar{e}} - \lambda_{\underline{e}}) \left[\mathbb{E}_G[X] - L\right]$, is greater than the entrepreneur's private benefits, $b + \Delta b$, making promotion of effort socially valuable.[12]

Note that in conditions (A1)-(A3), the project is valued from the point of view of the passive investor, who is the agent with the greatest discount factor. This feature depends on the fact that, as we will show later, it is optimal for the passive investor to be the residual claimant of the project's cash flow (due to his greater discount factor). Finally, note that under conditions (A1)-(A3), monitoring and screening are strategic complements.

2.3 Financing Arrangements and Securities

The entrepreneur must finance the investment project by raising capital from outside investors. Because of universal risk neutrality, without loss of generality, we can restrict ourselves to studying financing arrangements with at most three investors: an *early-stage* active investor (for example,

[12] We will later show that if the monitoring costs c_M are not too large, due to the differences in discount factors, it is optimal to induce entrepreneurial effort by monitoring rather than by giving entrepreneurs high-powered incentives (i.e, a sufficiently large equity retention). In addition, because monitoring is socially valuable and the entrepreneur captures all the surplus, there will not be "over monitoring" as in Pagano and Röell (1998).

"series A" investors) monitoring the entrepreneur at $t = 0$ (the "*monitor*"), a *late-stage* active investor (for example, "series B" investors) generating at $t = 1$ the signal on project quality, Y, before the follow-up investment is made, and a passive investor who does not conduct any service but provides cheap capital (due to his lower cost of capital). Monitoring and screening tasks may also be performed by a single active investor. A key question of our paper is whether the functions of monitoring and screening should be concentrated on a single active investor or, instead, should be delegated to two independent, active investors.

Let Ω denote the set of outside investors involved in the project undertakings. With a single active investor, $\Omega = \Omega_1 \equiv \{A, P\}$, which includes an active investor, A, engaging in both monitoring and screening, and a passive investor, P. In the case of two active investors, $\Omega = \Omega_2 \equiv \{M, I, P\}$, where one active investor acts as the monitor, M, and a second one as the late-stage investor, I, in addition to the passive investor, P. The entrepreneur is denoted as E.

At $t = 0$, the entrepreneur sets up a firm to undertake the investment project. The firm then issues securities specifying the cash flow and liquidation rights for all active and passive investors. The entrepreneur then retains the residual claim of the project's cash flow. Depending on the number of active investors, there are either two or three securities (namely, one for each active investor plus the passive investor), in addition to the entrepreneur's residual claim.

For each outside investor $i \in \Omega$, a security is a set $\mathcal{S}_i = \{\tau_{i0}, L_i, \tau_{i1}, S_i(X), \ell_i\}$ which specifies: (1) a transfer $\tau_{i0} \geq 0$ made by the investor to the firm at $t = 0$; (2) a payment L_i received by the investor if the project is liquidated at $t = 1$; (3) if the project is continued, an additional transfer τ_{i1} made by the investor to the firm at the interim date $t = 1$; (4) a final payment $S_i(X)$ received by the investor at $t = 2$, contingent on the realized payoff of the project $X \in \{X_L, X_H\}$; and (5) the allocation of the right to liquidate the project at $t = 1$, where $\ell_i = 1$ if the investor has the right to liquidate the project, and $\ell_i = 0$ otherwise. Note that we do not restrict the sign of the interim transfer at $t = 1$, and investor i can either receive a payment, with $\tau_{i1} < 0$, or be required to contribute funding to the firm, with $\tau_{i1} > 0$. We will, however, require each investor's continuation payoff to be positive so that he will not exit the contract at $t = 1$. Finally, we assume that $\forall i \in \Omega$

the securities satisfy limited liability and monotonicity:

$$S_i(X) \in [0, X], \ \sum_{i \in \Omega} S_i(X) \leq X, \text{ for } X \in \{X_H, X_L\}, \tag{1}$$

$$S_i(X_L) \leq S_i(X_H), \ \ X_L - \sum_{i \in \Omega} S_i(X_L) \leq X_H - \sum_{i \in \Omega} S_i(X_H), \tag{2}$$

$$L_i \geq 0, \quad \sum_{i \in \Omega} L_i \leq L. \tag{3}$$

In addition, securities $\mathcal{S}_i$ must satisfy the feasibility constraints:

$$\tau_{Et} + \sum_{i \in \Omega} \tau_{it} - K_t \geq 0, \quad t \in \{0, 1\} \tag{4}$$

where τ_{Et} is the transfer made from the entrepreneur to the firm at t. Note that $\tau_{Et} \leq 0$, for $t \in \{0, 1\}$ because the entrepreneur is penniless and therefore has nothing to transfer to the firm. We define the set of *admissible securities*, denoted by $\mathcal{S}$, as the set $\{\mathcal{S}_i\}_{i \in \Omega}$ that satisfy (1)-(4).

2.4 Project Timing and Payoffs

Project implementation unfolds as follows:

- At $t = 0$, after setting up the firm, the entrepreneur decides on whether to have one or two active investors and designs a corresponding set of securities $\{\mathcal{S}_i\}_{i \in \Omega} \in \mathcal{S}$. Securities are offered to each outside investor as a take-it-or-leave offer conditional on acceptance by all investors, allowing the entrepreneur to capture the entire surplus from the project. After raising capital, the firm makes the initial investment K_0 and distributes the residual cash flow to the entrepreneur, who consumes it. An active investor privately decides whether or not to spend the cost c_M and monitor the entrepreneur. The entrepreneur makes the effort choice $e \in \{\underline{e}, \bar{e}\}$.

- At $t = 1$, an active investor decides whether or not to screen the project and to acquire the signal Y on project quality. The project is either continued or liquidated. If the project is liquidated, proceeds from project liquidation are distributed to the entrepreneur and investors according to $\{L_i\}_{i \in \Omega}$. If the project is continued, a second round of financing takes place, whereby investors contribute $\{\tau_{i1}\}_{i \in \Omega}$, and the follow-up investment K_1 is made.

- At $t = 2$, if the project is not liquidated, cash flow X is realized, and payments from securities are settled.

The set of securities $\mathcal{S} \equiv \{\mathcal{S}_i\}_{i\in\Omega}$ determines the incentives for active investors to monitor and screen and for the entrepreneur to exert effort; as such, they will effectively function as the agents' incentive contracts.

Given securities $\mathcal{S} \equiv \{\mathcal{S}_i\}_{i\in\Omega}$, the entrepreneur's payoff at $t = 0$ is

$$U_E(e) \equiv b(e) - \tau_{E0} + \delta_E \left[-\lambda_e \tau_{E1} + (1 - \lambda_e)L_E\right] + \delta_E^2 \lambda_e \mathbb{E}_G \left[S_E(X)\right], \tag{5}$$

where $e \in \{\underline{e}, \bar{e}\}$, and $\lambda_e \in \{\lambda_{\underline{e}}, \lambda_{\bar{e}}\}$. The first term in (5) represents the entrepreneur's private benefit, which depends on the level of effort, e, and on monitoring by an outside investor, where $b(\bar{e}) = 0$, while $b(\underline{e}) = b$ with monitoring and $b(\underline{e}) = b + \Delta b$ without monitoring. The second term, $-\tau_{E0} \geq 0$, is the payout to the entrepreneur of the residual cash left to the firm at $t = 0$. The third and fourth terms represent the present value (at $t = 0$) of the entrepreneur's expected payoffs at $t = 1$ and $t = 2$, respectively, both valued using the entrepreneur's discount factor δ_E.

Similarly, payoff to investor $i \in \Omega$ at $t = 0$ is

$$U_i(e) \equiv -\tau_{i0} + \delta_i \left[-\lambda_e \tau_{i1} + (1 - \lambda_e)L_i\right] + \delta_i^2 \lambda_e \mathbb{E}_G \left[S_i(X)\right] - c_i, \tag{6}$$

where for active investor $i = M$ (the early-stage investor) we have $c_i = c_M$ and $\delta_i = \delta < 1$; for the active investor $i = I$ (the late-stage investor), we have $c_i = \delta c_I$ and $\delta_i = \delta < 1$; and for the passive investor $i = P$, we have $c_i = 0$ and $\delta_i = 1$. When monitoring and screening are combined, we have $c_i = \delta c_I + c_M$ and $\delta_i = \delta$ for $i = A$.

2.5 Individual Rationality Constraints

Given security offerings $\{\mathcal{S}_i\}_{i\in\Omega}$ and expecting high effort $\bar{e}$, the maximum amount τ_{i0} that investor i is willing to transfer to the firm at the initial date satisfies

$$\tau_{i0} \leq \delta_i(1 - \lambda_{\bar{e}})L_i + \lambda_{\bar{e}}\delta_i \left(-\tau_{i1} + \delta_i \mathbb{E}_G[S_i(X)]\right) - c_i, \quad \forall i \in \Omega, \tag{7}$$

giving the investors' ex-ante Individual Rationality (IR) constraints for $t = 0$. Moreover, because investor i can always choose to exit the contract at the interim date, continuation payoffs must be non-negative as well:

$$\begin{aligned} -\tau_{i1} + \delta_i \mathbb{E}_G[S_i(X)] &\geq 0, \quad L_i \geq 0, \qquad \text{for } i \in \{M, P\} & (8) \\ \lambda_{\bar{e}} \left[-\tau_{i1} + \delta \mathbb{E}_G[S_i(X)]\right] - c_I &\geq 0, \quad L_i \geq 0, \qquad \text{for } i \in \{I, A\} & (9) \end{aligned}$$

giving the interim IR constraints at $t = 1$.

Because outside investors are risk-neutral and have deep pockets, with all the bargaining power the entrepreneur sets the required transfer τ_{i0} to extract the maximum possible surplus from each outside investor, making their $t = 0$ IR constraints (7) to bind.[13]

By substitution of (7) into (5), we obtain that in an optimal contract with high effort $e = \bar{e}$, monitoring and screening, the entrepreneur's payoff at $t = 0$ is given by

$$U_E(\bar{e}) \equiv V_N - \sum_{i \in \{E\} \cup \{\Omega \setminus P\}} \left\{ (1 - \delta_i) \left(-\lambda_{\bar{e}} \tau_{i1} + (1 - \lambda_{\bar{e}}) L_i\right) + (1 - \delta_i^2) \lambda_{\bar{e}} \mathbb{E}_G[S_i(X)] - (1 - \delta) c_I \right\} \tag{10}$$

where $V_N \equiv \lambda_{\bar{e}} \left[\mathbb{E}_G(X) - K_1\right] + (1 - \lambda_{\bar{e}}) L - c_M - c_I - K_0$. Entrepreneur's payoff $U_E(\bar{e})$ has two components. The first one, V_N, represents the present value of the project's expected cash flow, net of required investments, monitoring, and screening costs, that is, the overall Net Present Value of the project. For future reference, define

$$V_0 \equiv \lambda_{\bar{e}} \left[\mathbb{E}_G(X) - K_1\right] + (1 - \lambda_{\bar{e}}) L - c_I, \tag{11}$$

as the first-best "post-money" valuation (at $t = 0$) of the project, which represents its continuation value (which is net of the anticipated screening cost, c_I). Both V_N and V_0 are valued using the passive investor's discount factor.

The second component of the entrepreneur's payoff $U_E(\bar{e})$ reflects the impact of the differences of discount factors between the passive investor, $\delta = 1$, and the other agents, $\delta_i < 1$ for $i \in \{E\} \cup \{\Omega \setminus P\}$. The difference in discount factors has two opposing effects on the entrepreneurs' payoff. The first is negative and is due to the inefficiencies of delaying payments to the entrepreneur and active investors in their incentive contracts. Incentive provision requires a delay of the compensation either at the interim date, after the signal Y is observed, or at the last date, after project payoff is realized. Delaying such payments is costly because entrepreneurs and active investors value

[13] In contrast, it may happen that in incentive-compatible contracts the interim IR constraints (8) and (9) may not bind, allowing investors to earn a surplus rent at the interim date, which is then paid upfront to the entrepreneur, making (7) to bind.

delayed payments less than the passive investor (who is effectively the residual claimant). These inefficiencies represent a necessary cost of incentive provision in our model. The second effect, captured by the term $(1-\delta)c_I$, is positive and is the converse of the first one. It reflects the fact that the active investor screening the project incurs the screening cost c_I only at the interim period, $t=1$. Such delay is valuable because the passive investor pays the active investor at $t=0$ only δc_I as compensation for the anticipated screening cost (to satisfy the IR constraint).

Expression (10) plays the key role in our analysis: The entrepreneur's problem will be to minimize the overall cost of incentive provision necessary to undertake the project.

3 Rounds Financing: Separate Monitoring and Screening

We consider first the case where monitoring and screening are delegated to two different active investors, as in "rounds financing. We first characterize the solution to the corresponding optimal security design problem. In Section 3.3, we discuss its implementation through a sequential offering of securities.

3.1 Incentive constraints

The incentive compatibility (IC) constraints that incentivize high effort, monitoring, screening, and where projects are continued at $t=1$ only after a good signal are as follows.

(i) *Entrepreneur incentive constraint.* From (5), with monitoring, the IC constraint for the entrepreneur is

$$-\tau_{E1} + \delta_E \mathbb{E}_G[S_E(X)] - L_E \geq \frac{b}{\delta_E(\lambda_{\bar{e}} - \lambda_{\underline{e}})}. \tag{12}$$

Note that, in alternative to monitoring, the entrepreneur is induced to exert effort by setting

$$-\tau_{E1} + \delta_E \mathbb{E}_G[S_E(X)] - L_E \geq \frac{b+\Delta b}{\delta_E(\lambda_{\bar{e}} - \lambda_{\underline{e}})}, \tag{13}$$

and, thus, dispensing with the monitor. In Proposition 3 we will show that, if the monitoring costs c_M are not too large, it is optimal to monitor rather than addressing the moral hazard problem through high-powered incentives.

(ii) *Monitoring incentive constraint.* From (6), and setting $i=M$, the IC constraint for the early-stage investor to pay the monitoring cost, c_M, and monitor the entrepreneur is

$$-\tau_{M1} + \delta\mathbb{E}_G[S_M(X)] - L_M \geq \frac{c_M}{\delta(\lambda_{\bar{e}} - \lambda_{\underline{e}})}. \tag{14}$$

(iii) *Screening incentive constraint.* From (6), and setting $i = I$, the IC constraint for the late-stage investor to pay the screening cost, c_I, and generate the signal Y is

$$\lambda_{\bar{e}} \left[-\tau_{I1} + \delta\mathbb{E}_G[S_I(X)]\right] + (1 - \lambda_{\bar{e}})L_I - c_I \geq \max\left[-\tau_{I1} + \delta\mathbb{E}_{\bar{e}}[S_I(X)], L_I\right]. \tag{15}$$

Note that the left-hand side of (15) reflects the fact that, with screening, the project is continued if the investor receives a good signal, and is liquidated if he receives a bad signal. The right-hand side of (15) reflects the fact that, absent screening, the investor may either obtain the unconditional expected value of the contract upon uninformed continuation, $-\tau_{I1} + \delta\mathbb{E}_{\bar{e}}[S_I(X)]$, or receive L_I if the project is liquidated. Note that IC constraint (15) requires that the investor must prefer to continue the project after obtaining a good signal and to liquidate it otherwise, that is

$$-\tau_{I1} + \delta\mathbb{E}_B[S_I(X)] \leq L_I \leq -\tau_{I1} + \delta\mathbb{E}_G[S_I(X)]. \tag{16}$$

Condition (16), obtained from letting $c_I = 0$ in (15), implies that information production is valuable: if the project is either always continued or liquidated, independently of signal realization, there is no benefit from producing information, and the incentive constraint (15) cannot be satisfied.[14]

3.2 Optimal security design

We first establish some preliminary results that will greatly simplify the analysis.

Lemma 1 *Optimal securities* $\mathcal{S}_i = \{\tau_{i0}, L_i, \tau_{i1}, S_i(X), \ell_i\}_{i\in\Omega_2}$ *satisfy:*

(i) $S_E(X_L) = S_E(X_H) = 0$, *and* $L_E = 0$*;*

(ii) $S_M(X_L) = S_M(X_H) = 0$, *and* $L_M = 0$*;*

(iii) $S_I(X_L) = 0$, *and* $L_I = 0$*;*

(iv) $L_P = L$.

[14] Note also that the incentive constraint (15) can be written as

$$c_I \leq \lambda_{\bar{e}} \left[-\tau_{I1} + \delta\mathbb{E}_G[S_I(X)]\right] + (1 - \lambda_{\bar{e}})L_I - \max\left\{-\tau_{I1} + \delta\mathbb{E}_{\bar{e}}[S_I(X)]; L_I\right\}$$

The above expression has the natural interpretation that the late-stage investor has the incentive to produce information when the cost of information production, c_I, is no greater than the (Blackwell) value of information.

These properties reflect the feature that, due to differences in discount rates of passive investors and the other agents, the most efficient way to satisfy the incentive constraints (12)-(15) is to minimize delays in compensation. They may be seen as follows.

Property (i) derives from the fact that the entrepreneur's incentive constraint (12) depends on the difference of payoffs between continuing and liquidating the project: $-\tau_{E1}+\delta_E\mathbb{E}_G[S_E(X)]-L_E$. From the entrepreneur's payoff (10), delaying compensation to the last period is costly, making it optimal to set $S_E(X_L) = S_E(X_H) = 0$ and to meet the IC constraint by correspondingly reducing the interim transfer τ_{E1}, giving the first part of (i). This implies that interim payoff depends now on the difference $-\tau_{E1} - L_E$ and that it is again optimal to have $L_E = 0$, and to set τ_{E1} at the (maximum) value that satisfies the incentive constraint (12), giving the second part of (i). Similarly, the monitor's IC constraint (14) depends only on the difference $-\tau_{M1} + \delta\mathbb{E}_G[S_M(X)] - L_M$. This implies that it is again more efficient to meet the monitor's IC by setting $S_M(X_L) = S_M(X_L) = L_M = 0$, and then by setting τ_{M1} to the (maximum) value that satisfies (14), giving (ii).

The remaining properties are obtained as follows. After subtracting L_I from both sides of (15), we can rewrite the IC constraint for screening as

$$\begin{aligned} & \lambda_{\bar{e}}\left[-\tau_{I1} + \delta p_G[S_I(X_H) - S_I(X_L)] + \delta S_I(X_L) - L_I\right] - c_I \\ \geq \quad & \max\left\{-\tau_{I1} + \delta\pi_{\bar{e}}[S_I(X_H) - S_I(X_L)] + \delta S_I(X_L) - L_I; 0\right\}. \end{aligned} \tag{17}$$

Similar to the previous cases, the IC constraint (17) depends on the difference $-\tau_{I1}+\delta S_I(X_L)-L_I$. Due to the differences in time discount factors of active and passive investors, it is again more efficient to meet (17) by setting $L_I = S_I(X_L) = 0$. The IC constraint (15) then becomes

$$-\tau_{I1} + \delta\mathbb{E}_G[S_I(X)] \geq \frac{c_I}{\lambda_{\bar{e}}} + \frac{1}{\lambda_{\bar{e}}}\max\left\{-\tau_{I1} + \delta\mathbb{E}_{\bar{e}}[S_I(X)]; 0\right\}. \tag{18}$$

Note that, because we will show that in an optimal contract $\tau_{I1} > 0$, condition (18) requires that to induce screening the late-stage investor must have exposure to project risk in the last period, with $S_I(X_H) > 0$. Finally, part (iv) follows immediately from (i)-(iii).

Intuitively, Lemma 1 implies that delaying any payment to the entrepreneur or to the early-stage investor to the last period is suboptimal, given that both the entrepreneur and early-stage investor are more impatient than the passive investor, and that no new information on either effort or monitoring is obtained in the last period. In addition, offering any liquidation proceeds to the entrepreneur or to the active investors is also suboptimal, as it would weaken their incentives to exert effort, monitor, or screen.

Lemma 1 has the following appealing interpretation. Because the IR constraints will bind in an optimal contract, an increase in delayed payments in the "bad" states, L_i or $S_i(X_L)$ for $i \in \{M, I\}$, results in an increase of the up-front payments to the entrepreneur, τ_{i0}, making them equivalent to a "safe loan" (from an active investor to the entrepreneur). An increase of these delayed payments, however, leads to a corresponding reduction of payments to the passive investor (who is effectively a residual claimant) and thus to a lower up-front payment, τ_{P0}, to the entrepreneur (again a "safe loan," now from the entrepreneur to the passive investor). The combined effect is effectively a safe loan from active to passive investors. Because active investors have a lower discount factor than passive investors, such loans are inefficient.

Lemma 2 *The IC constraints for the early-stage investor (14) and the late-stage investor (18) are satisfied if and only if*

$$
\begin{aligned}
-\tau_{M1} + \delta \mathbb{E}_G[S_M(X)] &\geq \frac{c_M}{\delta(\lambda_{\bar{e}} - \lambda_{\underline{e}})}, &(19)\\
-\tau_{I1} + \delta \mathbb{E}_G[S_I(X)] &\geq \frac{c_I}{\lambda_{\bar{e}}}, &(20)\\
-\tau_{I1} + \delta \frac{p_B}{p_G} \mathbb{E}_G[S_I(X)] &\leq -\frac{c_I}{(1-\lambda_{\bar{e}})}. &(21)
\end{aligned}
$$

Constraint (19) requires that, to induce monitoring, the early-stage investor must expect to receive a minimum payoff if the project is continued. In contrast, constraints (20) and (21) require that the continuation payoff for the late-stage investor must be bounded from both below and above. The lower bound in (20) reflects the (rather obvious) fact that, if the continuation payoff is too low, the investor has too little at stake to be willing to sustain the screening cost c_I (note that the late-stage investor receives compensation for screening only if a good signal is observed, which happens with probability $\lambda_{\bar{e}}$). In contrast, the upper bound in (21) reflects the fact that if the continuation payoff is too high, the late-stage investor will have the incentive to continue the project as uninformed, and thus getting the continuation payoff for sure, rather than screening the project, and obtain the continuation payoff only with probability $\lambda_{\bar{e}}$. Constraints (20) and (21) are (respectively) represented in Figure 1.

Several additional features are worth noting. First, the upper bound in (21) can only be satisfied

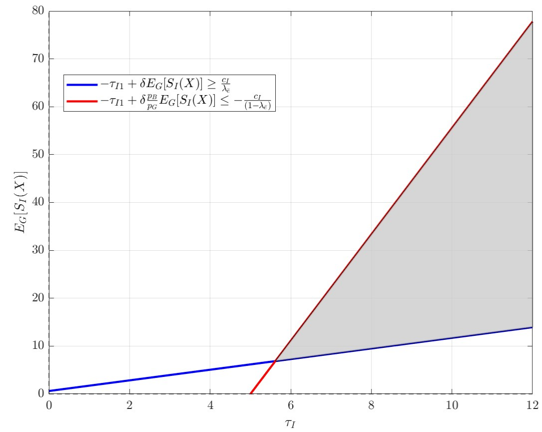

Figure 1: **IC Constraints of Late-Stage Investor.** The red line represents the constraint $-\tau_{I1} + \delta\mathbb{E}_G[S_I(X)] \geq \frac{c_I}{\lambda_{\bar{e}}}$, while the blue line represents the constraint $-\tau_{I1} + \delta\frac{p_B}{p_G}\mathbb{E}_G[S_I(X)] \leq -\frac{c_I}{(1-\lambda_{\bar{e}})}$. The x-axis corresponds to τ_{I1}, and the y-axis represents $\mathbb{E}_G[S_I(X)]$. The shaded region indicates the area where both IC constraints are simultaneously satisfied.

if $\tau_{I1} > 0$. This means that incentive compatibility for screening has a "certification" feature: the late-stage investor can credibly communicate that he received a good realization of the signal Y by being willing to make the up-front payment τ_{I1} to the firm. Further, note that the IC constraint (20) coincides with the interim IR constraint for the early investor (9).

Second, note that entrepreneurial effort affects in two opposing ways the incentives to screen the project and, thus, the cost of incentive provision for late-stage investors. On the one hand, entrepreneurial effort relaxes the lower bound in (20): greater effort increases the probability that the late-stage investor receives a good signal and, thus, that the project is continued (with a positive payoff to the investor) rather than liquidated (with zero payoff). This improves the investor's expected return from information production (i.e., the value of information), relaxing the incentive constraint. On the other hand, entrepreneurial effort tightens the upper bound in (21): greater effort increases the probability that the project has a high payoff, increasing the expected return from continuing the project as uninformed. The combined effect of entrepreneurial effort on the transfer payment τ_{I1} is therefore ambiguous.

Finally, from the IC constraints for the entrepreneur (12), and the early- and late-stage investors (19)-(20), define

$$\mathcal{R}_E \equiv \frac{b}{\delta_E(\lambda_{\bar{e}} - \lambda_{\underline{e}})}, \quad \mathcal{R}_M \equiv \frac{c_M}{\delta(\lambda_{\bar{e}} - \lambda_{\underline{e}})}, \quad \mathcal{R}_I \equiv \frac{c_I}{\lambda_{\bar{e}}}. \tag{22}$$

The terms $\mathcal{R}_i$, for $i \in \{E, M, I\}$, represent the minimum payoffs (i.e., the "rents") that are necessary to induce, respectively, the entrepreneur to exert high effort, the early-stage investor to monitor, the late-stage investor to screen the project. At $t = 0$ the entrepreneur solves the following optimal security design problem, denoted by $\mathbb{P}_2$:

$$\begin{aligned}
\min_{\{S_i\}\in\mathcal{S}} \mathbb{C}(S_i) \quad &\equiv \quad \sum_{i\in\{E,M,I\}} (1-\delta_i)\left(-\lambda_{\bar{e}}\tau_{i1} + (1-\lambda_{\bar{e}})L_i\right) + (1-\delta_i^2)\lambda_{\bar{e}}\mathbb{E}_G[S_i(X)] \\
s.t. \quad -\tau_{M1} + \delta\mathbb{E}_G[S_M(X)] \quad &\geq \quad \mathcal{R}_M \\
-\tau_{I1} + \delta\mathbb{E}_G[S_I(X)] \quad &\geq \quad \mathcal{R}_I \\
-\tau_{I1} + \delta\frac{p_B}{p_G}\mathbb{E}_G[S_I(X)] \quad &\leq \quad -\frac{c_I}{(1-\lambda_{\bar{e}})} \\
-\tau_{E1} + \delta_E\mathbb{E}_G[S_E(X)] \quad &\geq \quad \mathcal{R}_E \\
\tau_{i0} \quad &\leq \quad \delta_i(1-\lambda_{\bar{e}})L_i + \lambda_{\bar{e}}\delta_i\left(-\tau_{i1} + \delta_i\mathbb{E}_G[S_i(X)]\right) - c_i \quad \forall i \in \Omega_2 \\
-\tau_{i1} + \delta_i\mathbb{E}_G[S_i(X)] \quad &\geq \quad 0, \quad \forall i \in \{M, P\}, \\
\lambda_{\bar{e}}\left[-\tau_{i1} + \delta_i\mathbb{E}_G[S_i(X)]\right] - c_I \quad &\geq \quad 0, \quad i = I.
\end{aligned}$$

The solution to $\mathbb{P}_2$, $\mathcal{S}_2^{**} = \{\tau_{i0}^{**}, L_i^{**}, \tau_{i1}^{**}, S_i^{**}(X), \ell_i^{**}\}_{i\in\Omega_2}$ (where two "stars" denote the two active investors case) is characterized as follows.

Proposition 3 *The optimal security offering with two active investors, $\mathcal{S}_2^{**}$, that solves $\mathbb{P}_2$ has:*
(i) for the early-stage investor:

$$S_M^{**}(X_H) = S_M^{**}(X_L) = 0, \quad L_M^{**} = 0, \quad \ell_M^{**} = 0 \tag{23}$$

$$\tau_{M1}^{**} = -\mathcal{R}_M = -\frac{c_M}{\delta(\lambda_{\bar{e}} - \lambda_{\underline{e}})} \tag{24}$$

$$\tau_{M0}^{**} = -\delta\lambda_{\bar{e}}\tau_{M1}^{**} - c_M = \frac{\lambda_{\underline{e}}c_M}{\lambda_{\bar{e}} - \lambda_{\underline{e}}}; \tag{25}$$

(ii) for the late-stage investor:

$$S_I^{**}(X_L) = 0, \quad S_I^{**}(X_H) = \frac{c_I}{\delta\lambda_{\bar{e}}(1-\lambda_{\bar{e}})(p_G - p_B)}, \; L_I^{**} = 0, \quad \ell_I^{**} = 0 \tag{26}$$

$$\tau_{I1}^{**} = \delta p_G S_I^{**}(X_H) - \mathcal{R}_I = \frac{c_I}{(p_G - p_B)}\frac{\pi_{\bar{e}}}{\lambda_{\bar{e}}(1-\lambda_{\bar{e}})}, \tag{27}$$

$$\tau_{I0}^{**} = 0; \tag{28}$$

(iii) for the passive investor:

$$S_P^{**}(X_L) = X_L, \quad S_P^{**}(X_H) = X_H - S_I^{**}(X_H), \quad L_P^{**} = L, \quad \ell_P^{**} = 1 \tag{29}$$

$$\tau_{P1}^{**} = K_1 - \left(\tau_{E1}^{**} + \tau_{M1}^{**} + \tau_{I1}^{**}\right), \tag{30}$$

$$\tau_{P0}^{**} = \lambda_{\bar{e}}\left[X_L + p_G\left(X_H - S_I^{**}(X_H) - X_L\right) - \tau_{P1}^{**}\right] + (1-\lambda_{\bar{e}})L = \tag{31}$$

$$= V_0 - \lambda_{\bar{e}}\left[-\tau_{E1}^{**} - \tau_{M1}^{**} + (1-\delta)p_G S_I^{**}(X_H)\right], \tag{32}$$

where V_0 is the continuation value of the project, defined in (11);

(iv) for the entrepreneur:

$$S_E^{**}(X_H) = S_E^{**}(X_L) = 0, \quad L_E^{**} = 0, \quad \ell_E^{**} = 0 \tag{33}$$

$$\tau_{E1}^{**} = -\mathcal{R}_E = -\frac{b}{\delta_E(\lambda_{\bar{e}} - \lambda_{\underline{e}})} \tag{34}$$

$$\tau_{E0}^{**} = K_0 - \sum_{i\in\Omega_2} \tau_{i0}^{**}. \tag{35}$$

The entrepreneur's payoff is

$$U_E^{**} = -\tau_{E0}^{**} - \delta_E\lambda_{\bar{e}}\tau_{E1}^{**} = V_N - (1-\delta)\lambda_{\bar{e}}\left[\mathcal{R}_M + p_G S_I^{**}(X_H)\right] - (1-\delta_E)\lambda_{\bar{e}}\mathcal{R}_E.$$

The optimal contract is implementable if and only if

$$V_0 - c_M - (1-\delta)\lambda_{\bar{e}}\left[\mathcal{R}_M + p_G S_I^{**}(X_H)\right] - \lambda_{\bar{e}}\mathcal{R}_E \geq K_0 \tag{36}$$

$$X_H - S_I^{**}(X_H) \geq X_L. \tag{37}$$

In the optimal contract:

(i) The early-stage investor monitors the entrepreneur and receives at the interim date a positive cash flow, $-\tau_{M1}^{**}$, if the project is continued, and zero payoff if it is liquidated, $L_M^{**} = 0$. At $t = 0$, the early-stage investor pays the entrepreneur τ_{M0}^{**}, which is equal to the present value of the compensation expected at $t = 1$, discounted at the factor δ, net of the monitoring cost c_M. The early-stage investor receives no additional payoff from the project: he effectively "exits" the project as he has no more role to play in the project continuation.

(ii) At the interim date, the late-stage investor produces information and obtains the signal Y. If the signal is good, the investor contributes capital τ_{I1}^{**} toward the financing of the follow-up investment of the project, K_1, and receives a compensation at $t = 2$ only if the project has a high payoff, $S_I^{**}(X_H)$. The interim payment τ_{I1}^{**} is the present value of the compensation expected for time $t = 2$, discounted at the factor δ and reduced by the compensation needed to recover the screening cost, $\mathcal{R}_I = c_I/\lambda_{\bar{e}}$ (which happens only if the investor receives a good signal and the project is continued). If the signal is bad, $Y = B$, the late-stage investor does not contribute any capital to the firm. The project is liquidated and the investor receives no payment, $L_I^{**} = 0$. Finally, the late-stage investor makes no capital contributions at $t = 0$, and $\tau_{I0}^{**} = 0$. This feature is due to the fact that in the optimal contract both the IC constraint (20) and the interim IR constraint (9) bind, and the early investor is not required to make any up-front payment at the initial date, $t = 0$.

(iii) The passive investor contributes at the initial stage an amount τ_{P0}^{**} which is equal to the continuation value of the investment project V_0 (which, from (11), is already net of the screening cost c_I), and is reduced by the expected payment to entrepreneur and early-stage investor and by the dissipative costs of the compensation to the late-stage investor (due to the difference in their discount factors, $1-\delta$). At the interim stage, if the project is liquidated, the passive investor receives the full liquidation value $L_P^{**} = L$. If the project is continued, the passive investor contributes an amount of capital, τ_{P1}^{**}, that is necessary to compensate the entrepreneur, $-\tau_{E1}^{**}$, the early-stage investor, $-\tau_{M1}^{**}$, and the financing of the follow-up investment K_1, net of the capital contribution from the late-stage investor, τ_{I1}^{**}. The passive investor's payoff at the final date depends on the project payoff. If the project has a low payoff, X_L, the passive investor receives the full project value, $S_P^{**}(X_L) = X_L$. If the project has a high payoff, the passive investor pays the late-stage investor $S_I^{**}(X_H)$ and retains the residual value, $S_P^{**}(X_H) = X_H - S_I^{**}(X_H)$.

(iv) Finally, the entrepreneur exerts high effort and receives at the interim date a compensation $-\tau_{E1}^{**} > 0$ if the project is continued, and no payment of the project is liquidated, $L_E^{**} = 0$. Similar to the case of the early-stage investor, the entrepreneur receives no additional payoff from the project, and she also effectively "exits" the project as she has no more role to play in project continuation. At $t = 0$, the entrepreneur receives the residual cash flow from the firm, after the capital contributions from the active and passive investors and the initial capital expenditure are made, $-\tau_{E0}^{**} = \sum_{i \in \Omega_2} \tau_{i0}^{**} - K_0$. Finally, the total payoff to the entrepreneur, U_E^{**}, is equal to the overall net present value of the investment project as valued by the passive investor, V_N, reduced by the dissipative costs of delayed expected compensation to the early- and late-stage investors,

$(1-\delta)\lambda_{\bar{e}}\left[-\tau^{**}_{M1} + p_G S^{**}_I(X_H)\right]$, and for her own expected compensation, $(1-\delta_E)\lambda_{\bar{e}}(-\tau^{**}_{E1})$.

The optimal contract has the simple interpretation whereby the entrepreneur effectively "sells" at $t=0$ the project to the passive investor, who is the investor with the highest discount factor and, thus, valuing the project the most. The passive investor then "hires back" the entrepreneur to exert the initial effort e, and two active investors, where the early-stage investor is tasked with monitoring the entrepreneur, and the late-stage investor is tasked with screening the project. Note that the late-stage investor at the intermediate date $t=1$ must pay τ_{I1} to participate in the project.

The optimal contract (23)-(35) is implementable if conditions (36)-(37) are satisfied. Condition (36) ensures that the firm is able to raise at $t=0$ enough capital to cover the investment expenditure and, thus, that the net payment to the entrepreneur at the initial date is non-negative: $-\tau^*_{E0} = \sum_{i\in\Omega_1} \tau^*_{i0} - K_0 \geq 0$. It requires that the continuation value of the project, V_0, net of the monitoring cost, c_M, of the expected contractual compensation to the entrepreneur, $\lambda_{\bar{e}}\mathcal{R}_E$, and of the expected dissipative costs to the early- and late-stage investors, $(1-\delta)\lambda_{\bar{e}}[\mathcal{R}_M + p_G S^{**}_I(X_H)]$, is sufficient large to cover the initial investment K_0. This condition ensures that there is sufficient residual expected cash flow from the project that can be pledged to the passive investor, making him willing to provide sufficient financing at the initial stage. This condition also guarantees that the entrepreneur's payoff U^{**}_E is non-negative.

Condition (37) ensures that the monotonicity condition for the passive investor's payoff is satisfied. From (29), we have that $S^{**}_P(X_L) = X_L$, giving $S^{**}_P(X_H) = X_H - S^{**}_I(X_H) \geq X_L = S^{**}_P(X_L) \geq 0$. This condition also guarantees that the passive investor's limited liability constraint at $t=2$ is satisfied. The monotonicity and limited liability conditions for the late-stage investor are easily verified, because $S^{**}_I(X_H) > S^{**}_I(X_L) = 0$.

Implementability of optimal contracts is established in the following.

Proposition 4 *There are critical values $\hat{\delta}^{**}$ and $\hat{c}^{**}_I$ (both defined in the appendix) such that if $\delta \geq \hat{\delta}^{**}$ and $c_I \leq \hat{c}^{**}_I$, then contract $\mathcal{S}^{**}_2$ is implementable.*

Contract implementability requires that the active investors' discount factor δ is not too small, $\delta \geq \hat{\delta}^{**}$, and that the screening cost is not too large, $c_I \leq \hat{c}^{**}_I$. This happens because, from (26) and (22), discount factors that are too low or screening costs that are too large, lead to payoffs $\mathcal{R}_M$

and $S_I^{**}(X_H)$ that are sufficiently large to violate (36) and (37).

3.3 Implementation with securities

Of particular interest is the fact that the optimal contract can be implemented through a sequence of securities offerings, as follows.

At $t = 0$, the entrepreneur raises capital by selling the following securities to outside investors:

(i) *To the monitor*: A fraction equity $\alpha_M = \tau_{M1}^{**}/V_G$ to the early-stage investor M, at a value τ_{M0}^{**}, and retains the remainder $1 - \alpha_M$, where the term $V_G \equiv \mathbb{E}[X] - K_1 - (1-\delta)\mathbb{E}[S_I^{**}(X)]$ represents the (pre-money) continuation value of the project at $t = 1$ after a good signal is obtained.[15]

(ii) *To the passive investor*: A package of securities composed of (senior) secured debt, options, and a credit line with the following features: (senior) secured debt maturing at $t = 1$ with promised payment (face value) equal to L; the credit line has a total draw-down value τ_{P1}^{**}. Debt and credit line are secured by the firm's assets, and the passive investor has at $t = 1$ the liquidation rights, $\ell_P^{**} = 1$ (in which case, the credit line is terminated). The passive investor also has the option to buy shares held by the entrepreneur and the early-stage investor at the strike price $-\tau_{E1}^{**}$ and $-\tau_{M1}^{**}$, respectively. The combined value of the package of securities is τ_{P0}^{**}, which is paid up-front to the firm.

After the firm raises $\tau_{M0}^{**} + \tau_{P0}^{**}$ from investors, it makes the first-round investment K_0, and then pays the residual funds to the entrepreneur.

At $t = 1$, the following actions take place:

The firm must raise capital to finance the follow-up investment K_1, and it announces the intention to make an offer to issue preferred stock (mezzanine financing) with face value $F_I =$

[15]This may be seen by noting that, if a good signal is obtained at $t = 1$, the passive investor acquires 100% of the equity in the firm from the entrepreneur and the monitor, which he values at

$$V_G \equiv \mathbb{E}_G[X] - K_1 + \tau_{I1}^{**} - \mathbb{E}_G[S_I^{**}(X)] = \mathbb{E}_G[X] - K_1 - (1-\delta)\mathbb{E}[S_I^{**}(X)].$$

This valuation reflects the fact that the late-stage investor pays τ_{I1}^{**} to the firm while retaining a claim on the firm's final cash flow that the passive investor values at $\mathbb{E}_G[S_I^{**}(X)]$.

$S_I^{**}(X_H)$ at a price τ_{I1}^{**}.[16]

The late-stage investor must decide whether or not to subscribe to the preferred stock offer; he spends c_I to obtain the signal Y. If the signal is bad, $Y = B$, the late-stage investor does not subscribe to the offer of preferred equity, and no capital is contributed; in this case, the passive investor, who has the liquidation rights, decides to liquidate the firm. As a senior secured creditor, the passive investor's payoff is the full liquidation value, L. All other parties have zero payoff.

If the signal is good, $Y = G$, the late-stage investor subscribes to the newly issued preferred stock and pays the firm τ_{I1}^{**}. The passive investor decides not to liquidate the project (i.e., to let it continue) and exercises the option to buy shares held by the entrepreneur and the early-stage investor for $-\tau_{E1}^{**}$ and $-\tau_{M1}^{**}$, respectively. The firm draws τ_{P1}^{**} on the credit line and invests the follow-up capital K_1. Existing debt held by the passive investor is rolled over, and refinanced with new senior debt held by the passive investor with face value equal $F_P \in (L, X_H - S_I^{**}(X_H))$.[17]
At $t = 2$, final payoffs are realized. The passive investors hold both senior debt and 100% of equity. If $X = X_L$, the passive investor, holding senior debt with face value $F_P > X_L$, retains the full project payoff, X_L. The late-stage investor has zero payoff. If $X = X_H$, the passive investor will pay the late-stage investor the face value of the preferred stock (mezzanine), $S_I^{**}(X_H)$, and will retain the remainder, with payoff $F_P + (X_H - F_I - F_P) = X_H - S_I^{**}(X_H)$.

4 Milestone Financing: Combined Monitoring and Screening

Under "milestone financing" monitoring and screening are combined and delegated to a single active investor, denoted by A, and the set of outside investors is $\Omega = \Omega_1 \equiv \{A, P\}$.

4.1 Incentive constraints

An important difference with the two active investors case is that when monitoring and screening functions are combined in a single active investor, an additional IC constraint must be satisfied.

[16] Note that from (26) this security payoff in the good state (i.e., on the "upside") is a fixed payment which does not depend on project's payoff X_H. This feature makes this security closely resemble preferred stock rather than common stock, whose return depends on the realization of the project's payoff in the good state, X_H, as in (29).

[17] Since the passive investor holds both senior debt and common stock, the debt's face value becomes less relevant - it is effectively an internal transfer within the passive investor's own portfolio. Any face value F_P that falls within the range $(L, X_H - S_I^*(X_H))$ would be consistent with the implementation of the optimal contact.

Specifically, a monitoring constraint alone ensures that the active investor has the incentive to monitor at $t = 0$, if he anticipates that he will screen the project at $t = 1$. Similarly, a screening constraint alone ensures that, if the investor has monitored the project at $t = 0$, he will have the incentive to screen the project at $t = 1$. The additional constraint requires that the active investor has the incentives to *both* monitor *and* screen the project, rather than deviate and *neither* monitor *nor* screen. Under contract $S_A = \{\tau_{A1}, S_A(X), L_A, \ell_A\}$, this gives

$$\lambda_{\bar{e}}\delta\left[-\tau_{A1} + \delta\mathbb{E}_G[S_A(X)]\right] + (1-\lambda_{\bar{e}})L_A - c_M - \delta c_I \geq \delta \max\left\{-\tau_{A1} + \delta\mathbb{E}_{\underline{e}}[S_A(X)]; L_A\right\}. \tag{38}$$

Note that the active investor will continue the project as uninformed when

$$\max\left\{-\tau_{A1} + \delta\mathbb{E}_{\underline{e}}[S_A(X)]; L_A\right\} = -\tau_{A1} + \delta\mathbb{E}_{\underline{e}}[S_A(X)],$$

and liquidate it otherwise. We will denote (38) as the *double-deviation* constraint.

4.2 Optimal security design

Similar to the case of separating monitoring and screening, optimal securities satisfy the following.

Lemma 5 *Optimal securities* $\mathcal{S}_i = \{\tau_{i0}, L_i, \tau_{i1}, S_i(X), \ell_i\}_{i\in\Omega_1}$ *satisfy:*

(i) $S_A(X_L) = 0$, *and* $L_A = 0$

(ii) $S_E(X_L) = S_E(X_H) = 0$, *and* $L_E = 0$*;*

(iii) $L_P = L$.

Lemma 5 reflects the fact that, due to the differences in discount factors, it is again desirable to minimize delayed compensations. Property (i) may be seen as follows. After subtracting L_A on both sides of (38), the double-deviation constraint becomes

$$\begin{aligned} &\lambda_{\bar{e}}\delta\left[-\tau_{A1} + \delta p_G[S_A(X_H) - S_A(X_L)] + \delta S_A(X_L) - L_A\right] - c_M - \delta c_I \\ \geq\ &\delta \max\left\{-\tau_{A1} + \delta\pi_{\underline{e}}[S_A(X_H) - S_A(X_L)] + \delta S_A(X_L) - L_A; 0\right\}, \end{aligned}$$

which depends again on the difference $-\tau_{A1} + \delta S_A(X_L) - L_A$. Due to the difference in discount factors between active and passive investors, this implies again that in the optimal contract the active investor receives no payoff if the project is liquidated, $L_A = 0$, or if continued, when the project has a low payoff, $S_A(X_L) = 0$, giving (i). Properties (ii) and (iii) are derived as in the case of Lemma 1. From Lemma 5, the double-deviation constraint (38) simplifies to

$$\lambda_{\bar{e}}\delta\left[-\tau_{A1}+\delta p_G S_A(X_H)\right]-c_M-\delta c_I \geq \delta \max\left\{-\tau_{A1}+\delta\pi_{\underline{e}}S_A(X_H);0\right\}. \tag{39}$$

The remaining constraints for the active investor are

$$\begin{aligned}
-\tau_{A1}+\delta\mathbb{E}_G[S_A(X)] &\geq \frac{c_M}{\delta(\lambda_{\bar{e}}-\lambda_{\underline{e}})} && (40)\\
-\tau_{A1}+\delta\mathbb{E}_G[S_A(X)] &\geq \frac{c_I}{\lambda_{\bar{e}}}+\frac{1}{\lambda_{\bar{e}}}\max\left\{-\tau_{A1}+\delta\mathbb{E}_{\bar{e}}[S_A(X)];0\right\} && (41)\\
\lambda_{\bar{e}}\delta\left(-\tau_{A1}+\delta\mathbb{E}_G[S_A(X)]\right)-c_M-\delta c_I &\geq \tau_{A0}. && (42)
\end{aligned}$$

Constraint (40) ensures that the active investor is willing to monitor the entrepreneur at $t=0$ if he is also willing to screen the project at $t=1$. Constraint (41), which follows from (18), ensures that the active investor is willing to screen the project at $t=1$ if he has previously monitored the entrepreneur at $t=0$. Note that without screening, the project can either be continued, with payoff $-\tau_{A1}+\delta\mathbb{E}_{\bar{e}}[S_A(X)]$, or it can be liquidated, with payoff $L_A=0$. Lastly, (42) is the IR constraint for the active investor at $t=0$.

It is useful to note that some of the incentive constraints are redundant. Let

$$\mathcal{R}_A \equiv \max\left\{\frac{c_M}{\delta(\lambda_{\bar{e}}-\lambda_{\underline{e}})};\frac{c_M}{\delta\lambda_{\bar{e}}}+\frac{c_I}{\lambda_{\bar{e}}}\right\}=\max\left\{\mathcal{R}_M;\frac{c_M}{\delta\lambda_{\bar{e}}}+\mathcal{R}_I\right\}.$$

We have the following.

Lemma 6 *The IC constraints (39), (40), (41) are satisfied if and only if the following constraints are satisfied*

$$\begin{aligned}
-\tau_{A1}+\delta\mathbb{E}_G[S_A(X)] &\geq \mathcal{R}_A && (43)\\
-\tau_{A1}+\frac{p_B}{p_G}\delta\mathbb{E}_G[S_A(X)] &\leq -\frac{c_I}{1-\lambda_{\bar{e}}}. && (44)
\end{aligned}$$

The first constraint (43) ensures that the active investor receives the minimum payoff (the rent), $\mathcal{R}_A$, that is necessary to induce him to monitor *and* screen the project. This term depends on whether or not the double-deviation constraint (39) binds:

$$\begin{aligned}
\mathcal{R}_A &= \mathcal{R}_M \quad \text{if the double-deviation constraint does not bind;} && (45)\\
\mathcal{R}_A &= \frac{c_M}{\delta\lambda_{\bar{e}}}+\mathcal{R}_I \quad \text{if the double-deviation constraint binds.} && (46)
\end{aligned}$$

The second constraint (44) ensures that the active investor has the incentive to pay the information production cost c_I and to continue the project if a good signal is obtained, rather than continuing the project as uninformed. Constraints (43) and (44) are displayed in Figure 2.

The double-deviation constraint does not bind when monitoring cost, c_M, is sufficiently large relative to the screening cost, c_I, that is, for.

$$\frac{c_M}{(\lambda_{\bar{e}}-\lambda_{\underline{e}})} > \frac{c_M+\delta c_I}{\lambda_{\bar{e}}} \iff \frac{c_M}{\delta c_I} > \frac{\lambda_{\bar{e}}-\lambda_{\underline{e}}}{\lambda_{\underline{e}}}.$$

Intuitively, when monitoring cost c_M is large (relative to the information production cost c_I), delayed compensation for monitoring must be correspondingly large and is enough to guarantee (together with (44)) that the active investor has the incentive to screen the project as well, and the double-deviation constraint does not bind. In contrast, when information production cost c_I is large (relative to monitoring cost c_M), the final compensation must provide additional incentives to induce the active investor to screen the project, on top of those necessary to induce monitoring, leading to (46).

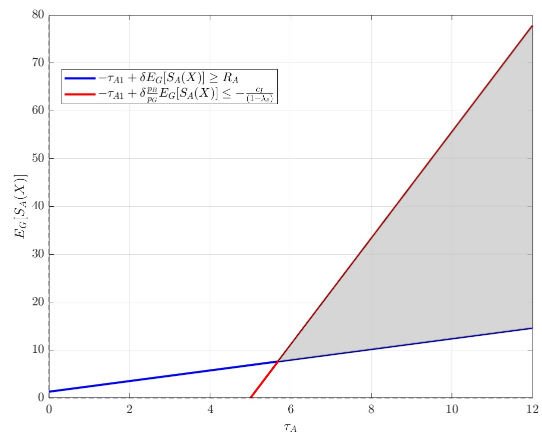

Figure 2: **IC Constraints of Active Investor.** The red line represents the constraint $-\tau_{A1}+\delta\mathbb{E}_G[S_A(X)] \geq \mathcal{R}_A$, while the blue line represents the constraint $-\tau_{A1}+\delta\frac{p_B}{p_G}\mathbb{E}_G[S_A(X)] \leq -\frac{c_I}{(1-\lambda_{\bar{e}})}$. The x-axis corresponds to τ_{I1}, and the y-axis represents $\mathbb{E}_G[S_I(X)]$. The shaded region indicates the area where both IC constraints are simultaneously satisfied.

Lemma 6 implies that combining monitoring and screening has important effects on the active investor's compensation structure.

First, note that constraint (44) implies that in an optimal contract $\tau_{A1} > 0$. This means that the active investor must make at the interim a payment to the firm date to continue the project. It also means that, different from the early-stage investor in the two-investor case, the single active investor does not receive any compensation at the interim date for monitoring the entrepreneur.

Rather, compensation for monitoring is delayed until the last period when the project payoff is realized (with the corresponding dissipative cost).

Second, note that if the double deviation constraint does not bind, as in (45), the incentive constraint for monitoring (40) binds, and the rent necessary to induce monitoring, $\mathcal{R}_M$, is also sufficient to induce screening. This implies that combining monitoring and screening within a single agent has two (opposing) effects on the cost of incentive provision: it has the benefit of rendering the screening constraint superfluous (with a saving of the rent $\mathcal{R}_I$), but it requires delaying the compensation for monitoring to the last period.

Third, if the double-deviation constraint binds, as in (46), incentive compatibility requires that the active investor expects to receive a payoff, $\mathcal{R}_A$, that sufficient to induce him to screen, given by the term $\mathcal{R}_I$, and to monitor, now given by the term $\frac{c_M}{\delta\lambda_{\bar{e}}}$. Note that the rent required to induce monitoring, $\frac{c_M}{\delta\lambda_{\bar{e}}}$, is lower that the rent necessary to satisfy the monitoring constraint alone in the two-agents case, $\mathcal{R}_M = \frac{c_M}{\delta(\lambda_{\bar{e}}-\lambda_{\underline{e}})}$.[18] Thus, combining monitoring and screening has again two (opposing) effects on the cost of incentive provision: it has the benefit of relaxing the monitoring constraint, but it requires again postponing the compensation for monitoring to the last period.

Finally, note that (43) implies that the interim participation constraint for the active investor, (9), is satisfied and, therefore, is lax. This means that active investor earns at the interim date a continuation payoff $\mathcal{R}_A$ which is strictly greater that the minimum payoff $\mathcal{R}_I = \frac{c_I}{\lambda_{\bar{e}}}$ that is necessary to satisfy the interim IR constraint Specifically, let

$$\mathcal{R}_{\mathcal{D}} \equiv \mathcal{R}_A - \frac{c_I}{\lambda_{\bar{e}}} > 0,$$

where

$$\mathcal{R}_{\mathcal{D}} \equiv \max\left\{\frac{c_M}{\delta(\lambda_{\bar{e}}-\lambda_{\underline{e}})} - \frac{c_I}{\lambda_{\bar{e}}}; \frac{c_M}{\delta\lambda_{\bar{e}}}\right\} = \max\left\{\mathcal{R}_M - \mathcal{R}_I; \frac{c_M}{\delta\lambda_{\bar{e}}}\right\} > 0.$$

The term $\mathcal{R}_{\mathcal{D}}$ represents the "*surplus rent*" that must be granted to the single active investor to satisfy the monitoring *and* double-deviation constraints that is in excess of the rent necessary to satisfy the interim IR constraint. It depends again on whether or not the double-deviation

[18]This happens because, with two active investors, if the early-stage investor does not monitor, the late-stage investor is expected to screen anyway, and the benefit of monitoring is proportional to $\lambda_{\bar{e}} - \lambda_{\underline{e}}$. In contrast, in the single active-investor case, if the active investor does not monitor, he will also refrain from screening, and the project will be liquidated. This means that the benefit of monitoring is now proportional to $\lambda_{\bar{e}} > \lambda_{\bar{e}} - \lambda_{\underline{e}}$, relaxing the corresponding IC constraint.

constraint binds:

$$\mathcal{R}_{\mathcal{D}} = \mathcal{R}_M - \mathcal{R}_I > 0 \quad \text{if the double-deviation constraint does not bind} \tag{47}$$

$$\mathcal{R}_{\mathcal{D}} = \frac{c_M}{\delta\lambda_{\bar{e}}} \quad \text{if the double-deviation constraint binds.} \tag{48}$$

At $t = 0$ the entrepreneur solves the following optimal security design problem, denoted $\mathbb{P}_1$:

$$\begin{aligned}
\min_{\{S_i\}\in\mathcal{S}} \mathbb{C}(S_i) &\equiv \sum_{i\in\{A,P\}} (1-\delta_i)\left(-\lambda_{\bar{e}}\tau_{i1} + (1-\lambda_{\bar{e}})L_i\right) + (1-\delta_i^2)\lambda_{\bar{e}}\mathbb{E}_G[S_i(X)] \\
s.t. \quad -\tau_{A1} + \delta\mathbb{E}_G[S_A(X)] &\geq \mathcal{R}_A \\
-\tau_{A1} + \delta\frac{p_B}{p_G}\mathbb{E}_G[S_A(X)] &\leq -\frac{c_I}{1-\lambda_{\bar{e}}} \\
-\tau_{E1} + \delta_E\mathbb{E}_G[S_E(X)] &\geq \mathcal{R}_E \\
\tau_{i0} &\leq \delta_i(1-\lambda_{\bar{e}})L_i + \lambda_{\bar{e}}\delta_i\left(-\tau_{i1} + \delta_i\mathbb{E}_G[S_i(X)]\right) - c_i \quad \forall i \in \Omega_1 \\
\lambda_{\bar{e}}\left[-\tau_{A1} + \delta\mathbb{E}_G[S_A(X)]\right] - c_I &\geq 0, \\
-\tau_{P1} + \mathbb{E}_G[S_P(X)] &\geq 0.
\end{aligned}$$

The solution to $\mathbb{P}_1$, $\mathcal{S}_1^* = \{\tau_{i0}^*, L_i^*, \tau_{i1}^*, S_i^*(X), \ell_i^*\}_{i\in\Omega_1}$, is characterized in the following proposition.

Proposition 7 *The optimal security offering with a single active investor, $\mathcal{S}_1^*$, that solves $\mathbb{P}_1$ has:*
(i) for the active investor:

$$S_A^*(X_L) = 0, \quad S_A^*(X_H) = \frac{1}{\delta(p_G - p_B)}\left[\frac{c_I}{\lambda_{\bar{e}}(1-\lambda_{\bar{e}})} + \mathcal{R}_{\mathcal{D}}\right], \quad L_A^* = 0, \quad \ell_A^* = 0 \tag{49}$$

$$\tau_{A1}^* = \delta p_G S_A^*(X_H) - \mathcal{R}_{\mathcal{A}} = \frac{1}{(p_G - p_B)}\left[\frac{\pi_{\bar{e}}}{\lambda_{\bar{e}}(1-\lambda_{\bar{e}})}c_I + p_B\mathcal{R}_{\mathcal{D}}\right] \tag{50}$$

$$\tau_{A0}^* = \delta\lambda_{\bar{e}}\left[-\tau_{A1}^* + \delta p_G S_A^*(X_H)\right] - c_M - \delta c_I; \tag{51}$$

(ii) for the passive investor:

$$S_P^*(X_L) = X_L, \quad S_P^*(X_H) = X_H - S_A^*(X_H), \quad L_P^* = L, \quad \ell_P^* = 1 \tag{52}$$

$$\tau_{P1}^* = K_1 - \left(\tau_{E1}^* + \tau_{A1}^*\right), \tag{53}$$

$$\tau_{P0}^* = \lambda_{\bar{e}}\left[X_L + p_G\left(X_H - S_A^*(X_H) - X_L\right) - \tau_{P1}^*\right] + (1-\lambda_{\bar{e}})L = \tag{54}$$

$$= V_0 - \lambda_{\bar{e}}\left[-\tau_{E1}^* + \mathcal{R}_{\mathcal{D}} + (1-\delta)p_G S_A^*(X_H)\right]; \tag{55}$$

(iii) for the entrepreneur:

$$S_E^*(X_H) = S_E^*(X_L) = 0, \quad L_E^* = 0, \quad \ell_E^* = 0 \tag{56}$$

$$\tau_{E1}^* = -\mathcal{R}_E = -\frac{b}{\delta_E(\lambda_{\bar{e}} - \lambda_{\underline{e}})} \tag{57}$$

$$\tau_{E0}^* = K_0 - \tau_{A0}^* - \tau_{P0}^* \tag{58}$$

The entrepreneur's payoff is

$$U_E^* = -\tau_{E0}^* - \delta_E \lambda_{\bar{e}} \tau_{E1}^* = V_N - (1-\delta)\lambda_{\bar{e}} \left[\mathcal{R}_{\mathcal{D}} + p_G S_A^*(X_H)\right] - (1-\delta_E)\lambda_{\bar{e}} \mathcal{R}_E. \tag{59}$$

The optimal contract is implementable if and only if

$$V_0 - c_M - (1-\delta)\lambda_{\bar{e}} \left[\mathcal{R}_{\mathcal{D}} + p_G S_A^*(X_H)\right] - \lambda_{\bar{e}} \mathcal{R}_E \geq K_0 \tag{60}$$

$$X_H - S_A^*(X_H) \geq X_L. \tag{61}$$

In the optimal contract: (i) The active investor monitors the entrepreneur at $t = 0$ and makes a payment τ_{A0}^* to the firm, contributing to the financing of the initial capital expenditure, K_0. Payment τ_{A0}^* represents the present value of the final payoff to the active investor, $\delta^2 \lambda_{\bar{e}} p_G S_A^*(X_H)$, net of the interim payment τ_{A1}^*, of the monitoring cost, c_M, and of the present value of the screening costs (which are incurred at the interim date, $t = 1$, and discounted at the active investor's discount factor, δ).

At $t = 1$, the active investor must decide whether or not to make an additional payment to the firm, τ_{A1}^*, which contributes to the financing of the follow-up investment, K_1. This decision is made after paying the cost c_I and privately observing the signal Y. If the investor receives a good signal, $Y = G$, he makes the interim payment, τ_{A1}^*, and receives a compensation at $t = 2$ only if the project has a high payoff, $S_A^*(X_H)$. The interim payment τ_{A1}^* is the present value of the compensation expected for time $t = 2$, reduced by the rent required for incentive provision, $\mathcal{R}_A$. Because $\mathcal{R}_A = c_I/\lambda_{\bar{e}} + \mathcal{R}_{\mathcal{D}}$, the latter include the compensation needed for the screening cost, $c_I/\lambda_{\bar{e}}$, and the surplus rent $\mathcal{R}_{\mathcal{D}}$ necessary to the satisfy the monitoring *and* the double-deviation constraints. If the investor receives a bad signal, $Y = B$, he refrains from making the interim payment τ_{A1}^*. The project is liquidated and the investor receives no payment, $L_A^* = 0$.

(ii) The passive investor makes at $t = 0$ a payment to the firm,τ_{P0}^*, providing the residual funds necessary to make the initial investment, K_0, and to pay the entrepreneur $-\tau_{E0}^*$. The payment τ_{P0}^*

is equal to the continuation value of the project V_0 (which, again, is net of the anticipated screening costs c_I due to the active investor), and is reduced by the expected value of the compensation to the entrepreneur, $-\tau_{E1}^*$, of the surplus rent due to the active investor, $\mathcal{R}_{\mathcal{D}}$, and of the dissipative costs of the delayed compensation to the active investor, $(1-\delta)p_G S_A^*(X_H)$. If the project is continued, at the interim date, the passive investor makes a second contribution of capital to the firm, τ_{P1}^*, which (in addition to the capital contribution from the active investor, τ_{A1}^*) is sufficient to finance the follow-up investment, K_1, and to pay the entrepreneur's compensation, $-\tau_{E1}^*$. If the project is liquidated, the passive investor receives again the full liquidation value of the project, $L_P^* = L$. At $t = 2$, if the project has a low payoff, the passive investor receives the full project value, X_L. If the project has a high payoff, the passive investor receives the residual value $X_H - S_A^*(X_H)$, after paying the active investor his compensation, $S_A^*(X_H)$.

(iii) Finally, the entrepreneur exerts at $t = 0$ high effort, and receives the residual cash flow from the firm after the contributions from the active and passive investors and the initial capital expenditure are made: $-\tau_{E0}^* = \sum_{i \in \Omega_2} \tau_{i0}^* - K_0 > 0$. At the interim date, $t = 1$, the entrepreneur receives compensation $-\tau_{E1}^*$ only if the project is continued, and no payment if the project is liquidated, $L_E^* = 0$. Similar to the case with two active investors, the entrepreneur receives no additional payoff from the project, and she "exits" again the project at the interim date. Finally, the total payoff to the entrepreneur is equal to the net present value of the investment project, V_N, reduced by the dissipative costs of delayed expected compensation to the active investor, $(1-\delta)\lambda_{\bar{e}}\left[\mathcal{R}_{\mathcal{D}} + p_G S_I^*(X_H)\right]$, and for her own expected compensation, $(1-\delta_E)\lambda_{\bar{e}}(-\tau_{E1}^{**})$.

The optimal contract can be interpreted again as one where the entrepreneur "sells" at $t = 0$ the firm to the passive investor, who then "hires back" the entrepreneur to exert effort and the active investor for both monitoring the entrepreneur and screening the project for continuation. The active investor pays the passive investor the value of his compensation contract.

Similar to the case with two active investors, the optimal contract (49)-(58) is implementable if and only if conditions (60)-(61) are satisfied. Condition (60) ensures that the firm is able to raise at $t = 0$ enough capital to cover the investment expenditure and, thus, that the net payment to the entrepreneur at the initial date is non-negative: $-\tau_{E0}^* = \sum_{i \in \Omega_2} \tau_{i0}^* - K_0 \geq 0$. It requires that the continuation value of the project, V_0 (inclusive of the anticipated screening costs

c_I) is sufficient to cover the monitoring cost, c_M, the expected contractual compensation to the entrepreneur, $\lambda_{\bar{e}}\mathcal{R}_E$, the expected dissipative costs for the compensation to the active investor, $(1-\delta)\lambda_{\bar{e}}\left[\mathcal{R}_D + p_G S_A^*(X_H)\right]$, and the initial investment K_0. This condition ensures that there is sufficient residual expected cash flow from the project that can be pledged to the passive investor, making him willing to provide financing at the initial stage. This condition also guarantees that the entrepreneur's payoff (59) is positive.

Condition (61) ensures again that the monotonicity condition for the passive investor's payoff is satisfied: from (52), we have that $S_P^*(X_L) = X_L$, giving $S_P^*(X_H) = X_H - S_A^*(X_H) \geq X_L = S_P^*(X_L) \geq 0$. This condition also guarantees that the passive investor's limited liability constraint at $t = 2$ is satisfied. The monotonicity and limited liability conditions are easily verified, because $S_A^*(X_H) > S_A^*(X_L) = 0$.

The implementability of optimal contracts is established in the following.

Proposition 8 *There are critical values $\hat{\delta}^*$ and $\{\hat{c}_I^*, \hat{c}_M^*\}$ (all defined in the appendix) such that if $\delta \geq \hat{\delta}^*$, $c_I \leq \hat{c}_I^*$ and $c_M \leq \hat{c}_M^*$ then contract $\mathcal{S}_1^*$ is implementable.*

Contract implementability requires that the active investors' discount factor δ is not too small, $\delta \geq \hat{\delta}^{**}$, and that the screening and monitoring costs are not too large: $c_I \leq \hat{c}_I^*$and $c_M \leq \hat{c}_M^*$. This happens again because, from (49) and (22), discount factors that are too low or screening and monitoring costs that are too large, leading to payoffs $\mathcal{R}_D$ and $S_I^{**}(X_H)$ that are sufficiently large to violate (60) and (61).

4.3 Implementation with securities

Similar to the case with two active investors, the optimal contract can be implemented through a sequence of securities offerings, as follows.

At $t = 0$, the entrepreneur raises capital by selling a package of securities to outside investors:

(i) *To the active investor*: The entrepreneur sells to a single active investor at a price τ_{A0}^* a warrant that gives the active investor the option to buy at $t = 1$ newly issued preferred equity (mezzanine financing) with face value $F_A = S_A^*(X_H)$. The exercise price of the warrant is τ_{A1}^*, which is paid to the firm at $t = 1$, and is used to finance the follow-up investment.

(ii) *To the passive investor*: The entrepreneur sells to the passive investor a package of securities which includes (senior) secured debt, a credit line, and options for a total value τ^*_{P0}. The senior debt has a face value $F_P = L$, and the credit line has a total draw-down value τ^*_{P1}. The passive investor also has the option to buy at $t = 1$ the shares held by the entrepreneur at $-\tau^*_{E1}$.

At $t = 1$, the following actions take place:

The active investor spends c_I and obtains the signal Y. If the signal is bad, $Y = B$, the active investor does not exercise the warrant on preferred equity and no capital is contributed. The passive investor terminates the project and liquidates the firm. The passive investor, as senior creditor, has a payoff equal to the full liquidation value, L. All other parties receive zero payoff.

If the signal is good, $Y = G$, the active investor exercises the warrant on newly issued preferred stock and pays τ^*_{A1} to the firm. The passive investor also exercises the option to buy shares from the entrepreneur at a price of $-\tau^*_{E1}$. The firm draws τ^*_{P1} on the credit line and invests in the follow-up capital K_1. The project is continued.

At $t = 2$, final payoffs are realized. The passive investors hold both senior debt and 100% of equity. If $X = X_L$, the passive investor, holding senior debt with face value $L > X_L$, obtains the full project payoff X_L. The active investor has zero payoff. If $X = X_H$, the passive investor will pay the active investor the face value of the preferred stock (mezzanine), $S^*_A(X_H)$, and will retain the remainder, with payoff $F_P + (X_H - F_A - F_P) = L + [X_H - S^*_A(X_H) - L] = X_H - S^*_A(X_H)$.

5 Optimal number of investors

Combining monitoring and screening has important effects on both the size and timing of active investors' compensation and transfers to the firm. Starting with compensation at $t = 2$, direct comparison of (26) with (49) reveals that

$$S^*_A(X_H) = S^{**}_I(X_H) + \Delta S, \quad \text{where} \tag{62}$$

$$\Delta S \equiv \frac{\mathcal{R}_D}{\delta(p_G - p_B)} > 0. \tag{63}$$

Combining monitoring and screening requires that the passive investor makes at the final date a larger payment to the active investor, $S^*_A(X_H)$, than that due to the late-stage investor, $S^{**}_I(X_H)$.

The incremental payment ΔS is due to the surplus rent $\mathcal{R}_D$ that is necessary to satisfy the double-deviation constraint.

Proceeding backward, direct comparison of (27) with (50) reveals that

$$\Delta\tau_1 \equiv \tau_{A1}^* - \tau_{I1}^{**} - \tau_{M1}^{**} = \delta p_G \Delta S - (\mathcal{R}_D - \mathcal{R}_M).$$

Combining monitoring and screening has two opposing effects on the active investor's payment to the firm at the interim date, τ_{A1}^*, relative to the combined payments required by the early- and late-stage investors, $\tau_{I1}^{**} + \tau_{M1}^{**}$. On the one hand, from (62), greater compensation ΔS received by the active investor at time $t = 2$ increases the up-front payment to the firm by its discounted expected value, $\delta p_G \Delta S$. On the other hand, the active investor must receive the surplus rent $\mathcal{R}_D$ in place of the compensation to the early-stage investor monitoring the firm, $\mathcal{R}_M$, decreasing τ_{A1}^*. The net effect depends on whether or not the double-deviation constraint binds, giving

$$\begin{aligned} \Delta\tau_1 &= \delta p_G \Delta S + \frac{c_I}{\lambda_{\bar{e}}} > 0 \quad \text{if the double-deviation constraint does not bind;} \\ \Delta\tau_1 &= \delta p_G \Delta S - \frac{c_M}{\delta \lambda_{\bar{e}}} \lesseqgtr 0 \quad \text{if the double-deviation constraint binds.} \end{aligned}$$

Finally, direct comparison of (51) with (25) and (28) reveals that

$$\Delta\tau_0 \equiv \tau_{A0}^* - \tau_{M0}^{**} = \delta\lambda_{\bar{e}} (\mathcal{R}_D - \mathcal{R}_M).$$

The effect of combining monitoring and screening on the active investors' payments to the firm at the initial date, τ_{A0}^*, is equal to the present value of the differential rent expected at the interim date, $\mathcal{R}_D - \mathcal{R}_M$. The overall effect is negative, and its size depends again on whether or not the double-deviation constraint binds, giving

$$\begin{aligned} \Delta\tau_0 &= -\delta c_I < 0 \quad \text{if the double-deviation constraint does not bind;} \\ \Delta\tau_0 &= -c_M - \delta\lambda_{\bar{e}}\mathcal{R}_M < 0 \quad \text{if the double-deviation constraint binds.} \end{aligned}$$

The impact of combining monitoring and screening on cash-flows to the passive investor is summarized in Table 1.[19] The top two rows display the total net payments received (or made, if negative) by the passive investor in the two configurations: $\mathbb{P}_1$ for the case of a single active investor, and $\mathbb{P}_2$ for the case of two active investors. The middle section presents the incremental cash flows,

[19] Note that, because in an optimal contract the passive investor is the residual claimant to the firm's cash flows, compensation to active investors represent a negative cash flow to the passive investor, while contributions to the firm represent a positive cash flow.

$\mathbb{P}_1 - \mathbb{P}_2$, when the double-deviation constraint does not bind. The bottom section presents the corresponding flows when this constraint binds.

Table 1: Net Cash Flows to Passive Investors at Different Dates and States

	$t = 0$	$t = 1, Y = G$	$t = 2, X = X_H$
$\mathbb{P}_1$	τ^*_{A0}	τ^*_{A1}	$-S^{**}_I(X_H) - \Delta S$
$\mathbb{P}_2$	τ^{**}_{M0}	$\tau^*_{M1} + \tau^*_{I1}$	$-S^*_I(X_H)$
Case 1: DD does no bind			
$\mathbb{P}_1$	$\tau^{**}_{M0} - \delta c_I$	$\delta p_G[S^{**}_I(X_H) + \Delta S] - \frac{c_M}{\delta(\lambda_{\bar{e}} - \lambda_{\underline{e}})}$	$-S^{**}_I(X_H) - \Delta S$
$\mathbb{P}_1$-$\mathbb{P}_2$	$-\delta c_I$	$\delta p_G \Delta S + \frac{c_I}{\lambda_{\bar{e}}}$	$-\Delta S$
Case 2: DD binds			
$\mathbb{P}_1$	0	$\delta p_G[S^{**}_I(X_H) + \Delta S] - \frac{c_I}{\lambda_{\bar{e}}} - \frac{c_M}{\delta \lambda_{\bar{e}}}$	$-S^{**}_I(X_H) - \Delta S$
$\mathbb{P}_1$-$\mathbb{P}_2$	$-\tau^{**}_{M0}$	$\delta p_G \Delta S - \tau^{**}_{M1}$	$-\Delta S$

The entrepreneur's decision on whether or not to combine monitoring and screening depends on which arrangement minimizes the overall cost of implementing the optimal contract. After substitution of (62) in the entrepreneur's payoff (59), we have

$$U^{**}_E = V_N - (1-\delta)\lambda_{\bar{e}} \left[-\tau^{**}_{M1} + p_G S^{**}_I(X_H)\right] + (1-\delta_E)\lambda_{\bar{e}}\tau^{**}_{E1}, \tag{64}$$

$$U^{*}_E = V_N - (1-\delta)\lambda_{\bar{e}} \left[\mathcal{R}_{\mathcal{D}} + p_G \left(S^{**}_I(X_H) + \Delta S\right)\right] + (1-\delta_E)\lambda_{\bar{e}}\tau^{*}_{E1}. \tag{65}$$

Because $\tau^*_{E1} = \tau^{**}_{E1}$, direct comparison of (64) and (65) gives that $U^*_E - U^{**}_E \geq 0$ if and only if

$$-\tau^{**}_{M1} - p_G \Delta S - \mathcal{R}_{\mathcal{D}} = \mathcal{R}_M - p_G \Delta S - \mathcal{R}_{\mathcal{D}} \geq 0. \tag{66}$$

Condition (66) has the following appealing interpretation. Because active and passive investors have deep pockets, and the entrepreneur has all the bargaining power, she will extract all the project's surplus, internalizing all costs and benefits. In addition, because the passive investor is effectively a residual claimant, the efficient contract minimizes the efficiency losses associated with active investors' incentive constraints. After substitution for $\mathcal{R}_{\mathcal{D}}$ from (47) and (48), condition (66) becomes:

$$\delta\left(c_I - \lambda_{\bar{e}} p_G \Delta S\right) \geq 0, \quad \text{if the double-deviation constraint does not bind} \tag{67}$$

$$\frac{\lambda_{\underline{e}} c_M}{\lambda_{\bar{e}} - \lambda_{\underline{e}}} - \lambda_{\bar{e}} p_G \delta \Delta S \geq 0, \quad \text{if the double-deviation constraint binds.} \tag{68}$$

If the double deviation constraint does not bind, combining monitoring and screening leads to incremental cash flows as in line $\mathbb{P}_1 - \mathbb{P}_2$, Case 1, in Table 1. The passive agent will have, in the initial period, a lower payment from the early-stage investor in the amount of δc_I, and will have to pay additional compensation to the active agent in the last period, ΔS. These cash flow reductions are balanced by a greater payment from the active investor at the interim date in the amount of $\delta p_G \Delta S + \frac{c_I}{\lambda_{\bar{e}}}$. The present value of the expected incremental cash flow is positive if

$$-\delta c_I + \lambda_{\bar{e}} \left[\delta p_G \Delta S + \frac{c_I}{\lambda_{\bar{e}}} \right] + \lambda_{\bar{e}} p_G \left[-\Delta S \right] \geq 0 \quad \Rightarrow \quad (1 - \delta) \left(c_I - \lambda_{\bar{e}} p_G \Delta S \right) \geq 0,$$

giving (67).

Similarly, if the double deviation constraint binds, combining monitoring and screening leads to incremental cash flows as in line $\mathbb{P}_1 - \mathbb{P}_2$, Case 2. The passive agent will have to forego receiving the initial payment from the monitor, τ_{M0}, and will have to pay an additional compensation to the active agent in the last period, ΔS. The reduction in cash flows is balanced by a greater payment from the active investor at the interim date in the amount of $\delta p_G \Delta S + \frac{\tau_{M0}^{**}}{\delta \lambda_{\bar{e}}}$. The present value of the expected incremental cash flow is positive if

$$-\tau_{M0}^{**} + \lambda_{\bar{e}} \left[\delta p_G \Delta S + \frac{\tau_{M0}^{**}}{\delta \lambda_{\bar{e}}} \right] + \lambda_{\bar{e}} p_G \left[-\Delta S \right] \geq 0 \quad \Rightarrow \quad \frac{(1 - \delta)}{\delta} \left(\frac{c_M}{\delta (\lambda_{\bar{e}} - \lambda_{\underline{e}})} - \lambda_{\bar{e}} p_G \delta \Delta S \right) \geq 0,$$

giving (68).

The following proposition characterizes the optimal arrangement of screening and monitoring.

Proposition 9 *The single-investor arrangement with milestone financing is better:*
(i) when the double-deviation constraint does not bind (for $\frac{\lambda_{\bar{e}} - \lambda_{\underline{e}}}{\lambda_{\underline{e}}} < \frac{c_M}{\delta c_I}$*) if:*

$$\frac{p_G - p_B}{p_G} \geq \frac{1}{\delta} \left[\frac{\lambda_{\bar{e}}}{\delta (\lambda_{\bar{e}} - \lambda_{\underline{e}})} \frac{c_M}{c_I} - 1 \right].$$

(ii) when double-deviation constraint binds (for $\frac{(\lambda_{\bar{e}} - \lambda_{\underline{e}})}{\lambda_{\underline{e}}} > \frac{c_M}{\delta c_I}$*) if:*

$$\frac{p_G - p_B}{p_G} \geq \frac{\lambda_{\bar{e}} - \lambda_{\underline{e}}}{\delta \lambda_{\underline{e}}}.$$

Furthermore, there is a $\delta_c \in [0, 1]$ *(defined in the appendix) such that the single-investor arrangement is better if and only if* $\delta \geq \delta_c$*.*

The characterization of the optimal number of active investors of Proposition 9 is displayed in Figure 3. The double deviation constraint does not bind, Case (i), when the human capital intensity of the project (as measured by $\lambda_{\bar{e}} - \lambda_{\underline{e}}$) is sufficiently small, with $\frac{\lambda_{\bar{e}} - \lambda_{\underline{e}}}{\lambda_{\underline{e}}} < \frac{c_M}{\delta c_I}$, and when the monitoring cost c_M is large compared to the screening cost c_I (leading to a larger threshold $\frac{c_M}{\delta c_I}$). In this case, the single-investor arrangement is optimal if the innovation risk of the project (as measured by $p_G - p_B$) is sufficiently large. This can be seen by noting that, from (47) and (63), we have that

$$\Delta S \equiv \frac{\mathcal{R}_{\mathcal{D}}}{\delta(p_G - p_B)} = \frac{1}{\delta(p_G - p_B)} \left[\frac{c_M}{\delta(\lambda_{\bar{e}} - \lambda_{\underline{e}})} - \frac{c_I}{\lambda_{\bar{e}}} \right].$$

This means that a larger value of either $p_G - p_B$ or $\lambda_{\bar{e}} - \lambda_{\underline{e}}$, reduces the incremental compensation ΔS that is necessary in the single investor case. The effect is to make condition (67) laxer and, thus, the single-investor arrangement more desirable. This effect is again more pronounced when the expected dissipative cost of the incremental pay ΔS is lower, that is, with a smaller success probability p_G and a greater discount factor δ. Finally, a greater value of $\lambda_{\bar{e}}$ increases the incremental compensation ΔS that the single late-stage investor receives at $t = 2$ and, thus, the corresponding payment that he must make at the interim date. In both cases, the effect is to make condition (67) more stringent and, thus, the single-investor arrangement less desirable.

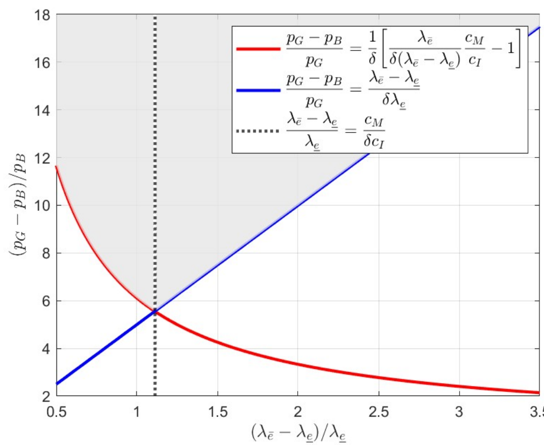

Figure 3: **Comparing Entrenreuer's Payoff.** The figure compares the entrepreneur's payoff under single and two active investors, illustrated by Proposition 9. The red line represents $\frac{p_G - p_B}{p_G} = \frac{1}{\delta}\left[\frac{\lambda_{\bar{e}}}{\delta(\lambda_{\bar{e}} - \lambda_{\underline{e}})}\frac{c_M}{c_I} - 1\right]$, the blue dashed line represents $\frac{p_G - p_B}{p_G} = \frac{\lambda_{\bar{e}} - \lambda_{\underline{e}}}{\delta\lambda_{\underline{e}}}$, and the black dash-dot line represents $\frac{\lambda_{\bar{e}} - \lambda_{\underline{e}}}{\lambda_{\underline{e}}} = \frac{c_M}{\delta c_I}$. The horizontal axis corresponds to $(\lambda_{\bar{e}} - \lambda_{\underline{e}})/\lambda_{\underline{e}}$, and the vertical axis corresponds to $(p_G - p_B)/p_B$. The shaded area plots in set of parameters under which the entrepreneur has a higher payoff under single-investor arrangement.

The double-deviation constraint binds, Case (ii), when the human capital intensity of the project is sufficiently large, with $\frac{\lambda_{\bar{e}} - \lambda_{\underline{e}}}{\lambda_{\underline{e}}} > \frac{c_M}{\delta c_I}$, and when the monitoring cost c_M is small compared to the screening cost c_I (leading to a smaller threshold $\frac{c_M}{\delta c_I}$). In this case, the single-investor arrangement is preferable if the innovation risk of the project is again sufficiently large with respect to its human-capital intensity. This may be seen as follows. From (48) and (63), we have that

$$\delta p_G \Delta S = \frac{\delta p_G c_M}{\delta^2 (p_G - p_B) \lambda_{\bar{e}}}.$$

An increase of the innovation risk, $p_G - p_B$, leads to a smaller incremental compensation ΔS and, thus, to a lower dissipative cost of the delay. The effect is to make condition (68) laxer and, thus, the single-investor case more desirable. This effect is more pronounced when the expected dissipative cost of the incremental pay ΔS is lower, that is, with a smaller success probability p_G, and a greater discount factor δ. In addition, because $\tau^*_{M0} = \frac{\lambda_{\underline{e}} c_M}{\lambda_{\bar{e}} - \lambda_{\underline{e}}}$, a smaller value of $\lambda_{\underline{e}}$ or a greater value of $\lambda_{\bar{e}} - \lambda_{\underline{e}}$ decrease the amount of the initial payment τ_{M0} that the early-stage investor must make in the two-investor case. The net effect is to make condition (68) tighter and, thus, the two-investor arrangement more desirable.

6 Conclusions

Our paper contributes to the theory of the optimal venture capital financing. Venture capital is affected by multiple sources of moral hazard: entrepreneurial rents seeking that may be mitigated by costly monitoring by external investors. In addition, a critical decision in the management of young firms is the decision of whether or not to continue their projects. This decision is made on the basis of costly information production, screening, which is also affected by moral hazard.

We examine the optimal contractual arrangement for the financing of start-up projects and their implementation through security offerings. We show that optimal financing contracts can be implement by a sequence of security offerings consisting of coomon and preferred equity, warrants and options, and a combination of senior debt and credit lines.

A specific question we address is to determine whether monitoring and screening should be delegated to two separate agents, and they can be combined in a single active investor. We find that combining monitoring and screening to a single active investor (milestone financing) is optimal when the innovation risk of a project is larger relative to its human capital intensity. In contrast, the two active investor case (round financing) is optimal when innovation risk is low and human capital intensity is either relatively very high or very low. Finally, we find that rounds financing is more beneficial when the market for informed capital is tighter.

References

Admati, A. R. and P. Pfleiderer (1994). Robust financial contracting and the role of venture capitalists. *The Journal of Finance 49*(2), 371–402.

Bergemann, D. and U. Hege (1998). Venture capital financing, moral hazard, and learning. *Journal of Banking & Finance 22*(6-8), 703–735.

Berglöf, E. and E.-L. Von Thadden (1994). Short-term versus long-term interests: Capital structure with multiple investors. *The quarterly journal of economics 109*(4), 1055–1084.

Bernstein, S., X. Giroud, and R. R. Townsend (2016). The impact of venture capital monitoring. *The Journal of Finance 71*(4), 1591–1622.

Cornelli, F. and O. Yosha (2003). Stage financing and the role of convertible securities. *The Review of Economic Studies 70*(1), 1–32.

Da Rin, M., T. Hellmann, and M. Puri (2013). A survey of venture capital research. In *Handbook of the Economics of Finance*, Volume 2, pp. 573–648. Elsevier.

Davis, J., A. Morse, and X. Wang (2020). The leveraging of silicon valley. Technical report, National Bureau of Economic Research.

Dewatripont, M. and E. Maskin (1995). Credit and efficiency in centralized and decentralized economies. *Review of Economic Studies 62*(4), 541–555.

Fulghieri, P. and M. Sevilir (2009a). Organization and financing of innovation, and the choice between corporate and independent venture capital. *Journal of Financial and Quantitative Analysis 44*(6), 1291–1321.

Fulghieri, P. and M. Sevilir (2009b). Size and focus of a venture capitalist's portfolio. *The Review of Financial Studies 22*(11), 4643–4680.

Gompers, P. A. (1995). Optimal investment, monitoring, and the staging of venture capital. In *Venture capital*, pp. 285–313. Routledge.

Hart, O. and J. Moore (1994). A theory of debt based on the inalienability of human capital. *The Quarterly Journal of Economics 109*(4), 841–879.

Hellmann, T. (1998). The allocation of control rights in venture capital contracts. *The Rand Journal of Economics*, 57–76.

Hellmann, T. and M. Puri (2002). Venture capital and the professionalization of start-up firms: Empirical evidence. *The journal of finance 57*(1), 169–197.

Hellmann, T. and V. Thiele (2015). Friends or foes? the interrelationship between angel and venture capital markets. *Journal of Financial Economics 115*(3), 639–653.

Holmstrom, B. and J. Tirole (1997). Financial intermediation, loanable funds, and the real sector. *Quarterly Journal of Economics 112*(3), 663–691.

Janeway, W. H., R. Nanda, and M. Rhodes-Kropf (2021). Venture capital booms and start-up financing. *Annual Review of Financial Economics 13*(1), 111–127.

Kaplan, S. N. and P. Strömberg (2003). Financial contracting theory meets the real world: An empirical analysis of venture capital contracts. *The review of economic studies 70*(2), 281–315.

Lerner, J. (1995). Venture capitalists and the oversight of private firms. In *Venture capital*, pp. 267–284. Routledge.

Lerner, J. and R. Nanda (2023). Venture capital and innovation. In *Handbook of the economics of corporate finance*, Volume 1, pp. 77–105. Elsevier.

Neher, D. V. (1999). Staged financing: an agency perspective. *The Review of Economic Studies 66*(2), 255–274.

Pagano, M. and A. Röell (1998). The choice of stock ownership structure: Agency costs, monitoring, and the decision to go public. *Quarterly Journal of Economics 113*(1), 187–225.

Repullo, R. and J. Suarez (2004). Venture capital finance: A security design approach. *Review of finance 8*(1), 75–108.

Schindele, I. (2006). Advice and monitoring: venture financing with multiple tasks. In *EFA 2004 Maastricht Meetings Paper*, Number 4637.

Appendix

A Proofs of Section 3

We supplement the proofs for all results in Section 3, including Lemma 1, 2, Proposition 3 and 4.

A.1 Constraints and Problem Summary

IC constraints

Let us define

$$\hat{L}_M \equiv \frac{L_M}{\delta}, \quad \hat{L}_I \equiv \frac{L_I}{\delta}, \quad \hat{c}_I \equiv \frac{c_I}{\delta}, \quad \hat{c}_M \equiv \frac{c_M}{\delta^2}, \quad \hat{\tau}_{M1} = \frac{\tau_{M1}}{\delta}, \quad \hat{\tau}_{I1} = \frac{\tau_{I1}}{\delta}.$$

1. The project is liquidated only after the bad signal

$$\begin{aligned} &\tau_{I1} + \delta\mathbb{E}_B[S_I(X)] \le L_I \le \tau_{I1} + \delta\mathbb{E}_G[S_I(X)] \\ \Rightarrow\ &\hat{\tau}_{I1} + \mathbb{E}_B[S_I(X)] \le \hat{L}_I \le \hat{\tau}_{I1} + \mathbb{E}_G[S_I(X)]. \end{aligned}$$

2. The investor has incentives to acquire signal at $t = 1$, given that the other investor has monitored.

$$\begin{aligned} &\lambda_{\bar{e}}\left(\tau_{I1} + \delta\mathbb{E}_G[S_I(X)]\right) + (1-\lambda_{\bar{e}})L_I - c_I \ge \max\{\tau_{I1} + \delta\mathbb{E}_{\bar{e}}[S_I(X)], L_I\} \\ \Rightarrow\ &\lambda_{\bar{e}}\left(\hat{\tau}_{I1} + \mathbb{E}_G[S_I(X)]\right) + (1-\lambda_{\bar{e}})\hat{L}_I - \hat{c}_I \ge \max\{\hat{\tau}_{I1} + \mathbb{E}_{\bar{e}}[S_I(X)], \hat{L}_I\}. \end{aligned}$$

3. IC constraints on monitoring. If the investor did not monitor, he would still expect the other one to acquire information

$$\begin{aligned} &\lambda_{\bar{e}}\left(\delta\tau_{M1} + \delta^2\mathbb{E}_G\left[S_m(X)\right]\right) + (1-\lambda_{\bar{e}})\delta L_M - c_M \ge \lambda_{\underline{e}}\left(\delta\tau_{M1} + \delta^2\mathbb{E}_G\left[S_m(X)\right]\right) + (1-\lambda_{\underline{e}})\delta L_M \\ \Rightarrow\ &\lambda_{\bar{e}}\left(\hat{\tau}_{M1} + \mathbb{E}_G\left[S_m(X)\right]\right) + (1-\lambda_{\bar{e}})\hat{L}_M - \hat{c}_M \ge \lambda_{\underline{e}}\left(\hat{\tau}_{M1} + \mathbb{E}_G\left[S_m(X)\right]\right) + (1-\lambda_{\underline{e}})\hat{L}_M \\ \Rightarrow\ &\hat{\tau}_{M1} + \mathbb{E}_G\left[S_m(X)\right] - \hat{L}_M \ge \frac{\hat{c}_M}{(\lambda_{\bar{e}} - \lambda_{\underline{e}})}. \end{aligned}$$

The IC constraints are satisfied if and only if

$$\begin{aligned} \hat{\tau}_{I1} + \mathbb{E}_G[S_I(X)] - \hat{L}_I &\ge \frac{\hat{c}_I}{\lambda_{\bar{e}}}. \\ \hat{L}_I - \hat{\tau}_{I1} - \mathbb{E}_B[S_I(X)] &\ge \frac{\hat{c}_I}{1-\lambda_{\bar{e}}}, \\ \hat{\tau}_{M1} + \mathbb{E}_G\left[S_m(X)\right] - \hat{L}_M &\ge \frac{\hat{c}_M}{(\lambda_{\bar{e}} - \lambda_{\underline{e}})}. \end{aligned}$$

Let us rewrite the set of constraints:

$$\hat{\tau}_{I1} + \mathbb{E}_B[S_I(X)] \leq \hat{L}_I \leq \hat{\tau}_{I1} + \mathbb{E}_G[S_I(X)]$$
$$\lambda_{\bar{e}}\left(\hat{\tau}_{I1} + \mathbb{E}_G[S_I(X)]\right) + (1-\lambda_{\bar{e}})\hat{L}_I - \hat{c}_I \geq \max\{\hat{\tau}_{I1} + \mathbb{E}_{\bar{e}}[S_I(X)], \hat{L}_I\}$$
$$\hat{\tau}_{M1} + \mathbb{E}_G\left[S_m(X)\right] - \hat{L}_M \geq \frac{\hat{c}_M}{(\lambda_{\bar{e}} - \lambda_{\underline{e}})}.$$

The second constraint includes

$$\lambda_{\bar{e}}\left(\hat{\tau}_{I1} + \mathbb{E}_G[S_I(X)]\right) + (1-\lambda_{\bar{e}})\hat{L}_I - \hat{c}_I \geq \hat{\tau}_{I1} + \mathbb{E}_{\bar{e}}[S_I(X)] = \hat{\tau}_{I1} + \lambda_{\bar{e}}\mathbb{E}_G[S_I(X)] + (1-\lambda_{\bar{e}})\mathbb{E}_B[S_I(X)]$$
$$\Rightarrow \hat{L}_I - \hat{\tau}_{I1} - \mathbb{E}_B[S_I(X)] \geq \frac{\hat{c}_I}{1-\lambda_{\bar{e}}},$$

and

$$\lambda_{\bar{e}}\left(\hat{\tau}_{I1} + \mathbb{E}_G[S_I(X)]\right) + (1-\lambda_{\bar{e}})\hat{L}_I - \hat{c}_I \geq \hat{L}_I$$
$$\Rightarrow \hat{\tau}_{I1} + \mathbb{E}_G[S_I(X)] - \hat{L}_I \geq \frac{\hat{c}_I}{\lambda_{\bar{e}}}.$$

Given so, the first constraint is redundant.

Summary This lemma above implies that some IC constraints are always slack; some may be slack; some always bind

- Always slack:
  - The project is liquidated after the bad signal but not after the good signal

$$\hat{\tau}_{I1} + \mathbb{E}_B[S_I(X)] \leq \hat{L}_I \leq \hat{\tau}_{I1} + \mathbb{E}_G[S_I(X)].$$

- Always binding:
  - The evaluation investor prefers to acquire information

$$\lambda_{\bar{e}}\left(\hat{\tau}_{I1} + \mathbb{E}_G[S_I(X)]\right) + (1-\lambda_{\bar{e}})\hat{L}_I - \hat{c}_I \geq \max\{\hat{\tau}_{I1} + \mathbb{E}_{\bar{e}}[S_I(X)], \hat{L}_I\}$$

  - The monitor investor prefers to monitor

$$\hat{\tau}_{M1} + \mathbb{E}_G\left[S_m(X)\right] - \hat{L}_M \geq \frac{\hat{c}_M}{(\lambda_{\bar{e}} - \lambda_{\underline{e}})}.$$

Entrepreneur's IC constraint

The passive investor lends τ_{P0} and the two active investors lends τ_{M0} and τ_{I0} at $t = 0$. Note that it must be $\tau_{P0} + \tau_{M0} + \tau_{I0} \geq K_0 - \tau_{E0}$. The entrepreneur's IC becomes

$$\lambda_{\bar{e}} \left(\delta_E \tau_{E1} + \delta_E^2 \mathbb{E}_G \left[X - \sum_{i \in \Omega} S_i(X) \right] \right) + (1 - \lambda_{\bar{e}}) \delta_E \left(L - \sum_{i \in \Omega} L_i \right)$$
$$\geq \lambda_{\underline{e}} \left(\delta_E \tau_{E1} + \delta_E^2 \mathbb{E}_G \left[X - \sum_{i \in \Omega} S_i(X) \right] \right) + (1 - \lambda_{\underline{e}}) \delta_E \left(L - \sum_{i \in \Omega} L_i \right) + b.$$

Let us define

$$\hat{b} = \frac{b}{\delta_E^2}$$
$$\hat{\tau}_{E1} = \frac{\tau_{E1}}{\delta_E}$$

With different discount rates, if the entrepreneur is the least patient, it should be that

$$L - \sum_{i \in \Omega} L_i = 0$$

so that the IC simplifies to

$$\hat{\tau}_{E1} + \mathbb{E}_G [X] - \sum_{i \in \Omega} (S_i(X_L) + p_G \Delta S_i) \geq \frac{\hat{b}}{\lambda_{\bar{e}} - \lambda_{\underline{e}}}.$$

Participation constraints

The passive investor's participation constraint is

$$\tau_{P0} \leq \lambda_{\bar{e}} (S_P(X_L) + p_G \Delta S_P + \tau_{P1}) + (1 - \lambda_{\bar{e}}) L_P.$$

Given that

$$L_P = L - \delta \hat{L}_M - \delta \hat{L}_I,$$

this constraint becomes

$$\tau_{P0} \leq \lambda_{\bar{e}} (S_P(X_L) + p_G \Delta S_P + \tau_{P1}) + (1 - \lambda_{\bar{e}})(L - \delta \hat{L}_M - \delta \hat{L}_I).$$

For the first active investor,

$$\tau_{M0} \leq \delta \lambda_{\bar{e}} \tau_{M1} + \delta^2 \lambda_{\bar{e}} (S_M(X_L) + p_G \Delta S_m) + \delta (1 - \lambda_{\bar{e}}) L_M - c_M$$
$$\Rightarrow \tau_{M0} \leq \delta^2 \left[\lambda_{\bar{e}} (S_M(X_L) + p_G \Delta S_m + \hat{\tau}_{M1}) + (1 - \lambda_{\bar{e}}) \hat{L}_M - \hat{c}_M \right].$$

For the second active investor,

$$\tau_{I0} \leq \delta\lambda_{\bar{e}}\tau_{I1} + \delta^2\lambda_{\bar{e}}(S_I(X_L) + p_G\Delta S_I) + \delta(1-\lambda_{\bar{e}})L_I - \delta c_I$$
$$\Rightarrow \tau_{I0} \leq \delta^2\left[\lambda_{\bar{e}}(S_I(X_L) + p_G\Delta S_I + \hat{\tau}_{I1}) + (1-\lambda_{\bar{e}})\hat{L}_I - \hat{c}_I\right].$$

The problem summary

The entrepreneur has the following objective function

$$\max \quad -\lambda_{\bar{e}}\delta_E^2\sum_{i\in\Omega}(S_i(X_L) + p_G\Delta S_i) + \sum_{i\in\Omega}T_i + \lambda_{\bar{e}}\delta_E^2\hat{\tau}_{E1}$$

subject to the following set of constraints:

$$\begin{aligned}
&S_I(X_L) + p_B\Delta S_I + \hat{\tau}_{I1} - \hat{L}_I \leq -\frac{\hat{c}_I}{1-\lambda_{\bar{e}}} \\
&\hat{L}_I - \hat{\tau}_{I1} - S_I(X_L) - p_G\Delta S_I \leq -\frac{\hat{c}_I}{\lambda_{\bar{e}}} \\
&-\hat{\tau}_{M1} - S_M(X_L) - p_G\Delta S_m + \hat{L}_M \leq -\frac{\hat{c}_M}{\lambda_{\bar{e}} - \lambda_{\underline{e}}} \\
&\sum_{i\in\Omega}(S_i(X_L) + p_G\Delta S_i) - \hat{\tau}_{E1} \leq \mathbb{E}_G\left[X\right] - \frac{\hat{b}}{\lambda_{\bar{e}} - \lambda_{\underline{e}}} \\
&\sum_{i\in\Omega}(S_i(X_L) + \Delta S_i) \leq X_H \\
&\sum_{i\in\Omega}S_i(X_L) \leq X_L \\
&\sum_{i\in\Omega}\Delta S_i \leq X_H - X_L \\
&L_M + L_I = \delta\hat{L}_M + \delta\hat{L}_I \leq L \\
&\tau_{P0} \leq \lambda_{\bar{e}}(S_P(X_L) + p_G\Delta S_P + \tau_{P1}) + (1-\lambda_{\bar{e}})(L - \delta\hat{L}_M - \delta\hat{L}_I) \\
&\tau_{M0} \leq \delta^2\left[\lambda_{\bar{e}}(S_M(X_L) + p_G\Delta S_m + \hat{\tau}_{M1}) + (1-\lambda_{\bar{e}})\hat{L}_M - \hat{c}_M\right] \\
&\tau_{I0} \leq \delta^2\left[\lambda_{\bar{e}}(S_I(X_L) + p_G\Delta S_I + \hat{\tau}_{I1}) + (1-\lambda_{\bar{e}})\hat{L}_I - \hat{c}_I\right] \\
&\tau_{P0} + \tau_{M0} + \tau_{I0} \geq K_0 - \tau_{E0} \\
&\delta_E\hat{\tau}_{E1} + \delta\hat{\tau}_{M1} + \delta\hat{\tau}_{I1} + \tau_{P1} = -K_1 \\
&S_i(X_L), \Delta S_i, \hat{L}_M, \hat{L}_I, \tau_{P0}, \tau_{M0}, \tau_{I0}, \hat{\tau}_{E1} \geq 0.
\end{aligned}$$

Note that the sixth and seventh constraint imply the fifth. To summarize, the problem is

$$\begin{aligned}
\max \quad & -\lambda_{\bar{e}}\delta_E^2 \sum_{i\in\Omega} (S_i(X_L) + p_G\Delta S_i) + \sum_{i\in\Omega} T_i + \lambda_{\bar{e}}\delta_E^2 \hat{\tau}_{E1} \\
s.t. & S_I(X_L) + p_B\Delta S_I + \hat{\tau}_{I1} - \hat{L}_I \leq -\frac{\hat{c}_I}{1-\lambda_{\bar{e}}} \\
& \hat{L}_I - \hat{\tau}_{I1} - S_I(X_L) - p_G\Delta S_I \leq -\frac{\hat{c}_I}{\lambda_{\bar{e}}} \\
& -\hat{\tau}_{M1} - S_M(X_L) - p_G\Delta S_m + \hat{L}_M \leq -\frac{\hat{c}_M}{\lambda_{\bar{e}} - \lambda_{\underline{e}}} \\
& \sum_{i\in\Omega}(S_i(X_L) + p_G\Delta S_i) - \hat{\tau}_{E1} \leq \mathbb{E}_G\left[X\right] - \frac{\hat{b}}{\lambda_{\bar{e}} - \lambda_{\underline{e}}} \\
& \sum_{i\in\Omega} S_i(X_L) \leq X_L \\
& \sum_{i\in\Omega} \Delta S_i \leq X_H - X_L \\
& \delta\hat{L}_M + \delta\hat{L}_I \leq L \\
& \tau_{P0} \leq \lambda_{\bar{e}}(S_P(X_L) + p_G\Delta S_P + \tau_{P1}) + (1-\lambda_{\bar{e}})(L - \delta\hat{L}_M - \delta\hat{L}_I) \\
& \tau_{M0} \leq \delta^2\left[\lambda_{\bar{e}}(S_M(X_L) + p_G\Delta S_m + \hat{\tau}_{M1}) + (1-\lambda_{\bar{e}})\hat{L}_M - \hat{c}_M\right] \\
& \tau_{I0} \leq \delta^2\left[\lambda_{\bar{e}}(S_I(X_L) + p_G\Delta S_I + \hat{\tau}_{I1}) + (1-\lambda_{\bar{e}})\hat{L}_I - \hat{c}_I\right] \\
& \tau_{P0} + \tau_{M0} + \tau_{I0} \geq K_0 - \tau_{E0} \\
& \delta_E\hat{\tau}_{E1} + \delta\hat{\tau}_{M1} + \delta\hat{\tau}_{I1} + \tau_{P1} = -K_1 \\
& S_i(X_L), \Delta S_i, \hat{L}_M, \hat{L}_I, \tau_{P0}, \tau_{M0}, \tau_{I0}, \hat{\tau}_{E1} \geq 0.
\end{aligned}$$

A.2 Deriving Feasibility Conditions

Define

$$\begin{aligned}
\hat{\wp}_G &\equiv E_G[X] - \frac{\hat{b}}{\lambda_{\bar{e}} - \lambda_{\underline{e}}} \\
w_s &\equiv \hat{\tau}_{I1} + S_I(X_L) + p_G\Delta S_I \\
w_m &\equiv \hat{\tau}_{M1} + S_M(X_L) + p_G\Delta S_m \\
w_p &\equiv \tau_{P1} + S_P(X_L) + p_G\Delta S_P,
\end{aligned}$$

The problem can be written as

$$
\begin{aligned}
\max \quad & -\lambda_{\bar{e}}\delta_E^2 \sum_{i\in\Omega}(S_i(X_L)+p_G\Delta S_i)+\sum_{i\in\Omega}T_i+\lambda_{\bar{e}}\delta_E^2\hat{\tau}_{E1} \\
& s.t. w_s-(p_G-p_B)\Delta S_I-\hat{L}_I \le -\frac{\hat{c}_I}{1-\lambda_{\bar{e}}} \\
& \hat{L}_I - w_s \le -\frac{\hat{c}_I}{\lambda_{\bar{e}}} \\
& -w_m+\hat{L}_M \le -\frac{\hat{c}_M}{\lambda_{\bar{e}}-\lambda_{\underline{e}}} \\
& \sum_{i\in\Omega}(S_i(X_L)+p_G\Delta S_i)-\hat{\tau}_{E1} \le \hat{\wp}_G \\
& \sum_{i\in\Omega}S_i(X_L) \le X_L \\
& \sum_{i\in\Omega}\Delta S_i \le X_H - X_L \\
& \delta\hat{L}_M+\delta\hat{L}_I \le L \\
& \tau_{P0} \le \lambda_{\bar{e}}w_p+(1-\lambda_{\bar{e}})(L-\delta\hat{L}_M-\delta\hat{L}_I) \\
& \tau_{M0} \le \delta^2\left[\lambda_{\bar{e}}w_m+(1-\lambda_{\bar{e}})\hat{L}_M-\hat{c}_M\right] \\
& \tau_{I0} \le \delta^2\left[\lambda_{\bar{e}}w_s+(1-\lambda_{\bar{e}})\hat{L}_I-\hat{c}_I\right] \\
& \tau_{P0}+\tau_{M0}+\tau_{I0} \ge K_0-\tau_{E0} \\
& \delta_E\hat{\tau}_{E1}+\delta\hat{\tau}_{M1}+\delta\hat{\tau}_{I1}+\tau_{P1} = -K_1 \\
& S_i(X_L), \Delta S_i, \hat{L}_M, \hat{L}_I, \tau_{P0}, \tau_{M0}, \tau_{I0}, \hat{\tau}_{E1} \ge 0.
\end{aligned}
$$

Note that the first and second constraints imply that

$$(p_G-p_B)\Delta S_I \ge w_s-\hat{L}_I+\frac{\hat{c}_I}{1-\lambda_{\bar{e}}} \ge \frac{\hat{c}_I}{\lambda_{\bar{e}}}+\frac{\hat{c}_I}{1-\lambda_{\bar{e}}} > 0$$

We are going to solve this problem recursively.

The $t = 1$ problem after the good signal

For this problem, we take w_p, w_m, w_s, $\hat{L}_M$, $\hat{L}_I$ as state variables and solve for the optimal transfers $\{\tau_{P1}, \hat{\tau}_{M1}, \hat{\tau}_{I1}, \hat{\tau}_{E1}\}$ and security $\{S_i(X_L), \Delta S_i\}$. Specifically, the continuation problem becomes

$$
\begin{aligned}
V_1(w_p, w_m, w_s, \hat{L}_M, \hat{L}_I) = \max\ & \hat{\tau}_{E1} - \sum_{i\in\Omega}(S_i(X_L) + p_G \Delta S_i) \\
s.t.\quad & -(p_G - p_B)\Delta S_I \le \hat{L}_I - w_s - \frac{\hat{c}_I}{1-\lambda_{\bar{e}}} \\
& \sum_{i\in\Omega}(S_i(X_L) + p_G\Delta S_i) - \hat{\tau}_{E1} \le \hat{\wp}_G \\
& \sum_{i\in\Omega} S_i(X_L) \le X_L \\
& \sum_{i\in\Omega} \Delta S_i \le X_H - X_L \\
& \delta_E \hat{\tau}_{E1} + \delta\hat{\tau}_{M1} + \delta\hat{\tau}_{I1} + \tau_{P1} = -K_1 \\
& S_i(X_L), \Delta S_i, \hat{\tau}_{E1} \ge 0.
\end{aligned}
$$

We use the fifth constraint and completely get rid of τ_{P1} by using $\tau_{P1} = -K_1 - (\delta_E\hat{\tau}_{E1} + \delta\hat{\tau}_{M1} + \delta\hat{\tau}_{I1})$. Moreover, we get rid of $S_i(X_L)$ by using

$$
\begin{aligned}
S_P(X_L) &= w_p - \tau_{P1} - p_G\Delta S_P = w_p - p_G\Delta S_P + K_1 + (\delta_E\hat{\tau}_{E1} + \delta\hat{\tau}_{M1} + \delta\hat{\tau}_{I1}) \\
S_M(X_L) &= w_m - \hat{\tau}_{M1} - p_G\Delta S_m \\
S_I(X_L) &= w_s - \hat{\tau}_{I1} - p_G\Delta S_I.
\end{aligned}
$$

Finally, we define

$$
\begin{aligned}
\tilde{w}_p &= w_p + K_1 \\
\tilde{V}_1(\tilde{w}_p, w_m, w_s, \hat{L}_M, \hat{L}_I) &= V_1(w_o, w_m, w_s, \hat{L}_M, \hat{L}_I) + (\tilde{w}_p + w_m + w_s)
\end{aligned}
$$

The problem thus becomes

$$
\begin{aligned}
\tilde{V}_1(\tilde{w}_p, w_m, w_s, \hat{L}_M, \hat{L}_I) = & \max_{\{\hat{\tau}_{E1}, \hat{\tau}_{M1}, \hat{\tau}_{I1}, \Delta S_i\}} \ \hat{\tau}_{E1}(1-\delta_E) + \hat{\tau}_{M1}(1-\delta) + \hat{\tau}_{I1}(1-\delta) \\
& \mu_1:\ -(p_G - p_B)\Delta S_I \le \hat{L}_I - w_s - \frac{\hat{c}_I}{1-\lambda_{\bar{e}}} \\
& \mu_2:\ -(1-\delta)\hat{\tau}_{M1} - (1-\delta)\hat{\tau}_{I1} - \hat{\tau}_{E1}(1-\delta_E) + \tilde{w}_p + w_m + w_s \le \hat{\wp}_G \\
& \mu_3: \delta_E\hat{\tau}_{E1} - (1-\delta)\hat{\tau}_{M1} - (1-\delta)\hat{\tau}_{I1} - p_G(\Delta S_P + \Delta S_m + \Delta S_I) \le X_L - (\tilde{w}_p + w_m + w_s) \\
& \mu_4:\ \sum_{i\in\Omega}\Delta S_i \le X_H - X_L \\
& \mu_5: p_G\Delta S_P - (\delta_E\hat{\tau}_{E1} + \delta\hat{\tau}_{M1} + \delta\hat{\tau}_{I1}) \le \tilde{w}_p \\
& \mu_6: \hat{\tau}_{M1} + p_G\Delta S_m \le w_m \\
& \mu_7: \hat{\tau}_{I1} + p_G\Delta S_I \le w_s \\
& \quad \Delta S_i, \hat{\tau}_{E1} \ge 0.
\end{aligned}
$$

Recall that we have shown it must hold $\Delta S_I > 0$. Therefore, the FOCs are:

$$\begin{aligned}
\Delta S_P &: -p_G\mu_3 + \mu_4 + p_G\mu_5 \geq 0 \\
\Delta S_m &: -p_G\mu_3 + \mu_4 + p_G\mu_6 \geq 0 \\
\Delta S_I &: -(p_G - p_B)\mu_1 - p_G\mu_3 + \mu_4 + p_G\mu_7 = 0 \\
\hat{\tau}_{M1} &: -(1-\delta) - (1-\delta)\mu_2 - (1-\delta)\mu_3 - \delta\mu_5 + \mu_6 = 0 \\
\hat{\tau}_{I1} &: -(1-\delta) - (1-\delta)\mu_2 - (1-\delta)\mu_3 - \delta\mu_5 + \mu_7 = 0 \\
\hat{\tau}_{E1} &: -(1-\delta_E) - (1-\delta_E)\mu_2 + \delta_E\mu_3 - \delta_E\mu_5 \geq 0.
\end{aligned}$$

The fourth equation implies

$$\mu_6 = (1-\delta) + (1-\delta)\mu_2 + (1-\delta)\mu_3 + \delta\mu_5 > 0.$$

The fifth equation implies

$$\mu_7 = (1-\delta) + (1-\delta)\mu_2 + (1-\delta)\mu_3 + \delta\mu_5 > 0.$$

The sixth inequality implies

$$\delta_E\mu_3 \geq (1-\delta_E) + (1-\delta_E)\mu_2 + \delta_E\mu_5 \geq 0.$$

The first inequality implies

$$\mu_4 \geq p_G(\mu_3 - \mu_5) \geq p_G\frac{1}{\delta_E}\left((1-\delta_E) + (1-\delta_E)\mu_2\right) > 0.$$

Moreover, the sixth inequality implies

$$\delta_E(1 + \mu_2 + \mu_3 - \mu_5) \geq 1 + \mu_2 > 0.$$

Therefore,

$$\mu_6 - \mu_5 = (1-\delta)(1 + \mu_2 + \mu_3 - \mu_5) > 0 \Rightarrow \mu_6 > \mu_5.$$

This result implies that the second inequality is slack given the first one, so that

$$\Delta S_m = 0.$$

Similarly,

$$\mu_7 - \mu_5 = (1-\delta)(1 + \mu_2 + \mu_3 - \mu_5) > 0 \Rightarrow \mu_7 > \mu_5 \geq 0.$$

This result implies that $\mu_1 > 0$. Otherwise if $\mu_1 = 0$, then the third equality becomes

$$-p_G\mu_3 + \mu_4 + p_G\mu_7 = 0,$$

which contradicts with the first inequality.

Taken together, these results imply that the constraints associated with $\{\mu_1, \mu_3, \mu_4, \mu_6, \mu_7\}$ are binding, i.e.,

$$
\begin{aligned}
-(p_G - p_B)\Delta S_I &= \hat{L}_I - w_s - \frac{\hat{c}_I}{1-\lambda_{\bar{e}}} \\
\delta_E \hat{\tau}_{E1} - (1-\delta)\hat{\tau}_{M1} - (1-\delta)\hat{\tau}_{I1} - p_G(\Delta S_P + \Delta S_m + \Delta S_I) &= X_L - (\tilde{w}_p + w_m + w_s) \\
\sum_{i\in\Omega} \Delta S_i &= X_H - X_L \\
\hat{\tau}_{M1} + p_G \Delta S_m &= w_m \\
\hat{\tau}_{I1} + p_G \Delta S_I &= w_s
\end{aligned}
$$

These results imply

$$
\begin{aligned}
\Delta S_I &= \frac{1}{p_G - p_B}(w_s + \frac{\hat{c}_I}{1-\lambda_{\bar{e}}} - \hat{L}_I). \\
\delta_E \hat{\tau}_{E1} &= \mathbb{E}_G[X] - (\tilde{w}_p + \delta w_m + \delta w_s) - p_G(1-\delta)\Delta S_I \\
\Delta S_P &= (X_H - X_L) - \Delta S_I \\
\hat{\tau}_{M1} &= w_m \\
\hat{\tau}_{I1} &= w_s - p_G \Delta S_I.
\end{aligned}
$$

From the definition, we know

$$
\begin{aligned}
S_M(X_L) &= w_m - \hat{\tau}_{M1} - p_G \Delta S_m = 0 \\
S_I(X_L) &= w_s - \hat{\tau}_{I1} - p_G \Delta S_I = 0 \\
S_P(X_L) &= X_L \\
\tau_{P1} &= -K_1 - \mathbb{E}_G[X] + \tilde{w}_p + p_G \Delta S_I.
\end{aligned}
$$

We need to make sure the conditions for μ_2, μ_5 and $\Delta S_i, \hat{\tau}_{E1} \geq 0$ hold:

$$
\begin{aligned}
-(1-\delta)\hat{\tau}_{M1} - (1-\delta)\hat{\tau}_{I1} - \hat{\tau}_{E1}(1-\delta_E) + \tilde{w}_p + w_m + w_s &\leq \hat{\wp}_G \\
p_G \Delta S_P - (\delta_E \hat{\tau}_{E1} + \delta \hat{\tau}_{M1} + \delta \hat{\tau}_{I1}) &\leq \tilde{w}_p.
\end{aligned}
$$

The first condition can be simplified into

$$
(1-\delta)\frac{p_G}{p_G - p_B}(w_s + \frac{\hat{c}_I}{1-\lambda_{\bar{e}}} - \hat{L}_I) \leq \mathbb{E}_G[X] - \frac{\delta_E \hat{b}}{\lambda_{\bar{e}} - \lambda_{\underline{e}}} - (\tilde{w}_p + \delta w_m + \delta w_s),
$$

which will always imply $\tau_{E1} \geq 0$ below. The second can be simplified into.

$$
\tilde{w}_p - X_L \leq \tilde{w}_p,
$$

which always holds. The positive constraints include

$$
\Delta S_P \geq 0 \Rightarrow (X_H - X_L) - \frac{1}{p_G - p_B}(w_s + \frac{\hat{c}_I}{1-\lambda_{\bar{e}}} - \hat{L}_I) \geq 0,
$$

and

$$\Delta S_I \geq 0 \Rightarrow \frac{1}{p_G - p_B}(w_s + \frac{\hat{c}_I}{1-\lambda_{\bar{e}}} - \hat{L}_I) \geq 0,$$

which will be redundant given the constraint $\hat{L}_I - w_s \leq -\frac{\hat{c}_I}{\lambda_{\bar{e}}}$ that will be used at the $t = 0$ problem. The last positive constraint is

$$\tau_{E1} \geq 0 \Rightarrow \mathbb{E}_G[X] - (\tilde{w}_p + \delta w_m + \delta w_s) - p_G(1-\delta)\frac{1}{p_G - p_B}(w_s + \frac{\hat{c}_I}{1-\lambda_{\bar{e}}} - \hat{L}_I) \geq 0,$$

which is implied by the first condition above.

Summary: the $t = 1$ problem after the good signal. Given any state variables $(\tilde{w}_p, w_m, w_s, \hat{L}_M, \hat{L}_I)$, the solutions are

$$\begin{aligned}
\Delta S_I &= \frac{1}{p_G - p_B}(w_s + \frac{\hat{c}_I}{1-\lambda_{\bar{e}}} - \hat{L}_I). \\
\Delta S_m &= 0 \\
\Delta S_P &= (X_H - X_L) - \Delta S_I \\
\hat{\tau}_{E1} &= \frac{1}{\delta_E}\left[\mathbb{E}_G[X] - (\tilde{w}_p + \delta w_m + \delta w_s) - p_G(1-\delta)\Delta S_I\right] \\
\hat{\tau}_{M1} &= w_m \\
\hat{\tau}_{I1} &= w_s - p_G \Delta S_I \\
\tau_{P1} &= -K_1 - \mathbb{E}_G[X] + \tilde{w}_p + p_G \Delta S_I \\
S_M(X_L) &= 0 \\
S_I(X_L) &= 0 \\
S_P(X_L) &= X_L.
\end{aligned}$$

The value functions are

$$\begin{aligned}
\delta_E \tilde{V}_1(\tilde{w}_p, w_m, w_s, \hat{L}_M, \hat{L}_I) &= (1-\delta_E)\mathbb{E}_G[X] - (1-\delta_E)\tilde{w}_p + (\delta_E - \delta)w_m + (\delta_E - \delta)w_s \\
&\quad - (1-\delta)\frac{p_G}{p_G - p_B}(w_s + \frac{\hat{c}_I}{1-\lambda_{\bar{e}}} - \hat{L}_I) \\
V_1(\tilde{w}_p, w_m, w_s, \hat{L}_M, \hat{L}_I) &= \frac{1}{\delta_E}\Bigg[(1-\delta_E)\mathbb{E}_G[X] - (\tilde{w}_p + \delta w_m + \delta w_s) \\
&\quad - (1-\delta)\frac{p_G}{p_G - p_B}(w_s + \frac{\hat{c}_I}{1-\lambda_{\bar{e}}} - \hat{L}_I)\Bigg].
\end{aligned}$$

Finally, the solutions require the following conditions

$$\begin{aligned}
&(1-\delta)\frac{p_G}{p_G - p_B}(w_s + \frac{\hat{c}_I}{1-\lambda_{\bar{e}}} - \hat{L}_I) \leq \mathbb{E}_G[X] - \frac{\delta_E \hat{b}}{\lambda_{\bar{e}} - \lambda_{\underline{e}}} - (\tilde{w}_p + \delta w_m + \delta w_s) \\
&w_s + \frac{\hat{c}_I}{1-\lambda_{\bar{e}}} - \hat{L}_I \leq (p_G - p_B)(X_H - X_L)
\end{aligned}$$

The $t = 0$ problem

Now we turn back to the $t = 0$ problem.

$$\begin{aligned}
&\max \quad \sum_{i\in\Omega} T_i + \lambda_{\bar{e}}\delta_E^2 V_1(\tilde{w}_p, w_m, w_s, \hat{L}_M, \hat{L}_I)\\
&(\tilde{w}_p + \delta w_m + \delta w_s) + (1-\delta)\frac{p_G}{p_G - p_B}(w_s + \frac{\hat{c}_I}{1-\lambda_{\bar{e}}} - \hat{L}_I) \le \mathbb{E}_G[X] - \frac{\delta_E \hat{b}}{\lambda_{\bar{e}} - \lambda_{\underline{e}}}\\
&w_s + \frac{\hat{c}_I}{1-\lambda_{\bar{e}}} - \hat{L}_I \le (p_G - p_B)(X_H - X_L)\\
&\hat{L}_I - w_s \le -\frac{\hat{c}_I}{\lambda_{\bar{e}}}\\
&-w_m + \hat{L}_M \le -\frac{\hat{c}_M}{\lambda_{\bar{e}} - \lambda_{\underline{e}}}\\
&\delta\hat{L}_M + \delta\hat{L}_I \le L\\
&\tau_{P0} \le \lambda_{\bar{e}} w_p + (1-\lambda_{\bar{e}})(L - \delta\hat{L}_M - \delta\hat{L}_I)\\
&\tau_{M0} \le \delta^2\left[\lambda_{\bar{e}} w_m + (1-\lambda_{\bar{e}})\hat{L}_M - \hat{c}_M\right]\\
&\tau_{I0} \le \delta^2\left[\lambda_{\bar{e}} w_s + (1-\lambda_{\bar{e}})\hat{L}_I - \hat{c}_I\right]\\
&\tau_{P0} + \tau_{M0} + \tau_{I0} \ge K_0 - \tau_{E0}\\
&\hat{L}_M, \hat{L}_I, \tau_{P0}, \tau_{M0}, \tau_{I0}, w_p, w_m, w_s \ge 0.
\end{aligned}$$

Clearly, the IR constraints of the investors should bind, and $w_m, w_s \ge 0$ are implied by the third and fourth constraints. Substituting τ_{P0}, τ_{M0}, and τ_{I0}, we get the objective function. Let us define

$$\begin{aligned}
Z_m \equiv \hat{L}_M - w_m \Rightarrow w_m = \hat{L}_M - Z_m\\
Z_s \equiv \hat{L}_I - w_s \Rightarrow w_s = \hat{L}_I - Z_s
\end{aligned}$$

which is equivalent to

$$\begin{aligned}
\lambda_{\bar{e}}(1-\delta_E)w_p + (1-\lambda_{\bar{e}})(-\delta\hat{L}_M - \delta\hat{L}_I) + \delta^2(\hat{L}_M - \lambda_{\bar{e}} Z_m) + \delta^2(\hat{L}_I - \lambda_{\bar{e}} Z_s)\\
+\lambda_{\bar{e}}\delta_E\left[-(\delta w_m + \delta w_s) + (1-\delta)\frac{p_G}{p_G - p_B} Z_s\right].
\end{aligned}$$

The first constraint must always bind; otherwise, we can always increase w_p. Therefore,

$$w_p = \mathbb{E}_G[X] - K_1 - \frac{\delta_E \hat{b}}{\lambda_{\bar{e}} - \lambda_{\underline{e}}} - (\delta w_m + \delta w_s) - (1-\delta)\frac{p_G}{p_G - p_B}(w_s + \frac{\hat{c}_I}{1-\lambda_{\bar{e}}} - \hat{L}_I).$$

We plug this into the problem and drop the constants (with very tedious algebra), so that the objective function becomes

$$-\delta(1-\delta)\hat{L}_M - \delta(1-\delta)\hat{L}_I + \lambda_{\bar{e}}\delta(1-\delta)Z_m + \lambda_{\bar{e}}(1-\delta)\left[\delta + \frac{p_G}{p_G - p_B}\right] Z_s.$$

Again, we are going to ignore the constraint $\tau_{P0}+\tau_{M0}+\tau_{I0} \geq K_0-\tau_{E0}$ by later imposing it as a parametric requirement. Therefore, the problem is equivalent to

$$\begin{aligned}
&\max \quad -\delta(1-\delta)\hat{L}_M - \delta(1-\delta)\hat{L}_I + \lambda_{\bar{e}}\delta(1-\delta)Z_m + \lambda_{\bar{e}}(1-\delta)\left[\delta + \frac{p_G}{p_G-p_B}\right]Z_s. \\
&s.t.\ \mathbb{E}_G[X] - K_1 - \frac{\delta_E \hat{b}}{\lambda_{\bar{e}}-\lambda_{\underline{e}}} - \left[\delta(\hat{L}_M - Z_m) + \delta(\hat{L}_I - Z_s)\right] - (1-\delta)\frac{p_G}{p_G-p_B}\left(\frac{\hat{c}_I}{1-\lambda_{\bar{e}}} - Z_s\right) \geq 0 \\
&-Z_s \leq (p_G-p_B)(X_H-X_L) - \frac{\hat{c}_I}{1-\lambda_{\bar{e}}} \\
&Z_s \leq -\frac{\hat{c}_I}{\lambda_{\bar{e}}} \\
&Z_m \leq -\frac{\hat{c}_M}{\lambda_{\bar{e}}-\lambda_{\underline{e}}} \\
&\delta\hat{L}_M + \delta\hat{L}_I \leq L \\
&\hat{L}_M, \hat{L}_I \geq 0.
\end{aligned}$$

Clearly, the solutions are

$$\begin{aligned}
Z_m &= -\frac{\hat{c}_M}{\lambda_{\bar{e}}-\lambda_{\underline{e}}} \\
Z_s &= -\frac{\hat{c}_I}{\lambda_{\bar{e}}} \\
\hat{L}_M &= 0 \\
\hat{L}_I &= 0 \\
\Rightarrow w_m &= \frac{\hat{c}_M}{\lambda_{\bar{e}}-\lambda_{\underline{e}}} \\
\Rightarrow w_s &= \frac{\hat{c}_I}{\lambda_{\bar{e}}} \\
w_p &= \mathbb{E}_G[X] - K_1 - \frac{\delta_E \hat{b}}{\lambda_{\bar{e}}-\lambda_{\underline{e}}} - \left(\delta\frac{\hat{c}_M}{\lambda_{\bar{e}}-\lambda_{\underline{e}}} + \delta\frac{\hat{c}_I}{\lambda_{\bar{e}}}\right) - (1-\delta)\frac{p_G}{p_G-p_B}\frac{\hat{c}_I}{\lambda_{\bar{e}}(1-\lambda_{\bar{e}})} \\
\tau_{P0} &= \lambda_{\bar{e}}\left[\mathbb{E}_G[X] - K_1 - \frac{\delta_E \hat{b}}{\lambda_{\bar{e}}-\lambda_{\underline{e}}} - \left(\delta\frac{\hat{c}_M}{\lambda_{\bar{e}}-\lambda_{\underline{e}}} + \delta\frac{\hat{c}_I}{\lambda_{\bar{e}}}\right) - (1-\delta)\frac{p_G}{p_G-p_B}\frac{\hat{c}_I}{\lambda_{\bar{e}}(1-\lambda_{\bar{e}})}\right] + (1-\lambda_{\bar{e}})L \\
\tau_{M0} &= \delta^2\frac{\lambda_{\underline{e}}}{\lambda_{\bar{e}}-\lambda_{\underline{e}}}\hat{c}_M \\
\tau_{I0} &= 0.
\end{aligned}$$

Moreover, the solutions must satisfy the other constraints, which require

$$\mathbb{E}_G[X] - K_1 - \frac{\delta_E \hat{b}}{\lambda_{\bar{e}}-\lambda_{\underline{e}}} - \left(\delta\frac{\hat{c}_M}{\lambda_{\bar{e}}-\lambda_{\underline{e}}} + \delta\frac{\hat{c}_I}{\lambda_{\bar{e}}}\right) - (1-\delta)\frac{p_G}{p_G-p_B}\frac{\hat{c}_I}{\lambda_{\bar{e}}(1-\lambda_{\bar{e}})} \geq 0$$

and

$$\frac{\hat{c}_I}{\lambda_{\bar{e}}} + \frac{\hat{c}_I}{1-\lambda_{\bar{e}}} \leq (p_G-p_B)(X_H-X_L).$$

Finally, the constraint $\tau_{P0} + \tau_{M0} + \tau_{I0} \geq K_0 - \tau_{E0}$ requires

$$\lambda_{\bar{e}} \left[\mathbb{E}_G [X] - K_1 - \frac{\delta_E \hat{b}}{\lambda_{\bar{e}} - \lambda_{\underline{e}}} - (\delta \frac{\hat{c}_M}{\lambda_{\bar{e}} - \lambda_{\underline{e}}} + \delta \frac{\hat{c}_I}{\lambda_{\bar{e}}}) - (1-\delta) \frac{p_G}{p_G - p_B} \frac{\hat{c}_I}{\lambda_{\bar{e}}(1-\lambda_{\bar{e}})} \right] + (1-\lambda_{\bar{e}})L + \delta^2 \frac{\lambda_{\underline{e}}}{\lambda_{\bar{e}} - \lambda_{\underline{e}}} \hat{c}_M \geq K_0$$

Summary: the $t = 0$ problem The solutions require the following conditions

$$\begin{aligned}
&\mathbb{E}_G [X] - K_1 - \frac{\delta_E \hat{b}}{\lambda_{\bar{e}} - \lambda_{\underline{e}}} - (\delta \frac{\hat{c}_M}{\lambda_{\bar{e}} - \lambda_{\underline{e}}} + \delta \frac{\hat{c}_I}{\lambda_{\bar{e}}}) - (1-\delta) \frac{p_G}{p_G - p_B} \frac{\hat{c}_I}{\lambda_{\bar{e}}(1-\lambda_{\bar{e}})} \geq 0 \\
&\frac{\hat{c}_I}{\lambda_{\bar{e}}} + \frac{\hat{c}_I}{1-\lambda_{\bar{e}}} \leq (p_G - p_B)(X_H - X_L). \\
&\lambda_{\bar{e}} \left[\mathbb{E}_G [X] - K_1 - \frac{\delta_E \hat{b}}{\lambda_{\bar{e}} - \lambda_{\underline{e}}} - (\delta \frac{\hat{c}_M}{\lambda_{\bar{e}} - \lambda_{\underline{e}}} + \delta \frac{\hat{c}_I}{\lambda_{\bar{e}}}) - (1-\delta) \frac{p_G}{p_G - p_B} \frac{\hat{c}_I}{\lambda_{\bar{e}}(1-\lambda_{\bar{e}})} \right] \\
&+ (1-\lambda_{\bar{e}})L + \delta^2 \frac{\lambda_{\underline{e}}}{\lambda_{\bar{e}} - \lambda_{\underline{e}}} \hat{c}_M \geq K_0.
\end{aligned}$$

We want to show that the last constraint implies the first one. From

$$S_I^*(X_H) = \frac{1}{\delta} \frac{1}{\lambda_{\bar{e}}(1-\lambda_{\bar{e}})} \frac{1}{p_G - p_B} c_I = \frac{1}{\delta} \frac{1}{(1-\lambda_{\bar{e}})} \frac{1}{p_G - p_B} \mathcal{R}_I \equiv \xi \mathcal{R}_I,$$

where

$$\xi \equiv \frac{1}{\delta} \frac{1}{(1-\lambda_{\bar{e}})} \frac{1}{p_G - p_B}.$$

Let us rewrite the three conditions

$$\begin{aligned}
\underbrace{[\mathbb{E}_G[X] - K_1 - \mathcal{R}_E]}_{\equiv Z_1} - \underbrace{\frac{K_0 - (1-\lambda_{\bar{e}})L}{\lambda_{\bar{e}}}}_{\equiv Z_2} &\geq \underbrace{\left[(1-\delta) + \frac{\delta(\lambda_{\bar{e}} - \lambda_{\underline{e}})}{\lambda_{\bar{e}}} \right]}_{d} \mathcal{R}_M + \underbrace{[1 + (1-\delta)p_G \xi]}_{b} \mathcal{R}_I \\
\underbrace{[\mathbb{E}_G[X] - K_1 - \mathcal{R}_E]}_{\equiv Z_1} &\geq \mathcal{R}_M + \underbrace{[1 + (1-\delta)p_G \xi]}_{b} \mathcal{R}_I \\
X_H - X_L &\geq \xi \mathcal{R}_I.
\end{aligned}$$

We examine the first two conditions:

$$\begin{aligned}
Z_1 - Z_2 &\geq d\mathcal{R}_M + b\mathcal{R}_I \\
Z_1 &\geq \mathcal{R}_M + b\mathcal{R}_I.
\end{aligned}$$

We know that $Z_1, Z_2 > 0, d < 1$. Therefore, if we draw a graph with $\mathcal{R}_M$ on x-axis and $\mathcal{R}_I$ on y-axis, it is clear that the second condition has a higher intercept at the y-axis. To compare the intercept at the x-axis, we are essentially comparing $\frac{Z_1 - Z_2}{d}$ with Z_1, which after some steps, become a comparison between

$$\lambda_{\underline{e}} [\mathbb{E}_G[X] - K_1] + (1-\lambda_{\bar{e}})L - \lambda_{\bar{e}} \mathcal{R}_E - K_0 \quad v.s. \quad 0,$$

which is implied by

$$\lambda_{\underline{e}}\left[\mathbb{E}_G[X]-K_1\right]+(1-\lambda_{\underline{e}})L-K_0<0,$$

B Proofs of Section 4

We supplement the proofs for all results in Section 4, including Lemma 5, 6, Proposition 7 and 8.

B.1 Constraints and Problem Summary

IC constraints

Let us define

$$\hat{L}_A \equiv \frac{L_A}{\delta}, \quad \hat{c}_I \equiv \frac{c_I}{\delta}, \quad \hat{c}_M \equiv \frac{c_M}{\delta^2}, \quad \hat{\tau}_{A1} \equiv \frac{\tau_{A1}}{\delta}.$$

1. The project is liquidated only after the bad signal

$$\begin{aligned} &\tau_{A1}+\delta\mathbb{E}_B[S_A(X)] \le L_A \le \tau_{A1}+\delta\mathbb{E}_G[S_A(X)] \\ &\Rightarrow \hat{\tau}_{A1}+\mathbb{E}_B[S_A(X)] \le \hat{L}_A \le \hat{\tau}_{A1}+\mathbb{E}_G[S_A(X)]. \end{aligned}$$

2. The investor has incentives to acquire signal at $t=1$, given that he has monitored at $t=0$

$$\begin{aligned} &\lambda_{\bar{e}}\big(\tau_{A1}+\delta\mathbb{E}_G[S_A(X)]\big)+(1-\lambda_{\bar{e}})L_A-c_I \ge \max\{\tau_{A1}+\delta\mathbb{E}_{\bar{e}}[S_A(X)],L_A\} \\ &\Rightarrow \lambda_{\bar{e}}\big(\hat{\tau}_{A1}+\mathbb{E}_G\left[S_A(X)\right]\big)+(1-\lambda_{\bar{e}})\hat{L}_A-\hat{c}_I \ge \max\{\hat{\tau}_{A1}+\mathbb{E}_{\bar{e}}[S_A(X)],\hat{L}_A\}. \end{aligned}$$

3. IC constraints on monitoring.

   - Single deviation: even if he did not monitor, he would still evaluate the project at $t=1$

$$\begin{aligned} &\lambda_{\bar{e}}\big(\delta\tau_{A1}+\delta^2\mathbb{E}_G\left[S_A(X)\right]\big)+(1-\lambda_{\bar{e}})\delta L_A-c_M \ge \lambda_{\underline{e}}\big(\delta\tau_{A1}+\delta^2\mathbb{E}_G\left[S_A(X)\right]\big)+(1-\lambda_{\underline{e}})\delta L_A \\ &\Rightarrow \lambda_{\bar{e}}\big(\hat{\tau}_{A1}+\mathbb{E}_G\left[S_A(X)\right]\big)+(1-\lambda_{\bar{e}})\hat{L}_A-\hat{c}_M \ge \lambda_{\underline{e}}\big(\hat{\tau}_{A1}+\mathbb{E}_G\left[S_A(X)\right]\big)+(1-\lambda_{\underline{e}})\hat{L}_A \\ &\Rightarrow \hat{\tau}_{A1}+\mathbb{E}_G\left[S_A(X)\right]-\hat{L}_A \ge \frac{\hat{c}_M}{(\lambda_{\bar{e}}-\lambda_{\underline{e}})}. \end{aligned}$$

   - Double deviation: if he did not monitor, then he would not evaluate the project either

$$\begin{aligned} &\lambda_{\bar{e}}\big(\delta\tau_{A1}+\delta^2\mathbb{E}_G\left[S_A(X)\right]\big)+(1-\lambda_{\bar{e}})\delta L_A-\delta c_I-c_M \ge \max\left\{\delta\tau_{A1}+\delta^2\mathbb{E}_{\underline{e}}\left[S_A(X)\right],\delta L_A\right\} \\ &\Rightarrow \lambda_{\bar{e}}\big(\hat{\tau}_{A1}+\mathbb{E}_G\left[S_A(X)\right]\big)+(1-\lambda_{\bar{e}})\hat{L}_A-\hat{c}_I-\hat{c}_M \ge \max\left\{\hat{\tau}_{A1}+\mathbb{E}_{\underline{e}}\left[S_A(X)\right],\hat{L}_A\right\}. \end{aligned}$$

The IC constraints are satisfied if and only if

$$\begin{aligned} &\hat{\tau}_{A1}+\mathbb{E}_G[S_A(X)]-\hat{L}_A \ge \frac{\phi}{\lambda_{\bar{e}}} \Rightarrow \tau_{A1}+\delta\mathbb{E}_G[S_A(X)]-L_A \ge \frac{\delta\phi}{\lambda_{\bar{e}}} \\ &\hat{L}_A-\hat{\tau}_{A1}-\mathbb{E}_B[S_A(X)] \ge \frac{\hat{c}_I}{1-\lambda_{\bar{e}}} \Rightarrow L_A-\tau_{A1}-\delta\mathbb{E}_B[S_A(X)] \ge \frac{c_I}{1-\lambda_{\bar{e}}}. \end{aligned}$$

where

$$\phi \equiv \max\left\{\hat{c}_M + \hat{c}_I, \frac{\lambda_{\bar{e}}}{\lambda_{\bar{e}} - \lambda_{\underline{e}}}\hat{c}_M\right\}.$$

The first IC has two cases. Case 1 is the double deviation constraint when $\phi = \hat{c}_M + \hat{c}_I$, and the binding constraint says if he did not monitor, he would always liquidate. The second case is the single deviation constraint. The second IC is the screening constraint, and the binding one says if he did not screen, he always continues. Let us rewrite the set of constraints:

$$\begin{aligned}
\hat{\tau}_{A1} + \mathbb{E}_B[S_A(X)] \leq \hat{L}_A &\leq \hat{\tau}_{A1} + \mathbb{E}_G[S_A(X)] \\
\lambda_{\bar{e}}\big(\hat{\tau}_{A1} + \mathbb{E}_G[S_A(X)]\big) + (1-\lambda_{\bar{e}})\hat{L}_A - \hat{c}_I &\geq \max\{\hat{\tau}_{A1} + \mathbb{E}_{\bar{e}}[S_A(X)], \hat{L}_A\} \\
\hat{\tau}_{A1} + \mathbb{E}_G\left[S_A(X)\right] - \hat{L}_A &\geq \frac{\hat{c}_M}{\lambda_{\bar{e}} - \lambda_{\underline{e}}} \\
\lambda_{\bar{e}}\big(\hat{\tau}_{A1} + \mathbb{E}_G\left[S_A(X)\right]\big) + (1-\lambda_{\bar{e}})\hat{L}_A - \hat{c}_I - \hat{c}_M &\geq \max\left\{\hat{\tau}_{A1} + \mathbb{E}_{\underline{e}}\left[S_A(X)\right], \hat{L}_A\right\},
\end{aligned}$$

The second constraint becomes

$$\begin{aligned}
\hat{L}_A - \hat{\tau}_{A1} - \mathbb{E}_B[S_A(X)] &\geq \frac{\hat{c}_I}{1-\lambda_{\bar{e}}} \\
\hat{\tau}_{A1} + \mathbb{E}_G[S_A(X)] - \hat{L}_A &\geq \frac{\hat{c}_I}{\lambda_{\bar{e}}} \qquad (69)
\end{aligned}$$

Given so, the first constraint is redundant. The fourth constraint implies

$$\hat{\tau}_{A1} + \mathbb{E}_G\left[S_A(X)\right] - \hat{L}_A \geq \frac{\hat{c}_I + \hat{c}_M}{\lambda_{\bar{e}}}, \qquad \text{double deviation always liquidate}$$

so that (69) is redundant. The fourth constraint also implies

$$\lambda_{\bar{e}}\mathbb{E}_G\left[S_A(X)\right] + (1-\lambda_{\bar{e}})(\hat{L}_A - \hat{\tau}_{A1}) - \hat{c}_I - \hat{c}_M \geq \lambda_{\underline{e}}\mathbb{E}_G\left[S_A(X)\right] + (1-\lambda_{\underline{e}})\mathbb{E}_B\left[S_A(X)\right]$$

which we are going to show is also redundant. Specifically,

$$\begin{aligned}
\lambda_{\bar{e}}\mathbb{E}_G\left[S_A(X)\right] + (1-\lambda_{\bar{e}})(\hat{L}_A - \hat{\tau}_{A1}) - \hat{c}_I - \hat{c}_M &\geq \lambda_{\bar{e}}\mathbb{E}_G\left[S_A(X)\right] + (1-\lambda_{\bar{e}})(\mathbb{E}_B[S_A(X)] + \frac{\hat{c}_I}{1-\lambda_{\bar{e}}}) - \hat{c}_I - \hat{c}_M \\
&= \lambda_{\bar{e}}\mathbb{E}_G\left[S_A(X)\right] + (1-\lambda_{\bar{e}})\mathbb{E}_B[S_A(X)] - \hat{c}_M,
\end{aligned}$$

so that is suffices to show

$$\begin{aligned}
\lambda_{\bar{e}}\mathbb{E}_G\left[S_A(X)\right] + (1-\lambda_{\bar{e}})\mathbb{E}_B[S_A(X)] - \hat{c}_M &\geq \lambda_{\underline{e}}\mathbb{E}_G\left[S_A(X)\right] + (1-\lambda_{\underline{e}})\mathbb{E}_B\left[S_A(X)\right] \\
\Rightarrow \mathbb{E}_G\left[S_A(X)\right] - \mathbb{E}_B\left[S_A(X)\right] &\geq \frac{\hat{c}_M}{\lambda_{\bar{e}} - \lambda_{\underline{e}}}. \qquad \text{single deviation}
\end{aligned}$$

This last condition holds because of the third constraint

$$\hat{\tau}_{A1} + \mathbb{E}_G\left[S_A(X)\right] - \hat{L}_A \geq \frac{\hat{c}_M}{\lambda_{\bar{e}} - \lambda_{\underline{e}}}$$

and

$$\hat{L}_A - \hat{\tau}_{A1} - \mathbb{E}_B[S_A(X)] \geq \frac{\hat{c}_I}{1-\lambda_{\bar{e}}}.$$

Summary This lemma above implies that some IC constraints are always slack; some may be slack; some always bind

- Always slack:
  - The project is liquidated after the bad signal but not after the good signal

$$\hat{\tau}_{A1} + \mathbb{E}_B[S_A(X)] \leq \hat{L}_A \leq \hat{\tau}_{A1} + \mathbb{E}_G[S_A(X)].$$

  - The active investor prefers acquiring information to always liquidating the project

$$\lambda_{\bar{e}}\hat{\tau}_{A1} + \lambda_{\bar{e}}\mathbb{E}_G[S_A(X)] + (1-\lambda_{\bar{e}})\hat{L}_A - \hat{c}_I \geq \hat{L}_A.$$

  - Double deviation: if he did not monitor he would not evaluate the project and always continue

$$\lambda_{\bar{e}}\mathbb{E}_G\left[S_A(X)\right] + (1-\lambda_{\bar{e}}))(\hat{L}_A - \hat{\tau}_{A1}) - \hat{c}_I - \hat{c}_M \geq \mathbb{E}_{\underline{e}}\left[S_A(X)\right]$$

- May be slack/binding (the stronger of the two binds)
  - Single deviation:

$$\mathbb{E}_G\left[S_A(X)\right] + \hat{\tau}_{A1} - \hat{L}_A \geq \frac{\hat{c}_M}{(\lambda_{\bar{e}} - \lambda_{\underline{e}})}$$

  - Double deviation: if he did not monitor, he would not evaluate the project and always liquidate

$$\lambda_{\bar{e}}\hat{\tau}_{A1} + \lambda_{\bar{e}}\mathbb{E}_G\left[S_A(X)\right] + (1-\lambda_{\bar{e}})\hat{L}_A - \hat{c}_I - \hat{c}_M \geq \hat{L}_A.$$

- Always binding:
  - The active investor prefers acquiring information to always continuing the project

$$\lambda_{\bar{e}}\mathbb{E}_G[S_A(X)] + (1-\lambda_{\bar{e}})(\hat{L}_A - \hat{\tau}_{A1}) - \hat{c}_I \geq \mathbb{E}_{\bar{e}}[S_A(X)].$$

Entrepreneur's IC constraint

The passive investor lends τ_{P0} and the active investor lends τ_{A0} at $t=0$. The entrepreneur's IC becomes

$$\begin{aligned}
&\lambda_{\bar{e}}\left(\delta_E\tau_{E1} + \delta_E^2\mathbb{E}_G\left[X - \sum_{i\in\Omega} S_i(X)\right]\right) + (1-\lambda_{\bar{e}})\delta_E\left(L - \sum_{i\in\Omega} L_i\right) \\
&\geq \lambda_{\underline{e}}\left(\delta_E\tau_{E1} + \delta_E^2\mathbb{E}_G\left[X - \sum_{i\in\Omega} S_i(X)\right]\right) + (1-\lambda_{\underline{e}})\delta_E\left(L - \sum_{i\in\Omega} L_i\right) + b.
\end{aligned}$$

Let us define

$$\hat{b} = \frac{b}{\delta_E^2}$$
$$\hat{\tau}_{E1} = \frac{\tau_{E1}}{\delta_E}$$

With different discount rates, if the entrepreneur is the least patient, it should be that

$$L - \sum_{i \in \Omega} L_i = 0$$

so that the IC simplifies to

$$\hat{\tau}_{E1} + \mathbb{E}_G[X] - \sum_{i \in \Omega} (S_i(X_L) + p_G \Delta S_i) \geq \frac{\hat{b}}{\lambda_{\bar{e}} - \lambda_{\underline{e}}}.$$

Participation constraints

The passive investor's participation constraint is

$$\tau_{P0} \leq \lambda_{\bar{e}}(S_P(X_L) + p_G \Delta S_P + \tau_{P1}) + (1 - \lambda_{\bar{e}}) L_0.$$

Given that

$$L_0 = L - \delta \hat{L}_A,$$

this constraint becomes

$$\tau_{P0} \leq \lambda_{\bar{e}}(S_P(X_L) + p_G \Delta S_P + \tau_{P1}) + (1 - \lambda_{\bar{e}})(L - \delta \hat{L}_A).$$

Turning to the active investor's participation constraint,

$$\tau_{A0} \leq \lambda_{\bar{e}}(\delta \tau_{A1} + \delta^2 (S_A(X_L) + p_G \Delta S_A)) + \delta(1 - \lambda_{\bar{e}}) L_A - \delta c_I - c_M$$
$$\Rightarrow \tau_{A0} \leq \delta^2 \left[\lambda_{\bar{e}}(\hat{\tau}_{A1} + S_A(X_L) + p_G \Delta S_A) + (1 - \lambda_{\bar{e}}) \hat{L}_A - \hat{c}_I - \hat{c}_M \right].$$

The limited liabilitiy constraint at $t = 1$ requires that

$$\hat{\tau}_{A1} + S_A(X_L) + p_G \Delta S_A \geq 0$$
$$\tau_{P1} + S_P(X_L) + p_G \Delta S_P \geq 0.$$

The problem summary

The entrepreneur has the following objective function

$$\max \quad -\lambda_{\bar{e}} \delta_E^2 \sum_{i \in \Omega} (S_i(X_L) + p_G \Delta S_i) + \sum_{i \in \Omega} T_i + \lambda_{\bar{e}} \delta_E^2 \hat{\tau}_{E1},$$

subject to the following set of constraints:

$$
\begin{aligned}
&\hat{L}_A - \hat{\tau}_{A1} - S_A(X_L) - p_G \Delta S_A \leq -\frac{\phi}{\lambda_{\bar{e}}} \\
&\hat{\tau}_{A1} + S_A(X_L) + p_B \Delta S_A - \hat{L}_A \leq -\frac{\hat{c}_I}{1-\lambda_{\bar{e}}} \\
&\sum_{i \in \Omega} (S_i(X_L) + p_G \Delta S_i) - \hat{\tau}_{E1} \leq E_G[X] - \frac{\hat{b}}{\lambda_{\bar{e}} - \lambda_{\underline{e}}} \\
&\sum_{i \in \Omega} (S_i(X_L) + \Delta S_i) \leq X_H \\
&\sum_{i \in \Omega} S_i(X_L) \leq X_L \\
&\sum_{i \in \Omega} \Delta S_i \leq X_H - X_L \\
&\delta \hat{L}_A \leq L \\
&\tau_{P0} \leq \lambda_{\bar{e}}(\tau_{P1} + S_P(X_L) + p_G \Delta S_P) + (1-\lambda_{\bar{e}})(L - \delta \hat{L}_A) \\
&\tau_{A0} \leq \delta^2 \left[\lambda_{\bar{e}}(\hat{\tau}_{A1} + S_A(X_L) + p_G \Delta S_A) + (1-\lambda_{\bar{e}})\hat{L}_A - \hat{c}_I - \hat{c}_M \right] \\
&\tau_{A0} + \tau_{P0} \geq K_0 - \tau_{E0} \\
&\delta_E \hat{\tau}_{E1} + \delta \hat{\tau}_{A1} + \tau_{P1} = -K_1 \\
&S_i(X_L), \Delta S_i, \hat{L}_A, \tau_{P0}, \tau_{A0}, \hat{\tau}_{E1} \geq 0.
\end{aligned}
$$

Note that the fifth and sixth constraint implies the fourth one. To summarize, the problem is

$$
\begin{aligned}
\max \quad & -\lambda_{\bar{e}} \delta_E^2 \sum_{i \in \Omega} (S_i(X_L) + p_G \Delta S_i) + \sum_{i \in \Omega} T_i + \lambda_{\bar{e}} \delta_E^2 \hat{\tau}_{E1}, \\
s.t. \, & \hat{L}_A - \hat{\tau}_{A1} - S_A(X_L) - p_G \Delta S_A \leq -\frac{\phi}{\lambda_{\bar{e}}} \\
& \hat{\tau}_{A1} + S_A(X_L) + p_B \Delta S_A - \hat{L}_A \leq -\frac{\hat{c}_I}{1-\lambda_{\bar{e}}} \\
& \sum_{i \in \Omega} (S_i(X_L) + p_G \Delta S_i) - \hat{\tau}_{E1} \leq E_G[X] - \frac{\hat{b}}{\lambda_{\bar{e}} - \lambda_{\underline{e}}} \\
& \sum_{i \in \Omega} S_i(X_L) \leq X_L \\
& \sum_{i \in \Omega} \Delta S_i \leq X_H - X_L \\
& \delta \hat{L}_A \leq L \\
& \tau_{P0} \leq \lambda_{\bar{e}}(\tau_{P1} + S_P(X_L) + p_G \Delta S_P) + (1-\lambda_{\bar{e}})(L - \delta \hat{L}_A) \\
& \tau_{A0} \leq \delta^2 \left[\lambda_{\bar{e}}(\hat{\tau}_{A1} + S_A(X_L) + p_G \Delta S_A) + (1-\lambda_{\bar{e}})\hat{L}_A - \hat{c}_I - \hat{c}_M \right] \\
& \tau_{A0} + \tau_{P0} \geq K_0 - \tau_{E0} \\
& \delta_E \hat{\tau}_{E1} + \delta \hat{\tau}_{A1} + \tau_{P1} = -K_1 \\
& S_i(X_L), \Delta S_i, \hat{L}_A, \tau_{P0}, \tau_{A0}, \hat{\tau}_{E1} \geq 0.
\end{aligned}
$$

B.2 Deriving Feasibility Conditions

Define

$$
\begin{aligned}
\hat{\wp}_G &\equiv \mathbb{E}_G[X] - \frac{\hat{b}}{\lambda_{\bar{e}} - \lambda_{\underline{e}}} \\
w_A &\equiv \hat{\tau}_{A1} + S_A(X_L) + p_G \Delta S_A \\
w_P &\equiv \tau_{P1} + S_P(X_L) + p_G \Delta S_P.
\end{aligned}
$$

The problem can be written as

$$
\begin{aligned}
\max \quad & -\lambda_{\bar{e}} \delta_E^2 \sum_{i \in \Omega} (S_i(X_L) + p_G \Delta S_i) + \sum_{i \in \Omega} T_i + \lambda_{\bar{e}} \delta_E^2 \hat{\tau}_{E1}, \\
s.t. & \hat{L}_A - w_A \leq -\frac{\phi}{\lambda_{\bar{e}}} \\
& w_A - (p_G - p_B)\Delta S_A - \hat{L}_A \leq -\frac{\hat{c}_I}{1 - \lambda_{\bar{e}}} \\
& \sum_{i \in \Omega} (S_i(X_L) + p_G \Delta S_i) - \hat{\tau}_{E1} \leq \hat{\wp}_G \\
& \sum_{i \in \Omega} S_i(X_L) \leq X_L \\
& \sum_{i \in \Omega} \Delta S_i \leq X_H - X_L \\
& \delta \hat{L}_A \leq L \\
& \tau_{P0} \leq \lambda_{\bar{e}} w_P + (1 - \lambda_{\bar{e}})(L - \delta \hat{L}_A) \\
& \tau_{A0} \leq \delta^2 \left[\lambda_{\bar{e}} w_A + (1 - \lambda_{\bar{e}}) \hat{L}_A - \hat{c}_I - \hat{c}_M \right] \\
& \tau_{A0} + \tau_{P0} \geq K_0 - \tau_{E0} \\
& \delta_E \hat{\tau}_{E1} + \delta \hat{\tau}_{A1} + \tau_{P1} = -K_1 \\
& S_i(X_L), \Delta S_i, \hat{L}_A, \tau_{P0}, \tau_{A0}, \hat{\tau}_{E1} \geq 0.
\end{aligned}
$$

We are going to write this problem recursively. Note that using the first constraint, the second constraint can be written as

$$
-(p_G - p_B)\Delta S_A \leq -w_A + \hat{L}_A - \frac{\hat{c}_I}{1 - \lambda_{\bar{e}}} \leq -\frac{\phi}{\lambda_{\bar{e}}} - \frac{\hat{c}_I}{1 - \lambda_{\bar{e}}} \leq 0
$$

so that

$$
\Delta S_A \geq \frac{1}{p_G - p_B} \left(\frac{\phi}{\lambda_{\bar{e}}} + \frac{\hat{c}_I}{1 - \lambda_{\bar{e}}} \right) > 0.
$$

The $t = 1$ problem after the good signal

For this problem, we take w_P, w_A, $\hat{L}_A$ as state variables and solve for the optimal transfers $\{\tau_{P1}, \hat{\tau}_{A1}, \hat{\tau}_{E1}\}$ and security $\{S_i(X_L), \Delta S_i\}$. Specifically, the continuation problem becomes

$$
\begin{aligned}
V_1(w_P, w_A, \hat{L}_A) = \max_{\{\tau_{P1}, \hat{\tau}_{A1}, \hat{\tau}_{E1}, S_i(X_L), \Delta S_i\}} & -\sum_{i\in\Omega} (S_i(X_L) + p_G \Delta S_i) + \hat{\tau}_{E1}, \\
s.t.\ w_A - (p_G - p_B)\Delta S_A - \hat{L}_A \leq & -\frac{\hat{c}_I}{1-\lambda_{\bar{e}}} \\
\sum_{i\in\Omega} (S_i(X_L) + p_G \Delta S_i) - \hat{\tau}_{E1} & \leq \hat{\wp}_G \\
\sum_{i\in\Omega} S_i(X_L) & \leq X_L \\
\sum_{i\in\Omega} \Delta S_i & \leq X_H - X_L \\
\delta_E \hat{\tau}_{E1} + \delta \hat{\tau}_{A1} + \tau_{P1} & = -K_1 \\
S_i(X_L), \Delta S_i, \hat{\tau}_{E1} & \geq 0.
\end{aligned}
$$

We use the fifth constraint and completely get rid of τ_{P1} by using $\tau_{P1} = -K_1 - (\delta_E \hat{\tau}_{E1} + \delta \hat{\tau}_{A1})$. Moreover, we get rid of $S_P(X_L)$ and $S_A(X_L)$ in the problem by using

$$
\begin{aligned}
S_A(X_L) &= w_A - \hat{\tau}_{A1} - p_G \Delta S_A \\
S_P(X_L) &= w_P - \tau_{P1} - p_G \Delta S_P = w_P + K_1 + (\delta_E \hat{\tau}_{E1} + \delta \hat{\tau}_{A1}) - p_G \Delta S_P.
\end{aligned}
$$

Finally, we define

$$
\begin{aligned}
\tilde{w}_P &= w_P + K_1 \\
\tilde{V}_1(\tilde{w}_P, w_A, \hat{L}_A) &= V_1(w_P, w_A, \hat{L}_A) + (\tilde{w}_P + w_A)
\end{aligned}
$$

so that the problem becomes

$$
\begin{aligned}
\tilde{V}_1(\tilde{w}_P, w_A, \hat{L}_A) = \max_{\{\hat{\tau}_{A1}, \hat{\tau}_{E1}, \Delta S_i\}} & \ \hat{\tau}_{A1}(1-\delta) + \hat{\tau}_{E1}(1-\delta_E) \\
\mu_1 :\ -(p_G - p_B)\Delta S_A & \leq \hat{L}_A - w_A - \frac{\hat{c}_I}{1-\lambda_{\bar{e}}} \\
\mu_2 :\ -(1-\delta)\hat{\tau}_{A1} - (1-\delta_E)\hat{\tau}_{E1} & \leq \hat{\wp}_G - \tilde{w}_P - w_A \\
\mu_3 : \delta_E \hat{\tau}_{E1} - (1-\delta)\hat{\tau}_{A1} - p_G(\Delta S_P + \Delta S_A) & \leq X_L - (\tilde{w}_P + w_A) \\
\mu_4 : \sum_{i\in\Omega} \Delta S_i & \leq X_H - X_L \\
\mu_5 :\ -(\delta_E \hat{\tau}_{E1} + \delta \hat{\tau}_{A1}) + p_G \Delta S_P & \leq \tilde{w}_P \\
\mu_6 : p_G \Delta S_A + \hat{\tau}_{A1} & \leq w_A \\
\Delta S_i, \hat{\tau}_{E1} & \geq 0.
\end{aligned}
$$

Recall that we have shown it must hold $\Delta S_A > 0$. Therefore, the FOCs are

$$\begin{aligned}
\Delta S_P: &\quad -\mu_3 p_G + \mu_4 + p_G \mu_5 \geq 0 \\
\Delta S_A: &\quad -(p_G - p_B)\mu_1 - p_G \mu_3 + \mu_4 + p_G \mu_6 = 0 \\
\hat{\tau}_{A1}: &\quad -(1-\delta) - (1-\delta)\mu_2 - (1-\delta)\mu_3 - \delta\mu_5 + \mu_6 = 0 \\
\hat{\tau}_{E1}: &\quad -(1-\delta_E) - (1-\delta_E)\mu_2 + \delta_E \mu_3 - \delta_E \mu_5 \geq 0.
\end{aligned}$$

The third equation tells us

$$\mu_6 = (1-\delta) + (1-\delta)\mu_2 + (1-\delta)\mu_3 + \delta\mu_5 > 0.$$

The last inequality tells us

$$\delta_E \mu_3 \geq (1-\delta_E) + (1-\delta_E)\mu_2 + \delta_E \mu_5 \geq 0,$$

and with this result, the first inequality tells us

$$\mu_4 \geq p_G(\mu_3 - \mu_5) \geq p_G \frac{(1-\delta_E) + (1-\delta_E)\mu_2}{\delta_E} > 0.$$

Moreover, the second equality becomes

$$\begin{aligned}
&p_G(1-\delta) - (p_G - p_B)\mu_1 + p_G(1-\delta)\mu_2 - p_G \delta\mu_3 + \mu_4 + p_G \delta\mu_5 = 0 \\
\Rightarrow (p_G - p_B)\mu_1 = &\, p_G(1-\delta) + p_G(1-\delta)\mu_2 + (1-\delta)\mu_4 + \delta(\underbrace{-p_G\mu_3 + \mu_4 + p_G\mu_5}_{\geq 0}) > 0,
\end{aligned}$$

Therefore, the constraints associated with μ_1, μ_3, μ_4, μ_6 must bind:

$$\begin{aligned}
&-(p_G - p_B)\Delta S_A = \hat{L}_A - w_A - \frac{\hat{c}_I}{1-\lambda_{\bar{e}}} \\
&\delta_E \hat{\tau}_{E1} - (1-\delta)\hat{\tau}_{A1} - p_G(\Delta S_P + \Delta S_A) = X_L - (\tilde{w}_P + w_A) \\
&\Delta S_P + \Delta S_A = X_H - X_L \\
&\hat{\tau}_{A1} = w_A - p_G \Delta S_A.
\end{aligned}$$

From here, we get the solutions

$$\begin{aligned}
\Delta S_A &= \frac{1}{p_G - p_B}(w_A + \frac{\hat{c}_I}{1-\lambda_{\bar{e}}} - \hat{L}_A) \\
\Delta S_P &= (X_H - X_L) - \frac{1}{p_G - p_B}(w_A + \frac{\hat{c}_I}{1-\lambda_{\bar{e}}} - \hat{L}_A) \\
\hat{\tau}_{A1} &= \frac{-p_B}{p_G - p_B} w_A - \frac{p_G}{p_G - p_B}(\frac{\hat{c}_I}{1-\lambda_{\bar{e}}} - \hat{L}_A) \\
\hat{\tau}_{E1} &= \frac{1}{\delta_E}\left(\mathbb{E}_G[X] - (\tilde{w}_P + w_A) + (1-\delta)\hat{\tau}_{A1}\right)
\end{aligned}$$

The solutions must satisfy the constraint associated with μ_2, μ_5 and the positive ones $\Delta S_i, \hat{\tau}_{E1} \geq 0$

$$
\begin{aligned}
&-(1-\delta)\hat{\tau}_{A1} - (1-\delta_E)\hat{\tau}_{E1} \leq \hat{\wp}_G - \tilde{w}_P - w_A \\
&-(\delta_E \hat{\tau}_{E1} + \delta \hat{\tau}_{A1}) + p_G \Delta S_P \leq \tilde{w}_P \\
&\frac{1}{p_G - p_B}(w_A + \frac{\hat{c}_I}{1-\lambda_{\bar{e}}} - \hat{L}_A) \geq 0 \\
&(X_H - X_L) - \frac{1}{p_G - p_B}(w_A + \frac{\hat{c}_I}{1-\lambda_{\bar{e}}} - \hat{L}_A) \geq 0 \\
&\frac{1}{\delta_E}\left(\mathbb{E}_G[X] - (\tilde{w}_P + w_A) + (1-\delta)\hat{\tau}_{A1}\right) \geq 0
\end{aligned}
$$

The first constraint can be simplified into

$$
\mathbb{E}_G[X] - (\tilde{w}_P + w_A) + (1-\delta)\hat{\tau}_{A1} \geq \frac{\delta_E \hat{b}}{\lambda_{\bar{e}} - \lambda_{\underline{e}}},
$$

which implies the last one. We can further show that this is equivalent to

$$
\tilde{w}_P + \frac{p_G - \delta p_B}{p_G - p_B} w_A - (1-\delta)\frac{p_G}{p_G - p_B}\hat{L}_A \leq \mathbb{E}_G[X] - \frac{\delta_E \hat{b}}{\lambda_{\bar{e}} - \lambda_{\underline{e}}} - (1-\delta)\frac{p_G}{p_G - p_B}\frac{\hat{c}_I}{1-\lambda_{\bar{e}}}
$$

The second can be simplified into

$$
-X_L \leq 0.
$$

The third is equivalent to

$$
w_A + \frac{\hat{c}_I}{1-\lambda_{\bar{e}}} - \hat{L}_A \geq 0,
$$

which is redundant given the constraint $\hat{L}_A - w_A \leq -\frac{\phi}{\lambda_{\bar{e}}}$ that will be used at the $t = 0$ problem. The fourth

$$
w_A + \frac{\hat{c}_I}{1-\lambda_{\bar{e}}} - \hat{L}_A \leq (p_G - p_B)(X_H - X_L)
$$

Summary: the $t = 1$ problem after the good signal Given any state variables $(w_P, w_A, \hat{L}_A)$, the solutions are

$$
\begin{aligned}
\Delta S_A &= \frac{1}{p_G - p_B}(w_A + \frac{\hat{c}_I}{1-\lambda_{\bar{e}}} - \hat{L}_A) \\
\Delta S_P &= (X_H - X_L) - \frac{1}{p_G - p_B}(w_A + \frac{\hat{c}_I}{1-\lambda_{\bar{e}}} - \hat{L}_A) \\
\hat{\tau}_{A1} &= \frac{-p_B}{p_G - p_B} w_A - \frac{p_G}{p_G - p_B}(\frac{\hat{c}_I}{1-\lambda_{\bar{e}}} - \hat{L}_A) \\
\hat{\tau}_{E1} &= \frac{1}{\delta_E}\left(\mathbb{E}_G[X] - (\tilde{w}_P + w_A) + (1-\delta)\hat{\tau}_{A1}\right) \\
\tau_{P1} &= -K_1 - (\mathbb{E}_G[X] - (\tilde{w}_P + w_A) + \hat{\tau}_{A1}) \\
S_A(X_L) &= w_A - \hat{\tau}_{A1} - p_G \Delta S_A \\
S_P(X_L) &= w_P + K_1 + (\delta_E \hat{\tau}_{E1} + \delta \hat{\tau}_{A1}) - p_G \Delta S_P.
\end{aligned}
$$

The value functions are

$$
\begin{aligned}
\tilde{V}_1(\tilde{w}_P, w_A, \hat{L}_A) =& \frac{1-\delta}{\delta_E}\left(\frac{-p_B}{p_G-p_B}w_A - \frac{p_G}{p_G-p_B}(\frac{\hat{c}_I}{1-\lambda_{\bar{e}}} - \hat{L}_A)\right) \\
&+ \frac{1-\delta_E}{\delta_E}\left(\mathbb{E}_G[X] - (\tilde{w}_P + w_A)\right) \\
V_1(\tilde{w}_P, w_A, \hat{L}_A) =& \frac{1-\delta}{\delta_E}\left(\frac{-p_B}{p_G-p_B}w_A - \frac{p_G}{p_G-p_B}(\frac{\hat{c}_I}{1-\lambda_{\bar{e}}} - \hat{L}_A)\right) \\
&+ \frac{1-\delta_E}{\delta_E}\left(\mathbb{E}_G[X] - (\tilde{w}_P + w_A)\right) - (\tilde{w}_P + w_A)
\end{aligned}
$$

Finally, the solutions require the following conditions

$$
\begin{aligned}
&\tilde{w}_P + \frac{p_G - \delta p_B}{p_G - p_B}w_A - (1-\delta)\frac{p_G}{p_G-p_B}\hat{L}_A \leq \mathbb{E}_G[X] - \frac{\delta_E \hat{b}}{\lambda_{\bar{e}} - \lambda_{\underline{e}}} - (1-\delta)\frac{p_G}{p_G-p_B}\frac{\hat{c}_I}{1-\lambda_{\bar{e}}} \\
&w_A + \frac{\hat{c}_I}{1-\lambda_{\bar{e}}} - \hat{L}_A \leq (p_G - p_B)(X_H - X_L).
\end{aligned}
$$

The $t = 0$ problem

Now we fold back to $t = 0$, and replace $\tilde{w}_P = w_P + K_1$

$$
\begin{aligned}
\max \quad & \sum_{i\in\Omega} T_i + \lambda_{\bar{e}}\delta_E^2 V_1(w_P, w_A, \hat{L}_A) \\
s.t. & w_P + K_1 + \frac{p_G - \delta p_B}{p_G - p_B}w_A - (1-\delta)\frac{p_G}{p_G-p_B}\hat{L}_A \leq \mathbb{E}_G[X] - \frac{\delta_E \hat{b}}{\lambda_{\bar{e}} - \lambda_{\underline{e}}} - (1-\delta)\frac{p_G}{p_G-p_B}\frac{\hat{c}_I}{1-\lambda_{\bar{e}}} \\
& w_A + \frac{\hat{c}_I}{1-\lambda_{\bar{e}}} - \hat{L}_A \leq (p_G - p_B)(X_H - X_L) \\
& \hat{L}_A - w_A \leq -\frac{\phi}{\lambda_{\bar{e}}} \\
& \delta \hat{L}_A \leq L \\
& \tau_{P0} \leq \lambda_{\bar{e}} w_P + (1-\lambda_{\bar{e}})(L - \delta \hat{L}_A) \\
& \tau_{A0} \leq \delta^2 \left[\lambda_{\bar{e}} w_A + (1-\lambda_{\bar{e}})\hat{L}_A - \hat{c}_I - \hat{c}_M\right] \\
& \tau_{A0} + \tau_{P0} \geq K_0 - \tau_{E0} \\
& \hat{L}_A, \tau_{P0}, \tau_{A0} \geq 0 \\
& w_P, w_A \geq 0.
\end{aligned}
$$

Note that the constraints $w_P, w_A \geq 0$ are added because implicitly we assume investors have no commitment at $t = 1$ and can choose to refuse to offer funding then. Clearly, the two IR constraints (fifth and sixth) must bind. Substituting τ_{P0} and τ_{A0} and with some tedious math, we get the objective function, which is equivalent to

$$
\lambda_{\bar{e}}(1-\delta_E)w_P - (1-\delta)\left[(1-\lambda_{\bar{e}})\delta - \lambda_{\bar{e}}\delta_E\frac{p_G}{p_G-p_B}\right]\hat{L}_A + \left[\lambda_{\bar{e}}\frac{(\delta^2-\delta_E)p_G - \delta(\delta-\delta_E)p_B}{p_G-p_B}\right]w_A.
$$

For now, we will ignore the constraint $\tau_{A0} + \tau_{P0} \geq K_0 - \tau_{E0}$ and later impose that as a parametric condition on $K_0 - \tau_{E0}$. By doing so, we can write the problem as

$$\begin{aligned}
\max \quad & \lambda_{\bar{e}}(1-\delta_E)w_P - (1-\delta)\left[(1-\lambda_{\bar{e}})\delta - \lambda_{\bar{e}}\delta_E \frac{p_G}{p_G - p_B}\right]\hat{L}_A + \left[\lambda_{\bar{e}}\frac{(\delta^2-\delta_E)p_G - \delta(\delta-\delta_E)p_B}{p_G - p_B}\right]w_A \\
s.t. w_P + & \frac{p_G - \delta p_B}{p_G - p_B}w_A - (1-\delta)\frac{p_G}{p_G - p_B}\hat{L}_A \leq \mathbb{E}_G[X] - K_1 - \frac{\delta_E \hat{b}}{\lambda_{\bar{e}} - \lambda_{\underline{e}}} - (1-\delta)\frac{p_G}{p_G - p_B}\frac{\hat{c}_I}{1-\lambda_{\bar{e}}} \\
& w_A + \frac{\hat{c}_I}{1-\lambda_{\bar{e}}} - \hat{L}_A \leq (p_G - p_B)(X_H - X_L) \\
& \hat{L}_A - w_A \leq -\frac{\phi}{\lambda_{\bar{e}}} \\
& \delta \hat{L}_A \leq L \\
& w_P, w_A, \hat{L}_A \geq 0.
\end{aligned}$$

The first constraint must bind; otherwise we can always increase w_P. Therefore,

$$w_P = \mathbb{E}_G[X] - K_1 - \frac{\delta_E \hat{b}}{\lambda_{\bar{e}} - \lambda_{\underline{e}}} - (1-\delta)\frac{p_G}{p_G - p_B}\frac{\hat{c}_I}{1-\lambda_{\bar{e}}} - \frac{p_G - \delta p_B}{p_G - p_B}w_A + (1-\delta)\frac{p_G}{p_G - p_B}\hat{L}_A.$$

We plug this into the problem and drop the constants (also divide the objective function by $1-\delta$)

$$\begin{aligned}
\max \quad & \left[\lambda_{\bar{e}}\frac{p_G}{p_G - p_B} - (1-\lambda_{\bar{e}})\delta\right]\hat{L}_A - \lambda_{\bar{e}}\left[\frac{p_G}{p_G - p_B} + \delta\right]w_A \\
s.t. & \frac{p_G - \delta p_B}{p_G - p_B}w_A - (1-\delta)\frac{p_G}{p_G - p_B}\hat{L}_A \leq \mathbb{E}_G[X] - K_1 - \frac{\delta_E \hat{b}}{\lambda_{\bar{e}} - \lambda_{\underline{e}}} - (1-\delta)\frac{p_G}{p_G - p_B}\frac{\hat{c}_I}{1-\lambda_{\bar{e}}} \\
& w_A + \frac{\hat{c}_I}{1-\lambda_{\bar{e}}} - \hat{L}_A \leq (p_G - p_B)(X_H - X_L) \\
& \hat{L}_A - w_A \leq -\frac{\phi}{\lambda_{\bar{e}}} \\
& \delta \hat{L}_A \leq L \\
& w_A, \hat{L}_A \geq 0.
\end{aligned}$$

Let us define

$$Z_A \equiv \hat{L}_A - w_A \Rightarrow w_A \equiv \hat{L}_A - Z_A.$$

Note that $w_A \geq 0$ is slack given that

$$Z_A \leq -\frac{\phi}{\lambda_{\bar{e}}}$$

and the problem becomes

$$
\begin{aligned}
\max \quad & -\delta\hat{L}_A + \lambda_{\bar{e}}\left[\frac{p_G}{p_G - p_B} + \delta\right] Z_A \\
s.t. \delta\hat{L}_A - & \frac{p_G - \delta p_B}{p_G - p_B} Z_A \le \mathbb{E}_G[X] - K_1 - \frac{\delta_E \hat{b}}{\lambda_{\bar{e}} - \lambda_{\underline{e}}} - (1-\delta)\frac{p_G}{p_G - p_B}\frac{\hat{c}_I}{1-\lambda_{\bar{e}}} \\
& - Z_A \le (p_G - p_B)(X_H - X_L) - \frac{\hat{c}_I}{1-\lambda_{\bar{e}}} \\
& Z_A \le -\frac{\phi}{\lambda_{\bar{e}}} \\
& \delta\hat{L}_A \le L \\
& \hat{L}_A \ge 0.
\end{aligned}
$$

Clearly, it must be that

$$
\begin{aligned}
\hat{L}_A &= 0 \\
Z_A &= -\frac{\phi}{\lambda_{\bar{e}}}.
\end{aligned}
$$

We need the following conditions to hold:

$$
\begin{aligned}
&\frac{p_G - \delta p_B}{p_G - p_B}\frac{\phi}{\lambda_{\bar{e}}} \le \mathbb{E}_G[X] - K_1 - \frac{\delta_E \hat{b}}{\lambda_{\bar{e}} - \lambda_{\underline{e}}} - (1-\delta)\frac{p_G}{p_G - p_B}\frac{\hat{c}_I}{1-\lambda_{\bar{e}}} \\
&\frac{\phi}{\lambda_{\bar{e}}} \le (p_G - p_B)(X_H - X_L) - \frac{\hat{c}_I}{1-\lambda_{\bar{e}}}.
\end{aligned}
$$

The solutions are therefore

$$
\begin{aligned}
\hat{L}_A &= 0 \\
w_A &= \frac{\phi}{\lambda_{\bar{e}}} \\
w_P &= \mathbb{E}_G[X] - K_1 - \frac{\delta_E \hat{b}}{\lambda_{\bar{e}} - \lambda_{\underline{e}}} - (1-\delta)\frac{p_G}{p_G - p_B}\frac{\hat{c}_I}{1-\lambda_{\bar{e}}} - \frac{p_G - \delta p_B}{p_G - p_B}\frac{\phi}{\lambda_{\bar{e}}} \\
\tau_{P0} &= \lambda_{\bar{e}}\left[\mathbb{E}_G[X] - K_1 - \frac{\delta_E \hat{b}}{\lambda_{\bar{e}} - \lambda_{\underline{e}}} - (1-\delta)\frac{p_G}{p_G - p_B}\frac{\hat{c}_I}{1-\lambda_{\bar{e}}} - \frac{p_G - \delta p_B}{p_G - p_B}\frac{\phi}{\lambda_{\bar{e}}}\right] + (1-\lambda_{\bar{e}})L \\
\tau_{A0} &= \delta^2\left[\phi - \hat{c}_I - \hat{c}_M\right].
\end{aligned}
$$

Finally, the condition $\tau_{A0} + \tau_{P0} \ge K_0 - \tau_{E0}$ requires

$$
\begin{aligned}
\lambda_{\bar{e}}\left[\mathbb{E}_G[X] - K_1 - \frac{\delta_E \hat{b}}{\lambda_{\bar{e}} - \lambda_{\underline{e}}} - (1-\delta)\frac{p_G}{p_G - p_B}\frac{\hat{c}_I}{1-\lambda_{\bar{e}}} - \frac{p_G - \delta p_B}{p_G - p_B}\frac{\phi}{\lambda_{\bar{e}}}\right] + (1-\lambda_{\bar{e}})L \\
+\delta^2\left[\phi - \hat{c}_I - \hat{c}_M\right] \ge K_0.
\end{aligned}
$$

Summary: the $t = 0$ problem The solutions require the following conditions

$$\frac{p_G - \delta p_B}{p_G - p_B}\frac{\phi}{\lambda_{\bar{e}}} \leq \mathbb{E}_G[X] - K_1 - \frac{\delta_E \hat{b}}{\lambda_{\bar{e}} - \lambda_{\underline{e}}} - (1-\delta)\frac{p_G}{p_G - p_B}\frac{\hat{c}_I}{1 - \lambda_{\bar{e}}}$$
$$\frac{\phi}{\lambda_{\bar{e}}} \leq (p_G - p_B)(X_H - X_L) - \frac{\hat{c}_I}{1 - \lambda_{\bar{e}}}$$
$$\lambda_{\bar{e}}\left[\mathbb{E}_G[X] - K_1 - \frac{\delta_E \hat{b}}{\lambda_{\bar{e}} - \lambda_{\underline{e}}} - (1-\delta)\frac{p_G}{p_G - p_B}\frac{\hat{c}_I}{1 - \lambda_{\bar{e}}} - \frac{p_G - \delta p_B}{p_G - p_B}\frac{\phi}{\lambda_{\bar{e}}}\right] + (1 - \lambda_{\bar{e}})L$$
$$+ \delta^2\left[\phi - \hat{c}_I - \hat{c}_M\right] \geq K_0.$$

Let us do some more rewriting. Note that

$$p_G - \pi_{\bar{e}} = (1 - \lambda_{\bar{e}})(p_G - p_B)$$
$$\pi_{\bar{e}} - \pi_{\underline{e}} = (\lambda_{\bar{e}} - \lambda_{\underline{e}})(p_G - p_B).$$

Specifically, define

$$\varphi = \max\left\{\left(1 - \frac{\lambda_{\underline{e}}}{\lambda_{\bar{e}}}\right)\delta c_I - \frac{\lambda_{\underline{e}}}{\lambda_{\bar{e}}}c_M, 0\right\} = \frac{\delta^2\phi}{\lambda_{\bar{e}}}(\lambda_{\bar{e}} - \lambda_{\underline{e}}) - c_M$$
$$\Rightarrow \frac{\phi}{\lambda_{\bar{e}}} = \frac{(\varphi + c_M)}{(\lambda_{\bar{e}} - \lambda_{\underline{e}})}/\delta^2$$
$$\mathcal{R}_e = \frac{b}{\Delta\lambda} = \frac{b}{\lambda_{\bar{e}} - \lambda_{\underline{e}}}$$
$$\mathcal{R}_m^{\{1\}} = \frac{c_M + \varphi}{\Delta\pi} = \frac{c_M + \varphi}{\pi_{\bar{e}} - \pi_{\underline{e}}}$$
$$\mathcal{R}_s^{\{1\}} = \frac{c_I}{p_G - \pi_{\bar{e}}}.$$

The three conditions can be written as

$$\mathbb{E}_G[X] - K_1 \geq \mathcal{R}_e/\delta_E + (p_G - \delta p_B)\mathcal{R}_m^{\{1\}}/\delta^2 + (1-\delta)p_G\mathcal{R}_s^{\{1\}}/\delta$$
$$\delta^2(X_H - X_L) \geq \mathcal{R}_m^{\{1\}} + \delta\mathcal{R}_s^{\{1\}}$$
$$K_0 + \delta c_I + c_M \leq \lambda_{\bar{e}}\left[\mathbb{E}_G[X] - K_1 - \mathcal{R}_e/\delta_E - (1-\delta)\left\{\left[p_G + \delta(p_G - p_B)\right]\mathcal{R}_m^{\{1\}}/\delta^2 + p_G\mathcal{R}_s^{\{1\}}/\delta\right\}\right] + (1 - \lambda_{\bar{e}})L.$$

Again, we show the first constraint is implied by the third one. To do that, we introduce the notation

$$\xi = \frac{1}{\delta(1 - \lambda_{\bar{e}})}\frac{1}{\Delta p}$$

and write

$$S_A^{**}(X_H) = S_I^*(X_H) + \frac{1}{\delta\Delta p}\mathcal{R}_D$$
$$= \frac{1}{\delta(1 - \lambda_{\bar{e}})}\frac{1}{\Delta p}\mathcal{R}_I + \frac{1}{\delta\Delta p}\mathcal{R}_D$$
$$= \xi\mathcal{R}_I + \frac{1}{\delta\Delta p}\mathcal{R}_D,$$

where

$$\Delta p \equiv p_G - p_B.$$

We also define

$$\Delta\lambda \equiv \lambda_{\bar{e}} - \lambda_{\underline{e}}.$$

Given this, we can rewrite the first condition as

$$\begin{aligned}
&E_G[X] - (1-\delta)p_G S_A^{**}(X_H) - \mathcal{R}_E - \mathcal{R}_A \geq K_1 \\
\Rightarrow &E_G[X] - \mathcal{R}_E - K_1 \geq (1-\delta)p_G(\xi\mathcal{R}_I + \frac{1}{\delta\Delta p}\mathcal{R}_D) + \mathcal{R}_E + \mathcal{R}_D \\
\Rightarrow &E_G[X] - \mathcal{R}_E - K_1 \geq \left[1 + (1-\delta)p_G\frac{1}{\delta\Delta p}\right]\mathcal{R}_D + \left[1 + (1-\delta)p_G\xi\right]\mathcal{R}_I.
\end{aligned}$$

Let us define

$$\begin{aligned}
Z_1 &\equiv E_G[X] - \mathcal{R}_E - K_1 \\
a &\equiv 1 + (1-\delta)p_G\frac{1}{\delta\Delta p} \\
b &\equiv 1 + (1-\delta)p_G\xi,
\end{aligned}$$

so that the first constraint can be written as

$$Z_1 \geq a\mathcal{R}_D + b\mathcal{R}_I.$$

Turning to the second constraint, note that

$$V_0 = \lambda_{\bar{e}}\left[E_G(X) - K_1\right] + (1-\lambda_{\bar{e}})L - c_M - c_I - K_0,$$

we can write it as

$$\begin{aligned}
&\lambda_{\bar{e}}\left[E_G[X] - K_1\right] + (1-\lambda_{\bar{e}})L - c_M - c_I - K_0 - \lambda_{\bar{e}}(1-\delta)[\mathcal{R}_D + p_G S_A^{**}(X_H)] - \lambda_{\bar{e}}\mathcal{R}_E \geq 0 \\
\Rightarrow &\lambda_{\bar{e}}\left[E_G[X] - K_1 - \mathcal{R}_E\right] + (1-\lambda_{\bar{e}})L - K_0 \geq \delta\Delta\lambda\mathcal{R}_M + \lambda_{\bar{e}}\mathcal{R}_I + \lambda_{\bar{e}}(1-\delta)\mathcal{R}_D + \lambda_{\bar{e}}(1-\delta)p_G\xi\mathcal{R}_I + \lambda_{\bar{e}}(1-\delta)p_G\frac{1}{\delta\Delta p}\mathcal{R}_D \\
\Rightarrow &E_G[X] - K_1 - \mathcal{R}_E - \frac{K_0 - (1-\lambda_{\bar{e}})L}{\lambda_{\bar{e}}} \geq \delta\frac{\Delta\lambda}{\lambda_{\bar{e}}}\mathcal{R}_M + \mathcal{R}_I + (1-\delta)\mathcal{R}_D + (1-\delta)p_G\xi\mathcal{R}_I + (1-\delta)p_G\frac{1}{\delta\Delta p}\mathcal{R}_D.
\end{aligned}$$

Let us define

$$\begin{aligned}
Z_2 &\equiv \frac{K_0 - (1-\lambda_{\bar{e}})L}{\lambda_{\bar{e}}} \\
c &\equiv \delta\frac{\Delta\lambda}{\lambda_{\bar{e}}},
\end{aligned}$$

so that the constraint can be rewritten as

$$Z_1 - Z_2 \geq c\mathcal{R}_M + (a-\delta)\mathcal{R}_D + b\mathcal{R}_I.$$

We now proceed to prove that given this constraint, the first constraint $Z_1 \geq a\mathcal{R}_D + b\mathcal{R}_I$ is redundant. There are two cases:

- When the double-deviation constraint binds so that $\mathcal{R}_D = \frac{\Delta\lambda}{\lambda_{\bar{e}}}\mathcal{R}_M$. Now the two constraints respectively become

$$\begin{aligned} Z_1 &\geq a\frac{\Delta\lambda}{\lambda_{\bar{e}}}\mathcal{R}_M + b\mathcal{R}_I \\ Z_1 - Z_2 &\geq a\frac{\Delta\lambda}{\lambda_{\bar{e}}}\mathcal{R}_M + b\mathcal{R}_I. \end{aligned}$$

  Clearly, the first constraint is redundant given that $Z_2 > 0$.

- When the double-deviation constraint is slack so that $\mathcal{R}_D = \mathcal{R}_M - \mathcal{R}_I$. Now the two constraints respectively become

$$\begin{aligned} Z_1 \geq& a\mathcal{R}_M + (b-a)\mathcal{R}_I \\ Z_1 - Z_2 \geq& (c+a-\delta)\mathcal{R}_M + (b-a+\delta)\mathcal{R}_I. \end{aligned}$$

  Given the linearity, we only need to compare the corners. We will show that both

$$\begin{aligned} \frac{Z_1}{a} &> \frac{Z_1 - Z_2}{c+a-\delta} \\ \frac{Z_1}{b-a} &> \frac{Z_1 - Z_2}{b-a+\delta} \end{aligned}$$

  hold, in which case the first constraint is again redundant.

  - Compare

$$\begin{aligned} \frac{Z_1}{a} &\text{ v.s. } \frac{Z_1 - Z_2}{c+a-\delta} \\ \Rightarrow (c+a-\delta)Z_1 &\text{ v.s. } a(Z_1 - Z_2) \\ \Rightarrow (c-\delta)Z_1 + aZ_2 &\text{ v.s. } 0 \\ \Rightarrow aZ_2 &\text{ v.s. } (\delta - c)Z_1 \\ \Rightarrow \left[1 + (1-\delta)p_G\frac{1}{\delta\Delta p}\right]\frac{K_0 - (1-\lambda_{\bar{e}})L}{\lambda_{\bar{e}}} &\text{ v.s. } \delta(1 - \frac{\Delta\lambda}{\lambda_{\bar{e}}})\left[E_G[X] - \mathcal{R}_E - K_1\right]. \end{aligned}$$

    Clearly, the LHS decreases with δ, whereas the RHS increases in δ. Let us compare the values at $\delta = 1$, which becomes

$$\begin{aligned} \frac{K_0 - (1-\lambda_{\bar{e}})L}{\lambda_{\bar{e}}} &\text{ v.s. } (1 - \frac{\Delta\lambda}{\lambda_{\bar{e}}})\left[E_G[X] - \mathcal{R}_E - K_1\right] \\ \Rightarrow K_0 - (1-\lambda_{\bar{e}})L &\text{ v.s. } \lambda_{\underline{e}}\left[E_G[X] - \mathcal{R}_E - K_1\right]. \end{aligned}$$

    Note that the LHS should be higher because

$$\lambda_{\underline{e}}\left[E_G[X] - \mathcal{R}_E - K_1\right] + (1-\lambda_{\bar{e}})L - K_0 < 0$$

    should hold, if we assume

$$\lambda_{\underline{e}}\left[E_G[X] - K_1\right] + (1-\lambda_{\underline{e}})L - K_0 < 0.$$

– Compare

$$
\begin{aligned}
\frac{Z_1}{b-a} &\text{ v.s. } \frac{Z_1 - Z_2}{b-a+\delta} \\
\Rightarrow (b-a)Z_1 + \delta Z_1 &\text{ v.s. } (b-a)Z_1 - (b-a)Z_2 \\
\Rightarrow (b-a)Z_2 + \delta Z_1 &\text{ v.s. } 0 \\
\Rightarrow (1-\delta)p_G(\xi - \frac{1}{\delta \Delta p})Z_2 + \delta Z_1 &\text{ v.s. } 0 \\
\Rightarrow \frac{(1-\delta)p_G}{\delta \Delta p}\left(\frac{1}{(1-\lambda_{\bar{e}})} - 1\right) Z_2 + \delta Z_1 &\text{ v.s. } 0.
\end{aligned}
$$

Clearly, the LHS is higher.

Therefore, we can conclude that the first constraint is redundant given the second.

C Proofs of Section 5

This section adds the proof of Proposition 9. The first two results follow from the discussion right above the Proposition. Regarding the last result, we know the single-investor arrangement is better if

$$
\frac{p_G - p_B}{p_G} \geq \max\left\{\frac{1}{\delta}\left[\frac{\lambda_{\bar{e}}}{\delta(\lambda_{\bar{e}} - \lambda_{\underline{e}})}\frac{c_M}{c_I} - 1\right], \frac{\lambda_{\bar{e}} - \lambda_{\underline{e}}}{\delta \lambda_{\underline{e}}}\right\}.
$$

From

$$
\frac{p_G - p_B}{p_G} \geq \frac{1}{\delta}\left[\frac{\lambda_{\bar{e}}}{\delta(\lambda_{\bar{e}} - \lambda_{\underline{e}})}\frac{c_M}{c_I} - 1\right],
$$

we get

$$
\delta \geq \delta_{c1} := \frac{-1 + \sqrt{1 + 4\frac{p_G - p_B}{p_G}\left(\frac{\lambda_{\bar{e}}}{\lambda_{\bar{e}} - \lambda_{\underline{e}}}\frac{c_M}{c_I}\right)}}{2\frac{p_G - p_B}{p_G}}.
$$

From

$$
\frac{p_G - p_B}{p_G} \geq \frac{\lambda_{\bar{e}} - \lambda_{\underline{e}}}{\delta \lambda_{\underline{e}}},
$$

we get

$$
\delta \geq \delta_{c2} := \frac{p_G(\lambda_{\bar{e}} - \lambda_{\underline{e}})}{\lambda_{\underline{e}}(p_G - p_B)}.
$$

Therefore, let

$$
\delta_c := \max\{\delta_{c1}, \delta_{c2}\},
$$

we have the last result.